

# **DETERMINING LUMP-SUM VALUES FOR ANNUAL LICENCE FEES FOR UK BROADBAND'S 3.4 GHZ AND 3.6 GHZ SPECTRUM**

**PREPARED FOR HUTCHISON 3G UK  
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# 1. Executive Summary

Power Auctions welcomes this opportunity to comment, on behalf of Hutchison 3G UK (“H3G”), upon appropriate methodology for determining lump-sum values for UK Broadband’s 3.4 GHz and 3.6 GHz spectrum.

The most appropriate figure for the lump-sum value of this spectrum is, on average, in a range of £17.1 – £17.8 million per 5 MHz block, or approximately 55 to 57% of the £31.1 million value proposed by Ofcom. The sources of this discrepancy are as follows:

- (1) In calculating the “marginal opportunity cost to other users” from the PSSR Auction of 2018, Ofcom correctly removed H3G’s losing bids, but incorrectly failed to remove H3G’s winning bids. We consider this to be an oversight by Ofcom, and we consider our adjustment merely to be a minor correction. This correction, by itself, reduces the value calculation from £31.1 million to £29.6 million. We demonstrate that all of H3G’s bids—winning and losing—should be removed in Section 3.
- (2) Ofcom should base the lump-sum value on the *opportunity cost*, not the *marginal opportunity cost*, to other users. We first demonstrate that opportunity cost is the appropriate measure on which to base the ALF, and that setting ALF according to marginal opportunity cost will often lead to a lump-sum value exceeding auction prices, in Section 4. We then consider how to measure opportunity cost in the actual empirical scenario, where the overall supply of 3.4 GHz and 3.6 GHz spectrum is being offered in two successive auctions of approximately equal sizes and where the first (PSSR) auction used essentially a uniform-price auction format. We show that this requires applying a descending sequence of losing bids to determine a descending sequence of lump-sum values. The highest relevant losing bid is £29.6 million and the lowest relevant losing bid is £9.0 million. This reduces the weighted average of lump-sum values from £29.6 million to £19.1 million. The model of two sequential multi-unit auctions is presented and analysed in Section 5, with proofs in an Appendix. Meanwhile, Section 6 shows that the opponents’ highest losing bids from the first auction (corresponding to the quantity of prior holdings) are an appropriate measure of the true opportunity cost when there are two sequential multi-unit auctions.
- (3) Finally, when two sequential auctions are held two years apart, one should expect that bidders would discount their spectrum purchases in the second auction. In the substantive situation of the 3.4 GHz and 3.6 GHz bands, obtaining spectrum in the 2018 auction gives the bidder a first-mover advantage in establishing 5G service and gives the bidder “bragging rights” that it is one of the first adopters. As such, the bidder’s value in the second auction would be reduced by a discount factor,  $\delta < 1$ . Based on Frontier Economics’ analysis, we could conservatively take  $\delta = 0.9$ . Since one-third (40 MHz) of the UKB spectrum is in the 3.4 GHz band and two-thirds (80 MHz) is in the 3.6 GHz band,

this would lead us to apply a factor of  $1 \times 1/3 + 0.9 \times 2/3 = 0.9333$  to the lump-sum value derived solely from the first auction. Alternatively, one may more aggressively take  $\delta = 0.85$ , and this would lead us to apply a factor of  $1 \times 1/3 + 0.85 \times 2/3 = 0.90$  to the lump-sum value derived solely from the first auction. This step reduces the value from £19.1 million to the range of £17.2 – £17.8 million. We argue for this adjustment in Section 7.

We also agree with some of Ofcom’s conclusions in the consultation document. In particular, we demonstrate that a variety of public policy considerations and practical concerns favour basing the lump-sum value solely on the results of the 2018 auction (as opposed to a weighted average of the results of the 2018 and 2020 auctions) in Section 8. We also provide support for Ofcom’s view “that setting ALFs at a conservative estimate of the market value of the spectrum will best meet our statutory duties” in Section 9.

## 2. Introduction: An Outline of our Reasoning

In this section, we give a short introduction to the analysis in this report by providing a brief outline of the overall reasoning.

Ofcom seeks to determine an appropriate lump-sum value to place on UK Broadband's (UKB's) 40 MHz of 3.4 GHz spectrum and UKB's 80 MHz of 3.6 GHz spectrum, for purposes of calculating an Annual License Fee (ALF). We argue that Ofcom's answer should be the best estimate that can be determined of the opportunity cost of UKB's 120 MHz of spectrum.<sup>1</sup>

In principle, a fully correct approach that could have been taken for letting the market determine this opportunity cost was for Ofcom to run a Vickrey-Clarke-Groves (VCG) mechanism (or a CCA) in which the total supply of 390 MHz of 3.4 GHz and 3.6 GHz spectrum was offered in a single auction. H3G would only be allowed to bid on packages that included at least 120 MHz of spectrum (corresponding to the prior holding of 120 MHz in UKB spectrum). Every other potential user would be entitled to submit bids expressing its value for any part of the remaining 270 MHz of spectrum. The winning bids (and the allocation of spectrum) would be determined by solving the main Winner Determination Problem (WDP): calculating the value-maximizing combination of bids subject to the constraint that H3G must win at least 120 MHz. The opportunity cost of each given bidder would then be determined by: (a) solving a second WDP in which the given bidder is "absent" (i.e. all of the given bidder's bids are excluded and, in H3G's case, the constraint on minimum winnings is removed); and (b) calculating the value difference between the WDP with the given bidder present and the given bidder absent (i.e. the difference between the main WDP and the second WDP of part (a)).

To the extent that all of the users express diminishing marginal values, the opportunity cost of a given bidder would equal the sum of the highest losing bids of the given bidder's opponents (summing the number of bids corresponding to the quantity won by the given bidder).<sup>2</sup> Note that in the case where H3G wins exactly 120 MHz, its opportunity costs would directly represent the lump-sum value of the 120 MHz of UKB spectrum, which could be used directly in setting the ALF. At the same time, if H3G would win  $X$  MHz of spectrum where  $X > 120$ , its opportunity costs would include both (1) the opportunity cost of its 120 MHz of UKB spectrum;

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<sup>1</sup> While Ofcom asserts that an appropriate measure of lump-sum value is the *marginal* opportunity cost of UKB's spectrum, we observe at the outset that: (1) while opportunity cost is one of the most fundamental and pervasive concepts in economics, the concept of marginal opportunity cost is hardly ever used; and (2) four of Ofcom's last five auctions (including the upcoming auction for 700 MHz and 3.6 GHz spectrum) have been combinatorial clock auctions (CCAs) that use opportunity cost pricing—since marginal opportunity costs are generally decreasing, basing lump-sum values on marginal opportunity costs would generally produce ALFs that exceed auction prices.

<sup>2</sup> Vickrey, William (1961), "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance*, 16(1): 8-37.

and (2) the opportunity cost of the newly-acquired (X - 120) MHz of spectrum. In this case, the lump-sum value for 120 MHz of UKB spectrum can be found by subtracting H3G's payment for the newly-acquired blocks from its overall opportunity costs.

The motivation for taking this auction approach is two-fold. First, it would determine the true opportunity cost of H3G's spectrum. Second, it would assure that the determined lump-sum value replicates what H3G would pay in a competitive auction—and not anything greater.

However, the actual data that Ofcom has to work with differs from the idealized exercise of the previous paragraph in four respects. Some of these differences are very easy to account for, while others are more difficult. The four differences are as follows:

- (1) First, the actual award process did not explicitly include the extra 120 MHz of spectrum that corresponded to UKB's prior holding. Accounting for this difference is straightforward. Suppose that H3G won  $Y$  MHz of spectrum in the actual award and that its opportunity cost in the award was given by  $P$ . The cumulative opportunity costs for  $(Y+120)$  MHz (denoted by  $P'$ ) can be calculated from the same bidding data by assuming that H3G won  $(Y+120)$  MHz instead of  $Y$  MHz and otherwise utilizing the standard VCG calculation. Then, the part of the opportunity cost that corresponds to the 120 MHz of UKB spectrum is given by  $P' - P$ , i.e. the incremental opportunity cost in going from  $Y$  MHz to  $(Y + 120)$  MHz.<sup>3</sup>
- (2) Second, the actual auction was an SMRA that was structured similarly to a uniform-price auction, rather than either a VCG mechanism or a CCA. Accounting for this difference is more difficult, but it can be done in a theoretical model using some careful game theory analysis. First, the true values of the bidders are specified. Second, equilibrium bid functions are derived for this model. Finally, the relevant data from the equilibrium (e.g. the opportunity cost based on the bids) can be compared with measures based on the true values of the bidders (e.g. the true opportunity cost based on the bidders' values). Proposition 1 of Section 5.1 (which is based on an Arizona State University economics dissertation from 2014) enables us to perform this exercise.
- (3) Third, the actual award process for the 3.4 GHz and 3.6 GHz spectrum is not a single combined auction, but rather a sequence of two multi-unit auctions, the first offering the 3.4 GHz band and the second offering the 3.6 GHz band. Accounting for this difference is *much* more difficult, but we have managed to generalize the ASU dissertation to handle two sequential multi-unit auctions. This novel analysis allows us, within the confines of our model, to compare the payment in the single combined

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<sup>3</sup> To the extent that all of the users express diminishing marginal values, H3G's opportunity cost would then equal a sum of losing bids of H3G's opponents, starting *after the highest  $Y$  MHz of opponents' losing bids (where  $Y$  MHz is the quantity acquired in the award)* and continuing for the next 120 MHz of opponents' losing bids.

auction with the sum of the payments in two sequential auctions. Proposition 2 of Section 5.2 constructs an elegant equilibrium in which the clearing prices in the two sequential auctions are equal. Proposition 3 of Section 5.3 establishes that each is greater than the clearing price in the combined auction, and Section 5.4 provides the intuition for this result. Moreover, as in the previous paragraph, one can solve for the relevant data from the equilibrium (e.g. the opportunity cost based on the losing bids) and compare it with measures based on the true values of the bidders (e.g. the true opportunity cost based on the bidders' values). Section 6 demonstrates that the two calculations exactly coincide in the model of Section 5.

- (4) Fourth, the actual second auction will occur two years after the first auction. Unlike the abstraction described in the previous paragraph—in which the second auction immediately follows the first—one should believe that there is significant discounting between the first and the second auction. If the appropriate discount factor  $\delta$  is conservatively taken to equal 0.9, and using the fact that one-third (40 MHz) of the UKB spectrum is in the 3.4 GHz band (first auction) and two-thirds (80 MHz) is in the 3.6 GHz band (second auction), this consideration would lead us to apply a factor of  $1 \times 1/3 + 0.9 \times 2/3 = 0.9333$  to the lump-sum value derived solely from the first auction. If the appropriate discount factor  $\delta$  is more aggressively taken to equal 0.85, this consideration would lead us to apply a factor of  $1 \times 1/3 + 0.85 \times 2/3 = 0.9$  to the lump-sum value derived solely from the first auction.

In the following sections, we will develop each component of the outlined argument.

