ISSUES IN BETA ESTIMATION FOR UK MOBILE OPERATORS: UPDATE

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1 Introduction and Summary

In July 2002 we prepared a report for the UK telecoms regulator (*Oftel*), examining issues in the estimation of Beta for UK mobile phone operators. *Oftel* have asked us to update our analysis with recent data for both Vodafone and mmO2. We also discuss some methodological issues that have arisen since the publication of our initial study.

1.1 Methodology

Oftel were advised by Charles Rivers Associates (CRA), on behalf of T-Mobile, to aggregate different estimates of a firm's beta into a composite 'average'. This is correct under certain circumstances, but is incorrect when using different methodologies to derive different estimates. Since we calculate multiple results from a set of different methodologies, *we do not calculate an average.*

We find that it is highly likely that the 'true' Vodafone beta has changed significantly over the past five years. *Using more than one year of data would compromise the assumptions underlying the CAPM*.

We also investigate the statistical robustness of the Dimson adjustments. We conclude that the use of symmetric and asymmetric lagged Dimson adjustments *is theoretically and statistically robust when calculating beta relative to the All Share index*. Dimson adjustments are *not* statistically robust when calculating beta relative to the All World index.

1.2 Results

Table 1 presents our beta estimates, calculated from daily data, for the year 26/09/02 to 25/09/03. Where applicable, we present a range of values.

Table 1: Summary of Beta Estimates (Daily Returns, 26/09/02 - 25/09/03)

	Vodafone	mmO2
FTSE All Share	1.15 to 1.25	1.01 to 1.31
FTSE All World	1.09	1.33

2 Methodology

We discuss three methodological issues relating to beta estimation:

- How to deal with multiple beta estimates;
- What time period to use;
- The statistical and theoretical robustness of the Dimson adjustments.

2.1 Multiple Beta Estimates

Charles River Associates (CRA) advised *Oftel* to aggregate different beta estimates:[1]

> *"It is a general principal that, when several estimates of a quantity are available, it is better – in the sense of minimizing estimation error – to combine them, rather than simply choosing the one that seems to be, in some sense, 'Best'."*[2]

CRA propose combining multiple estimates weighted in inverse proportion to their standard error. CRA's proposal presumes the following conditions:

- The estimates are derived by the *same methodology*, but from *different sample* data;
- The sample data come from the *same population*.

If these conditions do not hold then the results are not statistically comparable in the manner implied by CRA, and Oftel should not combine them. Therefore, when using different methodologies, or samples drawn from different populations, such a weighting technique is not applicable.[3]

2.2 Time Period

Choosing a long time period can permit greater statistical accuracy, due to an increase in data points, and the possibility of using longer sampling frequencies. However, a long time period may include information that does not reflect the current risk of an investment. The CAPM model assumes that *the underlying beta does not change during the time period across which the beta is estimated*[4]. Figure 1 shows estimates of the

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[1] "Cost of capital for T-Mobile (UK)", *July 2003*.

[2] Paragraph 5.47, *Ibid.*

[3] Note: a different methodology requires calculating the beta in a different method, such as using a different Dimson or Bayesian adjustment. Using a different time period, or sampling frequency, of data does not constitute a different methodology. However, different time periods may represent different 'populations' if the underlying value that we are attempting to estimate appears to change.

[4] The CAPM requires a linear relationship between returns on the market and returns on the asset in order for risk to be expressed as a single time-independent result, beta. To maintain this linear relationship, statisticians say that the true regressand coefficient must be 'stationary', or constant over time. See Greene, W.H. (2000), Econometric Analysis, 4th Edition, Prentice Hall, New Jersey; chapter 6.