### 3. The Marginal Opportunity Cost Should Exclude All of H3G's Bids

In calculating the “marginal opportunity cost to other users” from the PSSR Auction of 2018, Ofcom correctly removed H3G’s losing bids, but incorrectly failed to remove H3G’s winning bids. We consider this adjustment to be a simple correction. The correction, by itself, reduces the lump-sum value calculation from £31.1 million to £29.6 million. In this section, we demonstrate that all of H3G’s bids—both winning and losing—must be removed in order to assess correctly the opportunity costs to other users.

The concept of “opportunity cost” has a long tradition in economics. In its most rudimentary form, the opportunity cost of allocating scarce resources to a given economic agent is the incremental value that other economic agents would have received if the scarce resources were allocated optimally to them instead of to the given agent. By definition, this measure of opportunity cost never depends on the agent’s own values for the scarce resources in question. In the auction context, the celebrated Vickrey-Clarke-Groves (VCG) mechanism adopts opportunity cost as its principle for determining payments by winning bidders. The property that bidders’ payments never depend on their own bids, both winning and losing, is the sole reason why bidders in the VCG mechanism are incentivized to reveal their values truthfully (i.e. truthfully reveal their private information). Since the mechanism implements the value-optimizing allocation of resources based on these truthful reports, this implies that the mechanism achieves efficiency (i.e. it puts resources into the hands of the agents who value them the most).

On many occasions, Ofcom has utilized the combinatorial clock auction (CCA) format, whose pricing rule is based on a generalized form of the VCG mechanism. It accounts for collective opportunity costs (through a core adjustment) as well as for individual opportunity costs. And in performing any individual opportunity cost calculation for a given bidder in the CCA, Ofcom has always removed all bids that were made by this bidder.

However, in two instances, Ofcom has departed from the principles of “opportunity cost”. First, Ofcom deviated from opportunity cost when applying the Additional Spectrum Methodology (ASM) to auction data from the UK 4G auction of 2013 in order to derive lump-sum values for the ALF for 900 MHz and 1800 MHz spectrum.<sup>4</sup> In the ASM, Ofcom correctly eliminated all bids by the bidder for whom the market value is being derived. However, Ofcom simultaneously reduced the total supply of blocks available to other users by the winning quantity of the bidder, which is equivalent to putting the winning bid back into the calculation. Second, Ofcom

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<sup>4</sup> Second consultation on assessment of future mobile competition and proposals for the award of 800 MHz and 2.6 GHz spectrum and related issues, January 2012, paragraphs A13.64 – A13.75  
[https://www.ofcom.org.uk/\\_\\_data/assets/pdf\\_file/0024/63582/2nd\\_condoc\\_annexes\\_8-15.pdf](https://www.ofcom.org.uk/__data/assets/pdf_file/0024/63582/2nd_condoc_annexes_8-15.pdf)

implicitly did the same thing when deriving the “marginal opportunity cost to other users”, proposed for use in calculating the lump-sum values of UKB spectrum from the PSSR auction data. As a result, the opportunity cost calculation is incorrect in both instances, since the given bidder’s own winning bids directly affect the measure of lump-sum value.

In the current context, the calculation of the marginal opportunity cost to other users should exclude the impact of H3G’s winning bid for 20 MHz in the 3.4 GHz band. The correct calculation can be carried out using only losing bids. First, observe that Telefonica’s demand drop at £31.1 million does not correspond to the correct marginal opportunity cost since Telefonica reduced its demand from 55 MHz to 40 MHz for a total drop of 15 MHz (less than 20 MHz won by H3G). As a result, the next-highest losing bid must be considered. The next-highest losing bid was placed by Vodafone, which reduced its demand from 60 MHz to 50 MHz at £29.6m. Now the cumulative quantity of losing bids is 25 MHz (more than the 20 MHz won by H3G) and the price of £29.6m determines the correct marginal opportunity cost to other users.

To phrase it differently, Telefonica’s £31.1 million bid determines the marginal opportunity cost of the spectrum acquired in the PSSR auction. However, it does *not* demarcate the marginal opportunity cost of H3G’s prior holdings (the UKB spectrum), since without H3G’s winning bid in the auction, Telefonica’s £31.1 million bid could have been fulfilled without any of the UKB spectrum.

As a general policy matter, Ofcom should avoid basing ALF on auction data that includes bids of the bidder for whom the ALF is being determined, for two reasons:

- to eliminate incentives for distortions (Section 3.1); and
- to avoid extracting revenues in excess of a competitive tender (Section 3.2).

Finally, Ofcom defends its choice of adopting marginal opportunity costs to other users (£31.1 million) using an “efficient user of the spectrum” argument. However, Ofcom’s reasoning makes an implicit and unjustified assumption about the underlying spectrum holdings. In Section 3.3, we demonstrate that the same argument can be adopted to motivate finding that the marginal opportunity cost to other users equals £29.6 million.

### 3.1 Including winning bids creates distortions and inefficiencies

The first issue can be illustrated with a simple example. An auctioneer wishes to sell three homogeneous spectrum blocks to three bidders using a standard multi-unit Vickrey auction (VCG mechanism). In addition, the auctioneer seeks to set ALF for one block of spectrum (whose value is similar to the spectrum being auctioned) that is owned by Bidder 1. Suppose that bidders submit truthful bids as shown in Table 1. In the Vickrey auction, Bidder 1 wins one block and pays 15, while Bidders 2 and 3 also win one block each but pay 20.



**Table 1: Example 1**

Bidders	Bidder 1	Bidder 2	Bidder 3
Truthful bids	$b_1(1) = 20$ $b_1(2) = 40$	$b_2(1) = 30$ $b_2(2) = 45$	$b_3(1) = 30$ $b_3(2) = 35$
Outcome of the Vickrey Auction	$(q = 1, p = 15)$	$(q = 1, p = 20)$	$(q = 1, p = 20)$

Applying Ofcom’s proposed methodology to this example, the market clearing price is 20 and the marginal opportunity cost to other users of Bidder 1’s holding is 15 (set by the losing bid of Bidder 2). As a result, Bidder 1 would additionally pay 15 in ALF for its holding, for a total payment of 30.

Knowing that its winning bid will be used to derive the ALF for its holding, Bidder 1 will benefit by bidding less competitively. For example, Bidder 1 can refrain from participating in this auction. In this case, Bidder 1 would win nothing and Bidder 2 and 3 would win two blocks and one block, respectively. In this scenario, the marginal opportunity cost to other users of Bidder 1’s holding is now 5 (set by the losing bid of Bidder 3). Hence, Bidder 1’s ALF based on the new bidding data would just equal 5.

Observe that the non-participation strategy is profitable for Bidder 1, as the bidder thereby reduces its total payment by 25 (from 30 to 5) by sacrificing a block which it valued at only 20. Furthermore, the outcome of this auction is inefficient since Bidder 2 wins 2 blocks instead of 1.

The same critique applies if the auction format is a uniform-price auction, rather than a Vickrey auction.

The above argument might appear irrelevant since the 3.4 GHz award has already taken place and bidders cannot alter their bidding retroactively. At the same time, Ofcom’s final decision on whether to use winning bids for ALF calculations can set a precedent that will distort bidding in future auctions.

### 3.2 Including winning bids extracts more revenue than a competitive auction

In the Strategic Review of Spectrum Pricing consultation, Ofcom states that:

*“Administrative Incentive Pricing (AIP) acts as a proxy for market prices for scarce spectrum that has been assigned **administratively rather than auctioned**.”<sup>5</sup>*

<sup>5</sup> SRSP consultation, paragraph 1.12. [https://www.ofcom.org.uk/data/assets/pdf\\_file/0021/36804/srsp\\_condoc.pdf](https://www.ofcom.org.uk/data/assets/pdf_file/0021/36804/srsp_condoc.pdf)

AIP stands for “Administrative Incentive Pricing” and is equivalent to ALF. Furthermore, Ofcom goes to great lengths in the consultation to eliminate any second-order value differences that could arise from paying ALF in comparison with buying spectrum and making a lump-sum payment in a competitive auction. For example, paragraphs 3.63 and 3.64 of the consultation derive a tax adjustment factor (TAF) to eliminate a small tax benefit provided by ALF (and thereby bring the ALF and a lump-sum payment at auction into parity).

In light of such evidence, the proposed methodology for setting ALF appears to be excessive. It generally extracts more revenue than Ofcom would have collected if the corresponding spectrum had been subject to a competitive auction.

We can illustrate the issue with the following example (which differs from the example of the previous section only in Bidder 3’s bids). The auctioneer attempts to sell three homogeneous blocks of spectrum to three bidders using a standard Vickrey auction. In addition, the auctioneer seeks to set ALF for one block of spectrum (whose value is similar to the spectrum being auctioned) owned by Bidder 1. Suppose that the bidders submit truthful bids as shown in Table 2. In the Vickrey auction, Bidder 1 wins one block and pays 15, while Bidders 2 and 3 also win one block each but pay 20.

**Table 2: Example 2**

Bidders	Bidder 1	Bidder 2	Bidder 3
Truthful bids	$b_1(1) = 20$ $b_1(2) = 40$	$b_2(1) = 30$ $b_2(2) = 45$	$b_3(1) = 30$ $b_3(2) = 40$
Outcome of the Vickrey Auction	$(q = 1, p = 15)$	$(q = 1, p = 20)$	$(q = 1, p = 20)$

Applying Ofcom’s proposed methodology to this example, the market clearing price is 20 and the marginal opportunity cost to other users of Bidder 1’s holding is 15 (set by the losing bid of Bidder 2). As a result, Bidder 1 would additionally pay 15 in ALF for its holding, for a total payment of 30.

Now let’s calculate the auction revenues in the counterfactual situation where the fourth block is included in the auction. Suppose that the auctioneer wishes to sell four homogeneous blocks to three bidders. We assume that Bidder 1 has a very high value for the first block—in reality, this block is Bidder 1’s prior holding. Otherwise, all bidders have the same marginal values as they did in Table 2:

**Table 3: Example 3**

Bidders	Bidder 1	Bidder 2	Bidder 3
Truthful bids	$b_1(1) = 100$	$b_2(1) = 30$	$b_3(1) = 30$

	$b_1(1) = 120$ $b_1(2) = 140$	$b_2(2) = 45$	$b_3(2) = 40$
Outcome of the Vickrey Auction	$(q = 2, p = 25)$	$(q = 1, p = 20)$	$(q = 1, p = 20)$

We conclude that, if all four blocks were sold in an auction, Bidder 1 would pay only 25. By contrast, we have already seen that, if Bidder 1 buys one block at auction but pays ALF on the other, Ofcom’s proposed methodology makes Bidder 1’s total payment 30. It is unjust for Ofcom to set an ALF lump-sum value that exceeds the price that would be obtained at auction.

### 3.3 The “efficient user” argument

Ofcom describes one motivation for its proposed methodology for the ALF as establishing the “efficient user of the spectrum”:

*“In the case of the marginal opportunity cost to other users, the price signal would seek to ensure that the efficient user has the spectrum, ...”<sup>6</sup>*

In other words, setting ALF for UKB spectrum based on a bid of Telefonica at £31.1 million per 5 MHz block would ensure that H3G is the “efficient user” of the UKB spectrum. In case H3G marginal value for the UKB spectrum is lower than £31.1 million per 5 MHz, it would be optimal for H3G to divest some of this spectrum to another user, thus ensuring a transfer of the spectrum to a bidder with the highest value. According to Ofcom, the existence of such a bidder is guaranteed, since Telefonica was willing to buy an extra 15 MHz of 3.4 GHz spectrum in the auction at £31.1 million per 5 MHz.

On the surface, this argument may seem compelling. However, the argument must be interpreted with great care as it is highly sensitive to the structure of spectrum holdings at the time of the potential divestment. While making the argument, Ofcom makes an implicit assumption that the spectrum holdings will stay exactly the same as they were immediately after the PSSR auction of 2018. However, if the holdings were to change due to other spectrum trades or new spectrum releases (such as upcoming 700 MHz and 3.6-3.8 GHz auction), the “efficient user” level of ALF for UKB spectrum would change.

To further illustrate the sensitivity of the argument, consider a hypothetical scenario in which H3G needs to divest some of its spectrum holdings in the 3.4 GHz band. Ofcom’s proposed approach translates into an implicit assumption that the UKB spectrum will be the first spectrum that H3G divests. But that seems counterintuitive, since H3G also acquired 20 MHz of

<sup>6</sup> Annual Licence Fees for UK Broadband’s 3.4 GHz and 3.6 GHz spectrum, ¶13.31

3.4 GHz spectrum in the PSSR auction. There are many reasons why H3G would prefer to divest its “new” auction-acquired spectrum before divesting the UKB spectrum in the same band, including:

- In case H3G needs to raise capital, divesting 20 MHz of spectrum acquired in the auction would generate more cash than divesting 20 MHz of UKB spectrum, since it is free of ALF;
- The 20 MHz of spectrum acquired in the auction is not yet being used, whereas the UKB spectrum is being used by existing customers; and
- There is no equipment set up to use the 20 MHz of spectrum acquired in the auction and no equipment tied to those frequencies, whereas there is equipment set up to use the UKB spectrum.

By way of contrast, we do not see any reason why H3G would prefer to divest the UKB spectrum before the auction-acquired spectrum. On balance, it seems significantly more likely to us that H3G would divest the auction-acquired spectrum before the UKB spectrum.

Finally, observe that if H3G would divest the spectrum that it acquired in the PSSR auction before it divests any of the UKB spectrum, then the same “efficient user” argument made by Ofcom would now affirm our £29.6 million per 5 MHz block estimate of marginal opportunity cost. That is, Telefonica would be prepared to pay up to £31.1 million for three 5 MHz blocks of H3G’s auction-acquired 3.4 GHz spectrum, while Vodafone would be prepared to pay up to £29.6 million for the next 10 MHz (H3G’s fourth auction-acquired 5 MHz block and one of H3G’s 5 MHz blocks of UKB spectrum). But then, £29.6 million per 5 MHz block is the right lump-sum value to guarantee the “efficient user” for the marginal 5 MHz block of UKB spectrum. Therefore, the same argument that Ofcom puts forward to support its choice of £31.1 million can be made to support a figure of £29.6 million. The latter estimate has the advantage of being completely independent of H3G’s bids and, therefore, theoretically more correct.

## 4. The ALF Should be Based on Opportunity Cost, not on Marginal Opportunity Cost

### 4.1 ALF for 3.4 and 3.6 GHz spectrum should follow the principles of the SRSP, and need not follow the precedent of ALF for 900 and 1800 MHz spectrum

Ofcom has the power to impose fees for the use of spectrum, including fees greater than those necessary to recover Ofcom’s administrative costs, having regard in particular to Ofcom’s general duty to further the interests of citizens and consumers by securing the optimal use of the spectrum and its specific duties when carrying out its spectrum functions.<sup>7</sup> In order to meet these duties, Ofcom set out its general policy position for setting spectrum fees in its Strategic Review of Spectrum Pricing (the “SRSP”) in 2010, which Ofcom said would be used in the future as a guide to setting fees above administrative cost (which Ofcom referred to in the SRSP as administered incentive pricing or “AIP”).<sup>8</sup> In the SRSP, Ofcom stated:

*The purpose of AIP [a term that Ofcom uses synonymously with ALF]<sup>9</sup> is to provide users with a sustained long-term signal of the value of the spectrum **as indicated by its opportunity cost in the next highest use** and, as a result, to give them incentives to use it in a way that maximises benefits for society over time (emphasis added).<sup>10</sup>*

In December 2010, the Government directed Ofcom to award 4G licences in the 800 MHz and 2.6 GHz bands, and thereafter revise fees for mobile spectrum in the 900 MHz and 1800 MHz bands to reflect the full market value of those frequencies (the “Direction”).<sup>11</sup> Thus, when in September 2015, Ofcom set new ALFs for the 900 and 1800 MHz bands and, following litigation, when in December 2018, Ofcom again set ALFs for the 900 and 1800 MHz bands, Ofcom based the lump-sum values both on the policy it set in the SSRP and on what the Government instructed it to do in the Direction. However, in the current proceeding, wherein Ofcom is setting an ALF for UKB’s 3.4 GHz and 3.6 GHz spectrum, there is no Government instruction analogous to the Direction. Consequently, Ofcom should be setting the ALF for UKB’s 3.4 GHz and 3.6 GHz spectrum based solely on the principles of the SRSP, and Ofcom need not follow the precedent from the 900 and 1800 MHz bands.

<sup>7</sup> Ofcom (2018), *Annual Licence Fees for 900 MHz and 1800 MHz frequency bands*, ¶3.1.

<sup>8</sup> Ofcom (2018), *Annual Licence Fees for 900 MHz and 1800 MHz frequency bands*, ¶3.2.

<sup>9</sup> Ofcom (2018), *Annual Licence Fees for 900 MHz and 1800 MHz frequency bands*, ¶3.2.

<sup>10</sup> Ofcom (2010), *Strategic Review of Spectrum Pricing: The revised Framework for Spectrum Pricing*, ¶3.33.

<sup>11</sup> Ofcom (2018), *Annual Licence Fees for 900 MHz and 1800 MHz frequency bands*, ¶3.2.

## 4.2 The SRSP justifies why ALF should be based on opportunity cost

As noted above, Ofcom announced in ¶3.33 of the SRSP that the ALF should be based on opportunity cost. The rationale for the ALF is described in ¶3.34:

*The rationale for AIP may be simply stated. If the price charged for any limited resource, whether it is energy, raw materials, land or spectrum, does not reflect its opportunity cost, there will be less incentive to use it efficiently, it will not be available for alternative uses or other users that could produce additional value and society will be worse off. For example, faced with a choice between investing in more advanced equipment and using more spectrum businesses will naturally tend to choose the option with lower costs. If the cost of spectrum reflects its true opportunity cost, and the cost of equipment also reflects its true value (as would be expected in a well-functioning market for equipment) then business will make the trade-off between investment in spectrum and equipment in a way that maximises benefits generated from their use.*

The societal loss associated with pricing spectrum below opportunity cost is described in ¶3.35:

*If spectrum appears cheaper than its true opportunity cost, businesses will rationally use more spectrum, and invest less in equipment than the efficient balance. The result of this would be that fewer users overall will be able to access spectrum to generate benefits for society.*

Meanwhile, the societal loss associated with pricing spectrum above opportunity cost is described in ¶3.36:

*On the other hand, if spectrum appears more expensive than its true opportunity cost, businesses will be incentivised to over-economise in spectrum, leading either to users:*

- *using more complex (and therefore expensive) equipment, or alternatives to spectrum that are more costly than spectrum would be if charged at its “true” opportunity cost and which might translate into higher costs for consumers, or*
- *reducing, or ceasing altogether services they provided, resulting in reduced benefits to consumers and citizens and unused spectrum.*

All of these rationales require basing the ALF on opportunity cost, not on marginal opportunity cost.

## 4.3 Generally in economics, efficiency comes from the use of opportunity cost

A word search of the SRSP identifies 138 occurrences of the term “opportunity cost”, and **zero** occurrences of the term “marginal opportunity cost”. It is a bit difficult for us to understand how, in the current consultation document for the 3.4 and 3.6 GHz ALF, the balance has shifted so that there are now 43 occurrences of the term “marginal opportunity cost” and only 39

occurrences of “opportunity cost” not preceded by “marginal”. What makes the change in approach particularly mystifying is that the SRSP correctly attributes many desirable properties to using “opportunity cost”, but these properties no longer hold when one instead considers “marginal opportunity cost”.

Generally in economics, efficiency is obtained by charging for a resource according to its opportunity cost, *not* according to its marginal opportunity cost. As we have already seen in the introduction to Section 3, the celebrated Vickrey-Clarke-Groves (VCG) mechanism adopts opportunity cost—not marginal opportunity cost—as its principle for determining payments by winning bidders. Belabouring this point to Ofcom seems akin to carrying coals to Newcastle. After all, Ofcom has been the pioneering spectrum regulator, worldwide, in the use of the combinatorial clock auction, which utilises pricing based on the concept of opportunity cost. The consultation document on the award of the 700 MHz and 3.6 – 3.8 GHz spectrum bands, released on the same day as this consultation on the ALF for UKB’s spectrum, provides the following justification of the proposed auction design:

*The auction should be designed to best achieve our policy in this award of promoting optimal use of this spectrum in the interests of consumers and citizens by including coverage obligations. We are proposing to award the spectrum through a form of Combinatorial Clock Auction (CCA).<sup>12</sup>*

Further, the detailed description of the pricing rule states:

*Base prices for any bidders who are not assigned any lots in the principal stage are zero. Base prices to be paid by winning bidders who are assigned a non-zero package are based on **the concept of opportunity cost** (emphasis added).<sup>13</sup>*

The above text—as well as the detailed algorithm of the CCA—could instead say “marginal opportunity cost”, but it does not. It says “opportunity cost”. The reason, as Ofcom already knows, is that the VCG mechanism, which sets prices based on opportunity cost, maximizes revenues among all mechanisms that attain *ex post* efficiency in all states of the world.<sup>14</sup> If instead, base prices were based on marginal opportunity costs, the CCA would not attain efficient outcomes.

The source of confusion is that marginal opportunity cost is entirely appropriate, for example, in pricing a natural resource that is consumed only by atomistic consumers who are sufficiently small that each one’s consumption is an infinitesimal part of the total. The following quote reflects the usual consensus of the economics profession:

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<sup>12</sup> Ofcom (2018), “Award of the 700 MHz and 3.6-3.8 GHz spectrum bands,” paragraph 2.60.

<sup>13</sup> Ofcom (2018), “Award of the 700 MHz and 3.6-3.8 GHz spectrum bands,” paragraph A16.126.

<sup>14</sup> Krishna, V. and M. Perry (1998), “Efficient Mechanism Design,” available at SSRN: <https://ssrn.com/abstract=64934>.

*Although we have argued that MOC [marginal opportunity cost] is generally the right measure of scarcity, there are situations where the appropriate concept is not a marginal one. In particular, this will be so when the policies being considered involve large changes to the stocks of natural resources. Then the value of a small change in the resource, suitably scaled up, will not be an accurate measure and what is required is a comparison between the value of the total stock before and after the change.<sup>15</sup>*

In short, the concept breaks down entirely in a market where there are only four firms and in consideration of a fraction of 4/13 (i.e. 120 MHz/390MHz) of the total stock of a resource.