Vodafone beta for the previous five years, calculated from daily data and with a ±1 Dimson Adjustment.[5]

Figure 1: 90% Confidence Intervals for Annual Vodafone Beta vs. All Share (Daily Data, ±1 Dimson Adjustment)

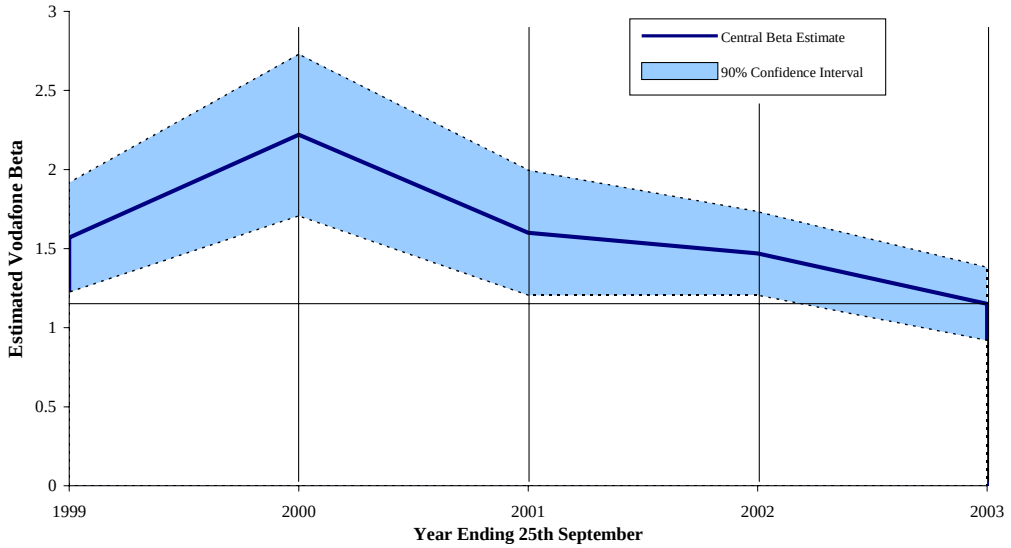

Figure 1 shows that estimates of the annual beta over the previous five years vary significantly. Even considering the apparently modest drop in estimated beta between 2002 and 2003, the 2003 beta lies outside the 90% confidence interval of the 2002 beta. Given the rapid changes in the mobile telephone industry, we conclude that using more than one year's worth of data to estimate the Vodafone beta would *compromise the CAPM's assumptions*. When using daily data, *we recommend using only the most recent year of available daily data to estimate Vodafone's beta.* We use the same time period and sampling frequency for mmO2 due to their relatively short trading period.

2.3 Dimson Adjustments

Problems with Daily Data

Using daily, rather than monthly, data can present a number of problems due to noise in the data created by the *institutional infrastructures of stock trading*. In particular:

- "*Bid-ask Bounce*": the artificial movement in reported prices over consecutive trades when the transaction goes through a 'market-maker';
- *Weekend heteroscedasticity*: the potential higher variance of returns on a Monday compared to other days of the week due to the increased amount of information (and time) available across the weekend;

[5] Excluding the Dimson adjustment does not change the important properties of the graph.

- "*Non-synchronous trading bias*": the problem that arises when the share whose beta is measured trades much more thinly or thickly than the market average;

- *Informational and Institutional delays*: the different speeds with which different agents and institutions digest information and adjust their holdings of different stocks.

In our previous study we investigated the "bid-ask bounce" and weekend heteroscadisticity, and *detected neither in the Vodafone stock price.*

"Non-synchronous trading bias" and information and institutional delays manifest themselves in statistically significant relationships between an asset's returns across the day *t*, and the returns on the market across days *t-x* and/or *t+x*. In our previous study we discovered evidence of such a relationship, *and recommended a correcting Dimson Adjustment*.

Theoretical Robustness of Dimson Adjustment

Beta measures the relationship between movements in the price of a stock compared to movements in the price of a market. The release or discovery of information presumably causes these movements: the price of either the market or the stock should not move otherwise.

A statistical analysis requires a Dimson adjustment if, by the close of trading, information affects the price of the stock but not the price of the market. This can happen if:

1. Some significant stocks in the market were *not traded* between the release of the information and the close of the market;

2. Complex business structures or high levels of uncertainty delayed the market's incorporation of price information;

3. Institutional investors hold disproportionately high quantities of some stocks, and institutional investors take *some time* to decide on trades.[6]

An analysis also requires a Dimson adjustment if the stock takes longer than the market to incorporate information, which can happen for reasons symmetrical to those listed above.

Informational and Institutional Delays

It may take more time to incorporate information into some stocks relative to others. Consider data that forecasts an increase in business investment and a decrease in consumer spending. The stock of a business-services supplier or high-street retail chain

[6] Reason one corresponds to the problem of "Non-Synchronous Trading"; reasons two and three correspond to the problem of Informational and Institutional Delays.

might incorporate such data more easily than Vodafone or mmO2. The market could react *more quickly* to some events.