#### **4.4 The “efficient user” argument is misplaced and, in any case, optimal use does not require setting the ALF equal to marginal opportunity cost**

The most compelling rationale for awarding spectrum by auction is efficiency: to put the spectrum in the hands of the user who values it the most. Nonetheless, there is typically no ALF imposed on spectrum that is awarded by auction, for the entire initial licence term. For example, licences in the 3.6 GHz band awarded in next year’s auction will be issued for an initial period of 20 years starting from the date of issue, and will not be subject to licence fees (other than the auction price) until after the initial 20-year period.

Observe that in awarding spectrum (as in setting an ALF), Ofcom is bound to follow the objectives set out in Article 8 of the Framework Directive, including “the promotion of competition in the provision of electronic communications networks and services by, amongst other things, ensuring there is no distortion or restriction of competition in the electronic communications sector and encouraging efficient use and effective management of radio frequencies” (Art. 8(2)). Apparently, Ofcom believes that the 3.6 GHz spectrum to be auctioned next year will not suffer from an “efficient user” problem despite having a zero ALF. Inconsistently, Ofcom believes that it is required to set an ALF based on opportunity cost on almost identical UKB 3.6 GHz spectrum “even where spectrum trading is possible, in order to meet our statutory duty of securing optimal use of the radio spectrum”.<sup>16</sup>

This is not to say that Ofcom is wrong in refraining from charging an ALF during the initial term of auctioned spectrum. Rather, Ofcom is severely understating the extent to which a user will account for opportunity cost in making use of its tradeable spectrum. Obviously, the UKB spectrum is eminently tradeable, since H3G just recently acquired it. And, with a zero ALF—or with an ALF set well below the marginal opportunity cost—there is still a strong incentive for tradeable spectrum to move into the hands of the optimal user.

<sup>15</sup> Pearce, D. & Markandya (1987), “Marginal opportunity cost as a planning concept in natural resource management,” *Annals of Regional Science*, 21(3), pp 18–32, at p. 24.

<sup>16</sup> Ofcom (2018), *Annual Licence Fees for UK Broadband’s 3.4 GHz and 3.6 GHz spectrum*, ¶3.7.



## 4.5 Setting ALF according to marginal opportunity cost will often lead to a lump-sum value that exceeds auction prices

In Section 3.2 above, we have already demonstrated that setting ALF based on the marginal opportunity cost (calculated without removing the winning bid of a bidder in question) will generally result in a higher lump-sum value than the corresponding auction revenue. In this section, we further show that even correctly computed marginal opportunity cost (calculated after removal of the winning bid of a bidder in question) will generally overstate the lump-sum value based on opportunity cost.

Let us consider a simple example that is patterned after the empirical 3.4 GHz and 3.6 GHz bands. The auctioneer is selling eight homogeneous blocks of spectrum to four bidders using a standard VCG mechanism or CCA. In addition, the auctioneer seeks to set ALF for four blocks of spectrum (whose value is similar to the spectrum being auctioned) owned by Bidder 4. Suppose that the bidders submit truthful bids as shown in Table 4. In the Vickrey auction, Bidder 1 wins three blocks and pays 100, Bidder 2 wins two blocks and pays 66, Bidder 3 wins two blocks and pays 68, and Bidder 4 wins one block and pays 34 (in Table 4, the winning bids are marked in red).

**Table 4: Example 4**

Bidders	Bidder 1	Bidder 2	Bidder 3	Bidder 4
Truthful bids	$b_1(1) = 40$ $b_1(2) = 78 (+38)$ $b_1(3) = 114 (+36)$ $b_1(4) = 143 (+29)$ $b_1(5) = 163 (+20)$	$b_2(1) = 40$ $b_2(2) = 76 (+36)$ $b_2(3) = 110 (+34)$ $b_2(4) = 141 (+31)$ $b_2(5) = 169 (+28)$	$b_3(1) = 40$ $b_3(2) = 76 (+36)$ $b_3(3) = 108 (+32)$ $b_3(4) = 136 (+28)$ $b_3(5) = 160 (+24)$	$b_4(1) = 40$ $b_4(2) = 74 (+34)$ $b_4(3) = 106 (+32)$ $b_4(4) = 136 (+30)$ $b_4(5) = 164 (+28)$
Outcome of the Vickrey Auction	$(q = 3, p = 100)$	$(q = 2, p = 66)$	$(q = 2, p = 68)$	$(q = 1, p = 34)$

Now let's calculate the auction revenues in the counterfactual situation where the four blocks are included in the auction. As a result, we will suppose that the auctioneer wishes to sell 12 homogeneous blocks to four bidders. We assume that Bidder 4's values for the first four blocks—corresponding to Bidder 4's prior holdings—are 44, 43, 42 and 41, respectively, to ensure that Bidder 4 wins them in the auction. Bids in the counterfactual auction are provided in Table 5.

**Table 5: Example 5**

Bidders	Bidder 1	Bidder 2	Bidder 3	Bidder 4
Truthful Bids for prior holding	n/a	n/a	n/a	$b_4(1) = 44$ $b_4(2) = 87 (+43)$ $b_4(3) = 129 (+42)$ $b_4(4) = 170 (+41)$
Truthful bids	$b_1(1) = 40$ $b_1(2) = 78 (+38)$ $b_1(3) = 114 (+36)$ $b_1(4) = 143 (+29)$ $b_1(5) = 163 (+20)$	$b_2(1) = 40$ $b_2(2) = 76 (+36)$ $b_2(3) = 110 (+34)$ $b_2(4) = 141 (+31)$ $b_2(5) = 169 (+28)$	$b_3(1) = 40$ $b_3(2) = 76 (+36)$ $b_3(3) = 108 (+32)$ $b_3(4) = 136 (+28)$ $b_3(5) = 160 (+24)$	$b_4(5) = 210 (+40)$ $b_4(6) = 244 (+34)$ $b_4(7) = 276 (+32)$ $b_4(8) = 306 (+30)$ $b_4(9) = 334 (+28)$
Outcome of the Vickrey Auction	$(q = 3, p = 100)$	$(q = 2, p = 66)$	$(q = 2, p = 68)$	$(q = 5, p = 154)$

Bidder 4 wins five blocks in this counterfactual auction. The lump-sum value for its prior holding (four blocks) is then derived by taking its payment of from the counterfactual auction and reducing it by its payment in the auction that did not include the four extra blocks:  $154 - 34 = 120$ .

Finally, in Table 6, we compare various methodologies for estimating the lump-sum value for the example above.

**Table 6: Comparison**

Methodology	Lump-Sum Value for 4 blocks	Overcharging relative to Corresponding Auction Revenue
Marginal Opportunity Cost (including the winning bid)	$34 \times 4 \text{ blocks} = 136$	16
Marginal Opportunity Cost (removing the winning bid)	$32 \times 4 \text{ blocks} = 128$	8
True Opportunity Cost	$32 \times 1 \text{ block} + 31 \times 1 \text{ block} + 29 \times 1 \text{ block} + 28 \times 1 \text{ block} = 120$	0

Suppose that due to the evolution of the UK wireless market, bidder 4 experienced a decline of 25% in its values for the spectrum blocks. Further suppose that the ALF had been based on the marginal opportunity cost (including the winning bid) of 34. Then observe that bidder 4's marginal values would now all be less than the ALF, and so bidder 4 would seek to divest all five blocks. Moreover, only bidder 2 would have a marginal value that was at least as great as the ALF—and only on the first block—and so only one block would be purchased from bidder 4.

Bidder 4's other four blocks would be relinquished to Ofcom and would go unused for an extended period of time.

A similar (although less extreme) scenario would occur if the ALF had been based on the marginal opportunity cost (removing the winning bid) of 32. Then all but two of bidder 4's marginal values would now all be less than the ALF, and so bidder 4 would seek to divest three blocks. Moreover, only bidders 2 and 3 would have marginal values that were at least as great as the ALF—and only on their first block—and so only two blocks would be purchased from bidder 4. Bidder 4's remaining block would be relinquished to Ofcom and would go unused for an extended period of time.

The inefficiency described in the previous two paragraphs is exactly the kind of “societal loss associated with pricing spectrum above opportunity cost” envisioned by Ofcom in paragraph 3.36 of the SRSF.

## 5. A Model of Sequential, Multi-Unit Auctions

In this section, we describe a novel, theoretical model of a multi-unit auction environment, in which half of the supply is offered in a first auction and the other half of the supply is offered in a second auction. Following the results of a 2014 Arizona State University Ph.D. dissertation, we derive an equilibrium of the second auction in Proposition 1. Then, using backward induction, we establish an equilibrium of the sequence of auctions in Proposition 2. We show that the clearing price in each stage of the sequential auction is greater than the clearing price of the combined auction in Proposition 3. The proofs are relegated to the Appendix.

### 5.1 A Model of a Single Uniform-Price Auction

An auctioneer seeks to sell a unit supply ( $S = 1$ ) of a homogeneous divisible good to a set of  $n$  bidders, denoted  $N = \{1, \dots, n\}$ . All bidders in  $N$  are assumed to be symmetric, each with a value function given by  $v(q) = q - \frac{1}{2}\beta q^2$ , where  $q \in [0,1]$  is the quantity of the good. Parameter  $\beta$  is assumed to be positive. Each bidder is assumed to have a quasilinear utility (i.e. each bidder's payoff from winning a quantity of  $q \in [0,1]$  and making a payment of  $r$  is given by  $v(q) - r$ ) and to have complete information about its opponents' utility functions.

As our first scenario, let us consider the auctioneer who allocates the entire unit of supply ( $S = 1$ ) in a single auction, using a standard uniform-price auction format. That is: (1) each bidder  $i$  submits a downward sloping demand function  $q_i(p)$  to the auctioneer; (2) the

auctioneer finds the clearing price  $\hat{p}$  by equating the aggregate demand and supply

$\sum_{i \in N} q_i(\hat{p}) = 1$ ; and (3) each bidder wins  $\hat{q}_i = q_i(\hat{p})$  quantity of the good and pays  $\hat{p} \cdot \hat{q}_i$  to the auctioneer.