In the case of large, high-profile, companies like Vodafone and mmO2, information about the companies themselves often moves the rest of the market, creating the potential for additional lags.

In addition, large institutional investors may hold disproportionately high amounts of Vodafone and mmO2 stock. Such large holdings force institutional investors to adjust their positions slowly, creating a level of inertia in the stock price.

Size of Lags and Leads

The possibility that information is incorporated into the market price, and not the stock price, is captured by regressing the return on the stock against both the contemporaneous return on the market, and the return on the market in previous periods. We call this a 'lag', and the number of previous periods in the analysis is the number of 'lags'. The possibility that information is incorporated in the stock price, and not the market price, is captured by regressing the return on the stock against both the contemporaneous return on the market, and the return on the market in future periods. We call this a 'lead', and the number of future periods to regress against is the number of 'leads'.

A key question is the number of lags and leads to add. In the addendum to our previous report, we concluded that *more than two lags or leads does not make theoretical sense*. We have further investigated the changing statistical significance of the lags and leads over time. Table 2 summarises the T-Statistics for various combinations of lags and leads.

Table 2: Statistical Significance of Dimson Adjustments, Vodafone vs. All Share, 2001-2003.

	2001	2002	2003
Dimson ±1	1.84	1.27	2.21
Dimson-1	1.98	2.32	1.94
Dimson+1	0.52	0.67	1.20
Dimson ±2	3.11	2.03	1.85
Dimson-2	2.69	2.75	2.01
Dimson+2	0.70	0.22	0.66

=Low Statistical Significance
=Statistically Significant at a 75% level of confidence
=Statistically Significant at a 90% level of confidence
=Statistically Significant at a 95% level of confidence

We conclude that the Dimson adjustments are statistically robust for all but the +1 lead and +2 lead adjustments.[7] This is not the case for estimates against the FTSE All-

[7] This result differs from CRA's erroneous assertion that the only "conceptually coherent version for a highly liquid stock is to regress the contemporaneous index and a single *leading* term" (*ibid.* para 5.36). The CRA assertion stems from an incomplete analysis of how an Ordinary Least Squares regression minimises errors under thick-trading.

World. Slightly higher standard errors, lower beta estimates, and greater variation in the Dimson adjustments prevent any clear pattern of statistical significance from emerging.

3 Results

3.1 Vodafone, All Share

Table 3 details our estimates of the Vodafone beta against the FTSE All Share. We have not included the results for the +1 and +2 leads since they do not appear to be consistently statistically significant.

Table 3: Vodafone vs FTSE All Share, 26/09/02 - 25/09/03. Daily Data.

			Symmetric Dimson		Asymmetric Dimson	
		No Adjustment	± 1 Period	± 2 Periods	- 1 Period	-2 Period
Vodafone	Adjusted Beta Estimate					
	Estimated Beta	1.42	1.15	1.17	1.25	1.17
	Standard Error	(0.07)	(0.14)	(0.18)	(0.11)	(0.14)
	Adjustment Details					
	Sum of Lags and Leads		-0.23	-0.3	-0.14	-0.23
	Standard Error		(0.1)**	(0.12)*	(0.07)*	(0.11)**

Notes: * indicates standard error on Dimson adjustments is small enough for the adjustment to be considered significant at the 90% level. ** indicates standard error on Dimson adjustments is small enough for the adjustment to be considered significant at the 95% level.

We conclude that a reasonable range of estimates would be between 1.15 and 1.25. Despite the low statistical significance of asymmetric leading terms reported earlier, it is still the case that both Vodafone and mmO2 are thickly traded relative to the market, and as such we recommend using symmetric Dimson adjustments, which favours the lower end of the scale.

For comparison purposes, detailed results from a range of regressions are included in Appendix 1.

3.2 mmO2, All Share

Table 3 details our estimates of the mmO2 beta against the FTSE All Share. We have not included the results for the +1 and +2 leads since they do not appear to be consistently statistically significant.

Table 4: mmO2 vs FTSE All Share, 26/09/02 - 25/09/03. Daily Data.