**Proposition 1.** Suppose that there are at least three bidders, i.e.  $n \geq 3$ . A profile of linear bid functions  $\{q_1^*(p), \dots, q_n^*(p)\}$ , where

$$q_i^*(p) = \frac{1}{\beta} \left( \frac{n-2}{n-1} \right) (1-p), \quad \forall i \in N, \quad (5.1)$$

forms a Nash equilibrium of the uniform-price auction. In this equilibrium, the clearing price is given by:

$$p^* = 1 - \frac{\beta}{n} \left( \frac{n-1}{n-2} \right), \quad (5.1)$$

and the equilibrium exists whenever:

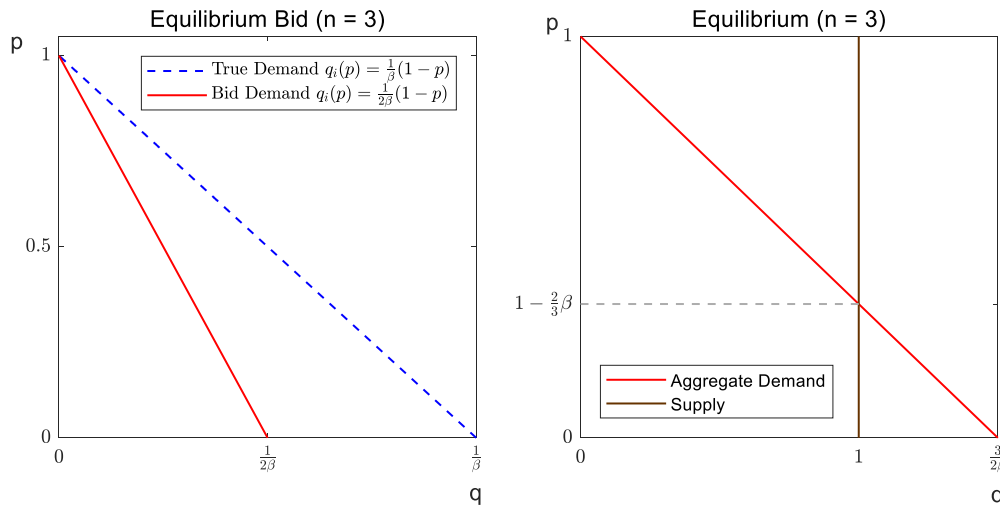
$$\beta \leq \frac{n(n-2)}{n-1}. \quad (5.1)$$

**Proof:** See the Appendix.

**Remark:** We have been treating the uniform-price auction as a game of complete information. If, instead, it is modeled as a game of incomplete information where the slope of the bidders' demand curves are commonly known but the intercepts are private information, the equilibrium of Proposition 1 becomes especially desirable, as it is the unique linear *ex post* equilibrium—see Wang, Mian (2014), “Share Auctions with Linear Demands,” Arizona State University doctoral dissertation, Proposition 5.

We illustrate the equilibrium bidding functions and the corresponding equilibrium clearing price in Figure 1.

**Figure 1: Illustration of the equilibrium from Proposition 1**



## 5.2 A Model of Sequential Uniform-Price Auctions

As a second scenario, we consider an auctioneer who divides the unit supply in half and allocates each half in a sequence of two uniform-price auctions. That is, in the first auction: (1) each bidder submits a downward sloping demand function  $q_i^1(p)$  to the auctioneer; (2) the auctioneer finds the clearing price  $\hat{p}^1$  by equating the aggregate demand and 50% of supply, i.e.  $\sum_{i \in N} q_i^1(\hat{p}^1) = \frac{1}{2}$ ; and (3) each bidder wins a quantity  $\hat{q}_i^1 = q_i^1(\hat{p}^1)$  of the good and pays  $\hat{p}^1 \cdot \hat{q}_i^1$  to the auctioneer. Then, in the second auction: (1) each bidder submits a downward sloping demand function  $q_i^2(p)$  to the auctioneer; (2) the auctioneer finds the clearing price  $\hat{p}^2$  by

equating the aggregate demand and 50% of supply  $\sum_{i \in N} q_i^2(\hat{p}^2) = \frac{1}{2}$ ; and (3) each bidder wins a quantity  $\hat{q}_i^2 = q_i^2(\hat{p}^2)$  of the good and pays  $\hat{p}^2 \cdot \hat{q}_i^2$  to the auctioneer.

To simplify the analysis of the sequential auctions, we will make the following assumptions:

- The parameter  $\beta$  is set to 1. In other words, the value function for each bidder in  $N$  is assumed to be  $v(q) = q - \frac{1}{2}q^2$ . This is done without loss of generality, for ease of exposition.
- Each bidder is restricted from buying more than  $L^1$  share of the available supply in the first auction where  $L^1$  is given by:

$$L^1 = \frac{2n-3}{n(n-2)}. \quad (5.1)$$

Note that for  $n = 3$ , the limit  $L^1$  is non-binding, while for  $n = 4$ , the limit  $L^1$  precludes any bidder from buying more than 62.5% of the available spectrum in the first auction. By comparison, Ofcom's overall spectrum cap for any operator is currently 37%.

**Proposition 2.** Suppose that there are at least three bidders, i.e.,  $n \geq 3$ . A profile of linear bid functions for the first auction  $\{q_1^{1*}(p), \dots, q_n^{1*}(p)\}$  and a profile of linear bid functions for the second auction  $\{q_1^{2*}(p, q_1^1), \dots, q_n^{2*}(p, q_n^1)\}$  (contingent on bidders' winnings in the first auction) where:

$$q_i^{1*}(p) = (n-2) \left( \frac{n-1}{n} - p \right), \quad \forall i \in N, \quad (5.1)$$

and:

$$q_i^{2*}(p, q_i^1) = \left( \frac{n-2}{n-1} \right) (1 - q_i^1 - p), \quad \forall i \in N, \quad (5.1)$$

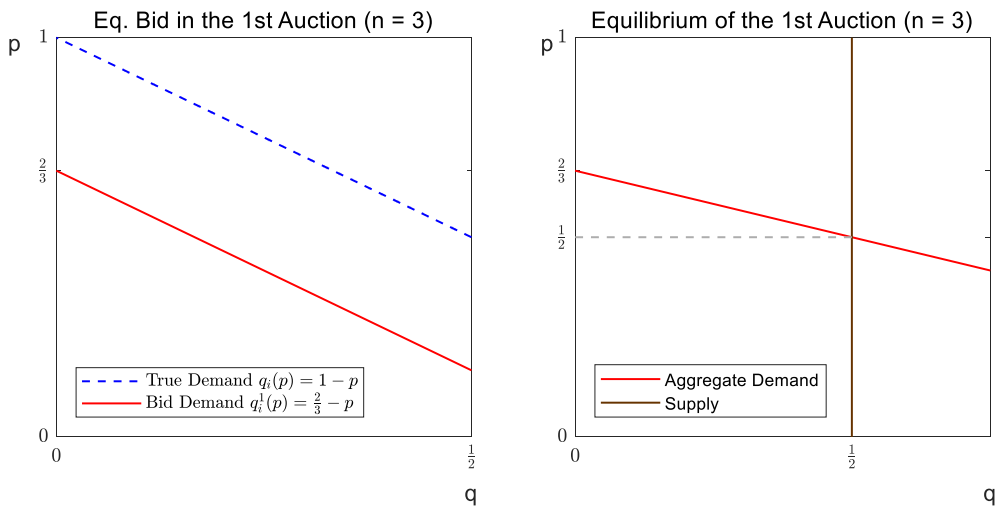
form a subgame perfect equilibrium of the sequential uniform-price auction. In this equilibrium, the clearing prices in both auctions are equal and are given by:

$$p^{1*} = p^{2*} = 1 - \frac{1}{n} \left( \frac{n-1}{2(n-2)} + \frac{1}{2} \right). \quad (5.2)$$

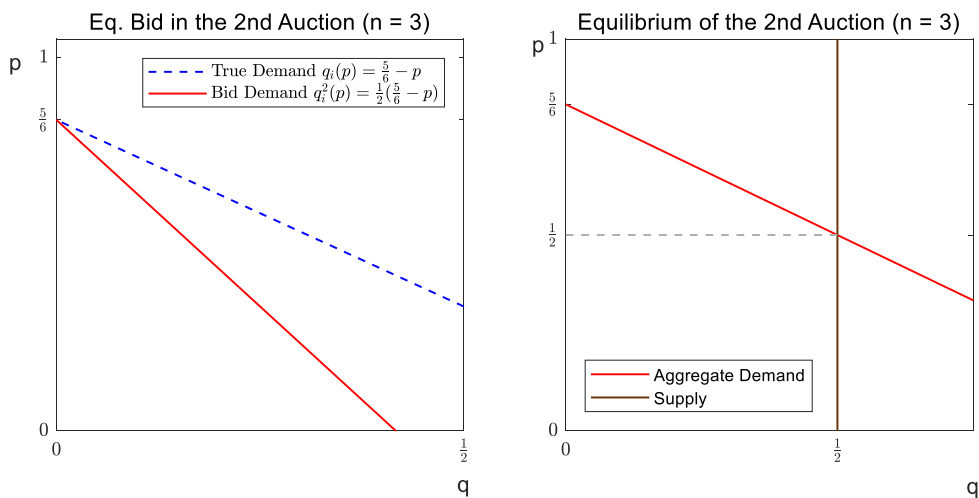
**Proof:** See the Appendix.

The equilibrium bidding functions and the corresponding equilibrium clearing prices are illustrated in Figure 2 and Figure 3.

**Figure 2: Illustration of Proposition 2 (1<sup>st</sup> Auction)**



**Figure 3: Illustration of Proposition 2 (2<sup>nd</sup> Auction)**



### 5.3 Ranking of Prices in the Single Auction and Sequential Auctions

Since Propositions 1 and 2 provide closed-form solutions of the equilibria of the single auction and sequential auctions, respectively, it is possible to rank the equilibrium prices of the two

award approaches. Proposition 3 shows that higher prices result when the supply is split between two auctions. We have:

**Proposition 3.** The equilibrium clearing prices in the two sequential uniform-price auctions (each for 50% of supply) exceed the clearing price in the single uniform-price auction for 100% of supply, i.e.

$$p^* < p^{1*} = p^{2*} \quad (5.2)$$

**Proof:** Trivially follows from Propositions 1 and 2, since:

$$\frac{n-1}{2(n-2)} + \frac{1}{2} < \frac{n-1}{n-2} \Leftrightarrow 1 < \frac{n-1}{n-2}.$$

QED

## 5.4 Intuition for Proposition 3

Proposition 3 establishes that, if the spectrum is awarded in two sequential uniform-price auctions, then in an equilibrium that arbitrages the clearing price between the first auction and the second auction, the clearing price is higher in each of the two sequential auctions than in the single combined auction. The intuition for this result is as follows. There are two effects shaping the comparison between the clearing price of the second sequential auction and that of the single combined auction. One effect is that each bidder has already purchased some of the spectrum in the first auction, implying that each bidder now bases its bid in the second auction only on its residual demand—this would tend to reduce the clearing price. The other effect is that there is a smaller supply in the second auction—this would tend to raise the clearing price. If bidding was non-strategic (i.e. if bidders merely bid their true demands), these two effects would exactly cancel each other out, since the supply in the second auction is reduced—as compared to a single combined auction—by exactly the sum of the amounts by which each bidder’s demand has been reduced. However, equilibrium bidding in a uniform-price auction is strategic and subject to demand reduction.<sup>17</sup> In this model, we found it possible to write a closed-form solution for equilibrium bid functions. Both in the second sequential auction and in the single combined auction, each bidder’s equilibrium bid function has the same vertical intercept as the bidder’s true demand, but it has a steeper slope. Moreover, the slope of the bid function is the same in the second sequential auction (Proposition 2) and in the single combined auction (Proposition 1). Thus, the bidders’ optimal bidding functions diverge

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<sup>17</sup> Ausubel, Lawrence, Peter Cramton, Marek Pycia, Marzena Rostek and Marek Weretka (2014), “Demand Reduction and Inefficiency in Multi-Unit Auctions,” *Review of Economic Studies*, 81(4): 1366-1400.



from their true demands, and consequently the amount by which the clearing price (taking demand reduction into account) is less than the clearing price (under non-strategic bidding) increases in the supply being auctioned. Since a smaller quantity is being auctioned in the second sequential auction than in the single combined auction, the clearing price is higher. By the price arbitrage condition, the clearing price in the first sequential auction is also higher than in the combined auction.

In short, the extent that the clearing price is reduced in a uniform-price auction depends on the supply. Consequently, dividing the spectrum into two sequential auctions blunts the effect of demand reduction and produces higher clearing prices.

## 6. Implications for the Opportunity Cost Calculation

We now apply the model and propositions of Section 5 to consider various possible measures of lump-sum value that could be computed from the first of a sequence of two multi-unit auctions for the same product. This will give us guidance as to what indices from the PSSR auction best reflect the true opportunity cost of 3.4 GHz and 3.6 GHz spectrum for other users.

### 6.1 Calibration of the Model

Our first step is to formulate the exact model to be considered. In the original model, recall that there are  $n$  bidders, each with a true demand curve of  $q = 1 - P$ , and there is a supply of  $S = 1$ . However, in the empirical scenario, H3G's UKB subsidiary had prior holdings of 120 MHz, which constituted 30.8% of the total supply of 390 MHz in the 3.4 and 3.6 GHz bands. In addition, H3G purchased 20 MHz (5.1% of the 3.4 and 3.6 GHz bands) in the PSSR auction, while Vodafone, BT/EE and O2 purchased 50 MHz, 40 MHz and 40 MHz respectively (12.8%, 10.3% and 10.3%, respectively, of the 3.4 and 3.6 GHz bands) in the PSSR auction.

We will now calibrate a slight modification of the original model to obtain a scenario whose equilibrium outcome comes strikingly close to reflecting the empirical reality. The only changes needed to the model of Proposition 2 are as follows:

- there are now three symmetric bidders ( $i = 1, 2, 3$ ) and a fourth asymmetric bidder ( $i = 4$ );
- bidder 4 enters the sequence of two auctions with prior holdings of 0.3;
- half of the remaining supply of 0.7 is offered in each of two uniform-price auctions;
- bidders ( $i = 1, 2, 3$ ) have true demand curves of  $q_i = 1 - P$ , as in the original model; and
- bidder 4 has a true demand curve of  $q_4 = \frac{47}{40} - P$ .

The higher intercept for bidder 4 is required so that, notwithstanding its prior holdings, bidder 4 makes positive purchases in the auctions. (If symmetry had been maintained, bidder 4's prior holdings of 0.3 would have exceeded its equal share of 0.25, and so bidder 4 would win zero quantity in the auction). Otherwise, we adhere to the modelling and conventions of Section 5.

Before solving the model in its exact modified form, we first calculate the opportunity cost of bidder 4's prior holdings (Section 6.2), the solution to the model if, counterfactually, there had been a single combined VCG mechanism or CCA (Section 6.3), and the solution to the model if, counterfactually, there had been a single combined uniform-price auction (Section 6.4).

### 6.2 Calculation of the Opportunity Cost to Other Users of the Prior Holdings

With the model as specified in Section 6.1, the true aggregate demand curve of the three symmetric bidders is given by  $Q = 3(1 - P)$ . For any supply  $S$  from 0.7 to 1, the marginal value to the symmetric bidders is therefore given by:

$$S = 3(1 - P) \Rightarrow MV = P = 1 - \frac{1}{3}S. \quad (6.0)$$

Therefore, the true *opportunity cost to other users* of the prior holdings is the integral of a declining sequence of marginal values, beginning at a supply of 0.7 and ending at the full supply of 1. Since the marginal values decline linearly, the opportunity cost is easily calculated to be:

$$\text{Opportunity cost to other users} = \int_{0.7}^1 (1 - \frac{1}{3}S) dS = \left[ S - \frac{1}{6}S^2 \right]_{0.7}^1 = \frac{43}{200} = 0.215. \quad (6.0)$$

Therefore, in the model as specified in Section 6.1, the true *opportunity cost to other users* of bidder 4's prior holdings is 0.215, the value determined by Eq. (6.0).

### 6.3 Outcome in a Single (Combined) VCG Mechanism or CCA

If the supply (excluding bidder 4's prior holdings) were auctioned in a single (combined) VCG mechanism or a CCA, the bidders would bid truthfully. Accounting for bidder 4's prior holdings of 0.3, bidder 4 enters the auction with a true residual demand curve of  $q = \frac{7}{8} - P$ . The supply (excluding bidder 4's prior holdings) is  $S = 0.7$ . Hence, the intersection of supply and demand is given by:

$$\frac{7}{10} = 3(1 - P) + (\frac{7}{8} - P) \Rightarrow P = \frac{31}{32} - \frac{7}{50} = 0.79375. \quad (6.0)$$

At this price, each of the three symmetric bidders would win quantities of 0.20625. Bidder 4 would win a quantity of 0.08125, giving bidder 4 post-auction holdings of 0.38125.

### 6.4 Outcome in a Single (Combined) Uniform-Price Auction

If the supply (excluding bidder 4's prior holdings) were auctioned in a single (combined) uniform-price auction, the bidders would bid as established by Proposition 1. Substituting  $n = 4$  into Eq. (5.1), we see that each of the three symmetric bidders would bid according to:

$$q_i(P) = \frac{2}{3}(1 - P), \text{ for } i = 1, 2, 3. \quad (6.0)$$

Meanwhile, accounting for bidder 4's prior holdings of 0.3, bidder 4 enters the auction with a true residual demand curve of  $q = \frac{7}{8} - P$ . Again using Eq. (5.1), bidder 4 would bid according to:

$$q_4(P) = \frac{2}{3}(\frac{7}{8} - P). \quad (6.0)$$

The supply (excluding bidder 4's prior holdings) is  $S = 0.7$ . Hence, the intersection of supply and demand is given by:

$$\frac{7}{10} = 3 \times \frac{2}{3}(1 - P) + \frac{2}{3}(0.875 - P) \Rightarrow P = \frac{113}{160} = 0.70625. \quad (6.0)$$

The clearing price would equal 0.70625. By Eq. (6.0), each of the three symmetric bidders would win quantities of 0.1958. By Eq. (6.0), bidder 4 would win a quantity of 0.1125, giving bidder 4

post-auction holdings of 0.4125. Observe that these quantities are similar to, but not identical to, what we would obtain in a single combined VCG mechanism or CCA.

## 6.5 Outcome in a Sequence of Two Uniform-Price Auctions

However, the analysis of Section 5 showed that the bids would be considerably higher if the items were auctioned in a sequence of two uniform-price auctions. Accounting for the prior holdings of 0.3 and for the higher intercept of bidder 4's true demand curve, we find the following equilibrium. In the first auction, similar to Eq. (5.1) (and substituting  $n = 4$ ), the equilibrium bid functions are:

$$\begin{aligned} q_i^1(P) &= 2\left(\frac{4}{5} - P\right), \quad \text{for } i = 1, 2, 3; \text{ and} \\ q_4^1(P) &= 2\left(\frac{31}{40} - P\right). \end{aligned} \quad (6.0)$$

In the second auction, the bid functions depend on the bidders' respective winnings,  $w_i^1$ , in the first auction. Following Eq. (5.1), the equilibrium bid functions are:

$$\begin{aligned} q_i^2(P) &= \frac{2}{3}(1 - w_i^1 - P), \quad \text{for } i = 1, 2, 3; \text{ and} \\ q_4^2(P) &= \frac{2}{3}\left(\frac{7}{8} - w_4^1 - P\right). \end{aligned} \quad (6.0)$$

The supply in the first auction is 0.35—i.e. half of the supply excluding prior holdings. Substituting from Eq. (6.0), the clearing price in the first auction is:

$$\frac{7}{20} = 3 \times 2\left(\frac{4}{5} - P\right) + 2\left(\frac{31}{40} - P\right) \Rightarrow P = \frac{3}{4}. \quad (6.0)$$

At this clearing price, by Eq. (6.0), each of the three symmetric bidders would win quantities of  $w_i^1 = 0.1$  (for  $i = 1, 2, 3$ ) and bidder 4 would win a quantity of  $w_4^1 = 0.05$ . Observe that these numbers (as fractions of 390 MHz) are strikingly close to the four operators' actual winnings in the PSSR auction.

Substituting from Eq. (6.0), the clearing price in the second auction is:

$$\frac{7}{20} = 3 \times \frac{2}{3}(1 - 0.1 - P) + \frac{2}{3}\left(\frac{7}{8} - 0.05 - P\right) \Rightarrow P = \frac{3}{4}. \quad (6.0)$$

At this clearing price, by Eq. (6.0), each of the three symmetric bidders would again win quantities of 0.1 (for  $i = 1, 2, 3$ ) and bidder 4 would again win a quantity of 0.05. For each of the three symmetric bidders, the winnings summed over the two auctions would be  $w_i = 0.2$  (for  $i = 1, 2, 3$ ) and for bidder 4, the winnings summed over the two auctions would be  $w_4 = 0.1$ .

Combined with its prior holdings, bidder 4's post-auction holdings would be 0.4.

Note also that Eqs. (6.0) and (6.0) give us consistency; with identical clearing prices, there is no incentive for intertemporal arbitrage between the first and second auctions.

## 6.6 Calculation of Opportunity Cost from Losing Bids in the First Auction

The methodology that we are proposing for calculating the opportunity cost to other users of H3G holding UKB's 3.4 GHz and 3.6 GHz spectrum, using the results of the PSSR auction, is as follows:

- (1) First, we remove all of H3G's bids from the PSSR auction. Note that this has two effects: it removes H3G's 20 MHz of winning bids, thereby converting the highest 20 MHz of other users' losing bids into winning bids; and it implies that when we sum the losing bids to calculate opportunity cost, we only consider the bids of other users.
- (2) Second, we sum the highest 120 MHz of other bidders' losing bids in the PSSR auction.

We will now implement this methodology for the model of Section 6.1. We will see that it performs exceedingly well in the model—indeed it replicates the exact opportunity cost that we have calculated, based on the true demand curves, in Eq. (6.0).

We implement this methodology for bidder 4 in the model as follows. From Eq. (6.0), we have seen that the aggregate bid function of the three symmetric bidders is given by  $Q = 3 \times 2 \left( \frac{4}{5} - P \right)$ . Rearranging terms gives us  $P = \frac{4}{5} - \frac{Q}{6}$ . Since a supply of  $S$  is sold in the auction and since all of bidder 4's bids (including its winning bids) have been removed, we are treating the highest  $S$  bids of the three symmetric bidders as winning bids and all subsequent (i.e. lower) bids as losing bids. Let  $h$  denote the quantity of bidder 4's prior holdings. The "sum" of the  $h$  highest losing bids of other bidders is calculated from the following integral:

$$\begin{aligned} \text{Sum of } h \text{ highest losing bids of other users} &= \int_S^{S+h} \left( \frac{4}{5} - \frac{1}{6}Q \right) dQ = \left[ \frac{4}{5}Q - \frac{1}{12}Q^2 \right]_S^{S+h} = \\ &= \frac{4}{5}h - \frac{1}{12}(S+h)^2 + \frac{1}{12}S^2 = \left( \frac{4}{5} - \frac{1}{6}S \right)h - \frac{1}{12}h^2. \end{aligned} \quad (6.0)$$

Evaluated at the supply  $S = 0.35$  and the prior holdings  $h = 0.3$ , this gives us a result that the sum of the  $h$  highest losing bids of other users equals 0.215.

That is, the sum of the  $h$  highest losing bids of opponents in the first sequential auction, as calculated in Eq. (6.0), coincides exactly with the true opportunity cost of the prior holdings to the other users in the model, as determined in Eq. (6.0) above!

The intuition for this result is an interplay of the following three effects:

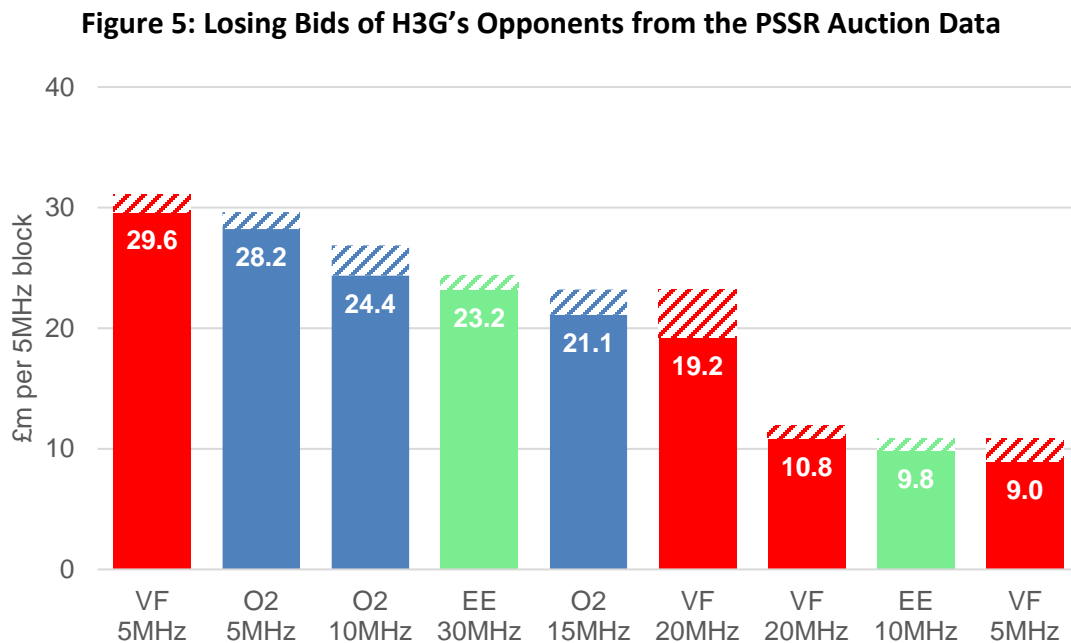
- Taking the sum of the  $h$  highest losing bids of opponents in a combined auction would clearly understate the true opportunity cost, due to strategic demand reduction in uniform-price auctions.
- However, as we have seen in Sections 5.3 and 5.4 above, auctioning the spectrum in two sequential auctions blunts the effect of demand reduction and produces a higher clearing price than in the combined auction.

- Moreover, not just the clearing price is higher. In addition, the equilibrium bid functions in the first sequential auction are flatter than the true demand curves—in contrast to the equilibrium bid functions in the second auction (which are steeper than the true demand curves). For example, with  $n = 4$  bidders, the slope of the bid function in the first auction is  $-\frac{1}{2}$  (where the horizontal axis is quantity and the vertical axis is price), while the slope of the true demand curve is  $-1$ . So even after accounting for the higher clearing price, the flattened bid function produces higher losing bids.

The first effect induces lower losing bids in the first auction, as compared to the associated true opportunity costs. However, the second and third effects induce higher losing bids in the first auction, as compared to the associated true opportunity costs. As established by Eqs. (6.0) and (6.0), the second and third effects together exactly cancel out the first effect in the model, and so the losing bids in the first sequential auction provide exactly the right measure of the true opportunity cost.

## 6.7 Actual Losing Bids of H3G’s Opponents in the PSSR Auction

Finally, we see what the methodology of Section 6.6 produces as applied to the actual PSSR auction data. In Figure 5, we depict the highest 120 MHz of losing bids by H3G’s opponents in the PSSR auction, after removing all of H3G’s bids:



As can be seen in Figure 5, the highest relevant losing bid is £29.6 million and the lowest relevant losing bid is £9.0 million. It is a straightforward calculation to see that the weighted average of the highest 120 MHz of losing bids by H3G's opponents in the PSSR auction, after removing all of H3G's bids, equals £19.1 million.

## 7. The Value of 3.6 GHz Spectrum Should be Discounted

In the consultation (paragraphs 2.10 – 2.18), Ofcom claims that the long-term value of 3.4 GHz spectrum and 3.6 GHz spectrum are the same. For example, paragraph 2.11 states that:

*“We auctioned 150 MHz of spectrum in the 3.4-3.6 GHz band earlier this year. In October 2017, we published our decision to remove fixed links and satellite earth station authorisations in the 3.6-3.8 GHz band. We expect to publish our proposals for the award of the 3.6-3.8 GHz band shortly. We therefore consider that the long-term value of the 3.6- 3.8 GHz band will be the same as the 3.4-3.6 GHz band.”*

and then paragraph 2.16 concludes that:

*“In the long-term we consider that the 3.4 GHz and 3.6 GHz spectrum will have the same value.”*

Ofcom’s justifications for this conclusion are provided in paragraphs 2.10 and 2.14:

(¶2.10) *UKB’s 3.4 and 3.6 GHz spectrum falls within the 3.4-3.8 GHz band, which is the primary band for 5G services in Europe.*

(¶2.14) *We also note that, following our recent decision to grant UKB’s 3.6 GHz licence variation request, the technical licence conditions for both UKB’s 3.4 GHz and 3.6 GHz licence have been aligned. This means that, in practice, there is no difference between the technology that H3G is able to deploy in its 3.4 GHz and 3.6 GHz spectrum.*

We agree with Ofcom’s assessment that the *long-term technical value* for 3.4 GHz and 3.6 GHz spectrum appear to be quite similar. However, the full market value of spectrum includes its commercial value as well as its technical value. Therefore, similar technical values do not necessarily translate into similar market values.

In the wireless industry, it is imperative for operators to innovate constantly and to introduce new services so as to retain (or increase) their market shares. Introducing the new services ahead of its competitors provides an operator with the following first-mover advantages:

- Initial pricing for 5G services can include a premium (higher revenues per subscriber and extra subscribers) to reflect the temporary exclusivity of a higher service level; and
- Higher long-term profitability and market share due both to “customer lock-in” associated with switching costs faced by the early adopters lured by the availability of 5G services and to “bragging rights” associated with being one of the first operators to offer 5G services.

When evaluating 3.4 GHz and 3.6 GHz spectrum from the perspective of the “early adoption of 5G technology”, we note that UKB’s 3.6 GHz spectrum faces a number of constraints on its use



until June 2020. At the same time, 3.4 GHz spectrum is already unencumbered and ready to be deployed. Therefore, there is a meaningful difference between the commercial values of 3.4 GHz and 3.6 GHz bands.

To put it succinctly, the long-term market value of 3.4 and 3.6 GHz spectrum, evaluated at mid-year 2020, are the same. However, the value of 3.4 GHz spectrum evaluated at mid-year 2018 exceeds the value of 3.6 GHz spectrum evaluated at mid-year 2020. Moreover, the difference is not just an omitted constant value generated from mid-2018 to mid-2020. Rather, it also includes a premium for being the first-mover in offering 5G services (“5G premium”). In short, auction prices for the 3.4 GHz band can be expected to be higher than auction prices for the 3.6 GHz band because of the existence of the 5G premium.

Ofcom recognises and proposes partial measures to deal with the “omitted value generated from mid-2018 to mid-2020” in paragraph 2.18:

*“In view of the short-term constraints on use of the 3.6 GHz spectrum however, we expect there will be some difference in value between 3.4 and 3.6 GHz spectrum in the short term. We discuss how we propose to take account of this difference, along with other considerations, when we consider the phasing in of the new licence fees in section 5.”*

However, Ofcom’s “phasing in” proposal only addresses the omitted constant value, and it does not address any implications of the 5G premium. Furthermore, the effects of the 5G premium in bids and prices from the 3.4 GHz award are being overlooked (see paragraph 2.17)

*We therefore consider that the bids and prices indicated in the 3.4 GHz award also provide a good indication of the value of UKB’s 3.6 GHz spectrum in the long term.*

Fortunately, it is very easy to account for the market value difference that arises from the 5G premium. According to the report prepared by Frontier Economics, the two-year delay in the availability of 3.6 GHz spectrum for 5G services reduces the market value of 3.6 GHz spectrum by around 10-15% relative to the 3.4 GHz spectrum that can be used for 5G now. Then, for the purposes of setting ALF fees for the UKB spectrum, the lump-sum value of 3.6 GHz spectrum should be discounted by 10-15% relative to the lump-sum value of 3.4 GHz spectrum inferred from bids in the PSSR auction.

Let  $\delta$  denote the discount factor to be applied to values of the 3.6 GHz spectrum, relative to the 3.4 GHz spectrum. Since one-third (40 MHz) of the UKB spectrum is in the 3.4 GHz band and two-thirds (80 MHz) is in the 3.6 GHz band, this leads us to apply a factor of  $\frac{1}{3} \times 1 + \frac{2}{3} \times \delta$  to the overall lump-sum value. With a conservative  $\delta = 0.9$ , this would reduce the overall ALF by a factor of 0.93333. With a more aggressive  $\delta = 0.85$ , this would reduce the overall ALF by a factor of 0.9. We take no opinion on whether the discount factor  $\delta$  should be applied selectively to the lump-sum value of UKB’s 3.6 GHz spectrum or whether a blended reduction factor (in the range of 0.9 to 0.93333) should be applied to all of the UKB spectrum.

## 8. The ALF Should Not Depend on the Upcoming Auction

Ofcom is proposing to set the ALF for all 120 MHz of UKB spectrum solely by reference to the auction results for the 3.4 GHz band in the public sector spectrum release (PSSR) auction of the 2.3 and 3.4 GHz bands completed in April 2018. Ofcom also states that it does not intend to revise the ALF based on the results of the forthcoming 700 MHz and 3.6-3.8 GHz spectrum auction.

*“... We would therefore be unlikely to review ALFs in the five years after implementation save in very exceptional circumstances, and would also propose to retain them beyond that date unless there were grounds to believe that a material misalignment had arisen between the level of these fees and the value of the spectrum, in keeping with our general policy on fee reviews. This proposed approach means that we do not intend to review the level of ALFs for UKB’s 3.4 GHz and 3.6 spectrum after the forthcoming auction for 700 MHz and 3.6-3.8 GHz spectrum.”<sup>18</sup>*

We strongly agree with the proposed approach. Any methodology that incorporates the bidding data from the 700 MHz and 3.6-3.8 GHz auction into the ALF calculation would necessarily distort bidding incentives in this auction, leading to an inefficient outcome and reducing revenues. More importantly, it would allow dominant MNOs to buy spectrum at uncompetitive prices and compromise the competition in the downstream market. In addition, Ofcom once again would face a challenging and nontransparent task of eliciting band-specific market value signals from bidding data of a multi-band combinatorial auction, similar to the challenge it faced in setting annual license fees for the 900 MHz and 1800 MHz bands using data from the 800 MHz and 2.6 GHz combinatorial clock auction (CCA) of 2013.

Next we provide a detailed account of our reservations towards using the data from the forthcoming 700 MHz and 3.6-3.8 GHz auction to calculate the ALF for the UKB spectrum.

### 8.1 Distortions in H3G’s Bidding

A dependency of the ALF on the outcome of the 700 MHz and 3.6-3.8 GHz auction (e.g. opportunity cost or marginal opportunity cost) would introduce an indirect tax on bids placed by H3G. In a typical ascending auction, a bidder compares the price it needs to pay to win a license with its intrinsic value when deciding whether to place a bid. In contrast, the cost of placing a bid for H3G would include two components here: (1) the price it needs to pay to win a license; and (2) the total increase in the ALF resulting from an increase in the market clearing price. Given the 120 MHz size of the UKB spectrum holdings, even a modest increase in the

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<sup>18</sup> Ofcom, “Annual Licence Fees for UK Broadband’s 3.4 GHz and 3.6 GHz spectrum”, December 2018, p. 40, ¶5.14.

clearing price of a 5 MHz block will translate into a huge impact on the ALF. Consequently, H3G's main objective in such auction would be to "keep prices low at all costs". To pursue this objective, H3G would be expected to withdraw its intrinsic demand at uncompetitive low prices, leading to an inefficient allocation of spectrum and reduced auction revenues.

## 8.2 Bidding Distortions for Dominant Operators

Unlike the previous instances of using auction data to derive the ALF, setting the ALF for UKB spectrum affects only H3G. This provides a unique opportunity for the dominant operators (EE, O2 and Vodafone) to reverse Ofcom's competitive measures and harm competition in the downstream market by competing aggressively during the auction. Ordinarily, a bidder would base its bids in this auction on the intrinsic value for 3.6 GHz spectrum. However, if it is known that the ALF for UKB spectrum will be derived from bidding data in the 3.6 GHz auction following Ofcom's general principles for setting the ALF, the dominant players would have strong incentives to bid higher so as to directly harm H3G via an inflated ALF. While this can potentially offset the revenue loss due to bidding distortions for H3G, it would have an anticompetitive effect in the downstream market and would harm consumers through higher prices.

## 8.3 Inference Problems in Using Data from the Upcoming Auction

Ofcom had major difficulties with extracting market value signals from the bidding data of the UK 4G Auction held in 2013. Indeed, the process of setting annual license fees for 900 MHz and 1800 MHz bands based on the bidding data from the 2013 auction dragged on until 2018. The fundamental source of these difficulties is that the auction included multiple bands and used the combinatorial clock auction (CCA) format. In this auction, bidders bid for packages of licences and payments for winners are assessed on the whole packages. As a result, the auction does not generate separate prices for different blocks leading to an inference problem if one needs to elicit band-specific prices.

Similar issues will arise in the 700 MHz and 3.6-3.8 GHz auction, for which the CCA format has been proposed. Moreover, Ofcom intends to include provisions for coverage obligation in this auction in a distorting way (Ofcom acknowledges this problem), further conflating the price signals generated by the auction.

### **Multi-band CCAs do not produce market signals for individual spectrum bands**

As Ofcom notes in previous consultations on setting ALFs, it is sometimes difficult for CCAs to produce market signals for individual spectrum bands.

*"Another source of complication is that three of the five winners of spectrum in the 4G auction acquired packages of spectrum in multiple bands. This raises the question of how to decompose these package prices by band. We set out below a decomposition of the auction prices, based on the nature of the highest losing bids from which they were*

*derived. This decomposition is well-defined for three of the five winning bidders. However, in the case of each of Niche's and Vodafone's auction price we have not identified a unique decomposition by band and instead we present alternatives.*<sup>19</sup>

Indeed, it is quite possible that there is no unique decomposition for any bidder. If only winners of 700 MHz spectrum win 3.6 GHz spectrum, as is likely if the 3.6 GHz band is used to inform the UKB spectrum ALF calculation, then there may be a wide range of possible decomposition values. In the extreme case, if winners win equal proportions of 700 MHz and 3.6 GHz spectrum and losing bidders use the same proportion for all supplementary bids, two of the valid decompositions of the spectrum band could place the value of either spectrum band at zero, in which case there would be no signal at all. This actually happened for portions of the 2.6 GHz calculation:

*"...The maximum and minimum attributable to 4xE can similarly be identified by attributing all or none of the synergy to 4xE."*<sup>20</sup>

The process is laborious and there is a multiplicity of methodologies to choose from:

*"...[W]e derive the market value of each of the 800 MHz and 2.6 GHz bands through analysis of a range of methods to assess the evidence from the UK 4G auction:*

- a) Prices in the 4G auction, which were determined as the higher of (i) reserve prices and (ii) the incremental bid value<sup>25</sup> of the bidder's highest losing bids for additional spectrum compared to that bidder's winning package.*
- b) Opportunity cost in the 4G auction, which is the incremental bid value for additional spectrum in the highest losing bids compared to the winning packages of the bidders submitting these highest losing bids (i.e. unlike the actual prices in the auction, they are not influenced by reserve prices).*
- c) Linear Reference Prices (LRPs), which estimate the linear prices that were closest to market-clearing prices (by a linear price we mean the same price per MHz in a given band, such as 800 MHz; to all operators and for all block sizes); and*
- d) Marginal bidder analysis to analyse opportunity cost by assessing the bids of the highest losing bidder for additional spectrum."*<sup>21</sup>

### **The coverage obligation will further obfuscate any price signals for 3.6 GHz spectrum**

The proposed design of the 700 MHz / 3.6 GHz auction further includes coverage obligations that will be sold in packages together with spectrum. First, this will further confuse whether

<sup>19</sup> Ofcom, 2015 Consultation Annual Licence Fee Statement, Annex 6, p. 9, ¶A6.5.

<sup>20</sup> Op. Cit., p. 12, ¶A6.15.

<sup>21</sup> Ofcom, "Annual Licence Fees for 900 MHz and 1800 MHz frequency bands" October 2013 Statement, pp. 12-13, ¶4.3.

payments are directed toward 700 MHz spectrum, 3.6 GHz spectrum, or a coverage obligation (i.e. there will be a third variable that needs to be disentangled). Second, Ofcom has acknowledged that the inclusion of the coverage obligation in the auction will, under some circumstances, result in a distortion of the auction outcome:

¶7.98 If the price of the spectrum a bidder has won is less than the discount for a coverage obligation, then the bidder would not be realising the full discount. In such circumstances, bidders may have an incentive to increase the amount of spectrum they are bidding on to increase their effective discount up to the full amount (or a large enough effective discount to offset their costs of the coverage obligation). This could result in a change in the allocation of spectrum compared to an auction with no coverage obligations.

It should further be observed that, since bidders would have an incentive to increase their quantities bid at given prices, this distortion would increase the prices observed for spectrum in the auction. It would be unacceptable to base lump-sum values used for the ALF on auction results that are known to be distorted upward. Moreover, due to the package bidding nature of the CCA, it will be difficult or impossible to disentangle the *bona fide* demand for spectrum from this distortion resulting from the coverage obligation in a reliable way, just as it will be difficult to disentangle demand expressed for the 700 MHz band from demand expressed for the 3.6 GHz band.

Even if there was a way to disentangle the value of the bids for 3.6 GHz spectrum, the bids will be hopelessly noisy and misleading. As we discussed above, any bids by H3G will be biased by the ALF dependence. The clock bids for the 3.6 GHz from the coverage obligation bidders will be inflated by the subsidy and the clock bids will only have a very loose relationship with intrinsic value. The revenue implied by accepting the winning clock bids could be a factor of two or three higher than the sum of the winning package bids. All of these distortions make an ALF calculation based on the 3.6 GHz auction data a case of “garbage in, garbage out”.

[3].

Given that Ofcom does not have the “power to accept negative bids” for the coverage obligation (for example in a negative price SMRA), there seems to be little scope for improving the mechanism of the 3.6 GHz auction to obtain a clean opportunity cost for 3.6 GHz spectrum.<sup>22</sup>

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<sup>22</sup> Ofcom (2018), “Award of the 700 MHz and 3.6-3.8 GHz spectrum bands,” ¶7.10.

## 9. The ALF should be set at a conservative estimate of market value

Ofcom makes a good case for taking the best unbiased estimate of the fair price for the ALF and reducing it to reflect the asymmetric risks. The harm of the ALF being set too low is small compared to the harm of the ALF being set too high (i.e. the spectrum laying fallow or the 3.4/3.6 GHz services market being less competitive due to H3G not being able to compete as fiercely on service quality or pricing). Ofcom summarizes many of these points as follows:

*...[T]he risk that ALFs set too high could threaten the optimal use of spectrum. In particular, it noted that ALFs set too high could incentivise companies to over-economise on spectrum, leading companies to either use more complex and therefore expensive equipment as an alternative to spectrum, or to reduced services for consumers. We also consider that, on balance, there is a greater risk to optimal use of spectrum from inadvertently setting fees above market value than below. A further effect of higher ALFs is that H3G might seek to recover any higher costs from consumers through higher retail prices.<sup>23</sup>*

Any reasonable loss function would support Ofcom's intent to err on the side of an ALF that is lower than an unbiased estimate, in order to mitigate the costlier risks. This could suggest further reducing the ALF from the correct unbiased calculation of opportunity cost. We note that the risk of the ALF being set too high is similar to the risk of the reserve price in the auction being too high. If there is excess supply at the reserve price of the auction, there is fallow spectrum. It is an international best practice to set reserve prices very conservatively since the inefficiency associated with a delay to reauction the spectrum is a much bigger risk to efficiency than too low a starting price.

In the auction consultation, Ofcom has proposed selecting a reserve price for the 3.6 GHz spectrum in the range of £15 to £25 million per 5 MHz block.<sup>24</sup> Ofcom would find a suitable candidate for the ALF in the lower part of this range. Indeed, our reasoning in this paper has suggested an average lump-sum value of between £17.2 and £17.8 million. We conclude by noting that if the ALF is any higher than the 2020 auction's reserve price, Ofcom risks the unseemly outcome that the fourth largest MNO in the UK would be required to pay more for its 3.6 GHz spectrum than any of its three larger rivals.

<sup>23</sup> Consultation, 3.33, p. 16.

<sup>24</sup> Ofcom (2018), "Award of the 700 MHz and 3.6-3.8 GHz spectrum bands", ¶17.248.