			Symmetric Dimson		Asymmetric Dimson	
		No Adjustment	± 1 Period	± 2 Periods	- 1 Period	-2 Period
mmO2	Adjusted Beta Estimate					
	Estimated Beta	1.58	1.24	1.01	1.31	1.22
	Standard Error	(0.11)	(0.2)	(0.26)	(0.16)	(0.2)
	Adjustment Details					
	Sum of Lags and Leads		-0.3	-0.64	-0.24	-0.34
	Standard Error		(0.15)**	(0.16)**	(0.11)**	(0.16)**

Notes: * indicates standard error on Dimson adjustments is small enough for the adjustment to be considered significant at the 90% level. ** indicates standard error on Dimson adjustments is small enough for the adjustment to be considered significant at the 95% level.

mmO2 was more volatile than Vodafone during the period of analysis, and exhibits a greater range in beta estimates. The standard errors of the estimates are also higher.

Nonetheless, a beta in the range of 1.01 to 1.31 would appear reasonable, which overlaps completely with the estimates for Vodafone.

3.3 Vodafone and mmO2, All World

Table 3 details our estimates of both the Vodafone and mmO2 betas against the FTSE All World. We show estimates using all our Dimson adjustments, since none of the adjustments seem to be consistently more significant than the others.

Table 5: Vodafone and mmO2 vs FTSE All World, 26/09/02 - 25/09/03. Daily Data.

			Symmetric Dimson		Asymmetric Dimson			
		No Adjustment	± 1 Period	± 2 Periods	- 1 Period	- 2 Period	+ 1 Period	+ 2 Period
Vodafone	Adjusted Beta Estimate							
	Estimated Beta	1.09	0.98	1.2	1.07	0.84	0.97	1.16
	Standard Error	(0.1)	(0.18)	(0.22)	(0.15)	(0.18)	(0.15)	(0.18)
	Adjustment Details							
	Sum of Lags and Leads		-0.09	-0.1	0.01	-0.26	-0.11	0.08
	Standard Error		(0.14)	(0.15)	(0.1)	(0.15)*	(0.11)	(0.15)
mmO2	Adjusted Beta Estimate							
	Estimated Beta	1.33	1.04	1.03	1.19	0.95	1.16	1.11
	Standard Error	(0.13)	(0.22)	(0.28)	(0.19)	(0.22)	(0.19)	(0.22)
	Adjustment Details							
	Sum of Lags and Leads		-0.28	-0.58	-0.13	-0.4	-0.17	-0.23
	Standard Error		(0.18)	(0.19)*	(0.13)	(0.19)*	(0.13)	(0.19)

Notes: * indicates standard error on Dimson adjustments is small enough for the adjustment to be considered significant at the 90% level.

We conclude that a reasonable range of estimates for mobile phone operators against a world market portfolio is between 1.09 and 1.33.

3.4 Summary of Beta Estimates

Table 6 summarises our beta estimates.

Table 6: Summary of Beta Estimates (Daily Returns, 26/09/02 - 25/09/03)

	Vodafone	mmO2
FTSE All Share	1.15 to 1.25	1.01 to 1.31
FTSE All World	1.09	1.33

Appendix 1: Detailed Results

Table 7, Table 8, and Table 9 provide a large range of beta estimates for comparison with the recommended betas published in the main body of the report.

Table 7: Beta estimates for comparisons, Vodafone vs. All Share

	Ending 25th September				
	1999	2000	2001	2002	2003
12 Months	**Normal: 1.5 (0.12)**	**Normal: 2.07 (0.18)**	**Normal: 1.96 (0.13)**	**Normal: 1.63 (0.09)**	**Normal: 1.42 (0.07)**
	Dimson ±1: 1.57 (0.21)	Dimson ±1: 2.22 (0.31)	Dimson ±1: 1.6 (0.24)*	Dimson ±1: 1.47 (0.16)	Dimson ±1: 1.15 (0.14)*
	Dimson-1: 1.57 (0.17)	Dimson-1: 1.92 (0.25)	Dimson-1: 1.71 (0.18)*	Dimson-1: 1.42 (0.13)*	Dimson-1: 1.25 (0.11)*
	Dimson+1: 1.5 (0.17)	Dimson+1: 2.36 (0.25)*	Dimson+1: 1.9 (0.18)	Dimson+1: 1.7 (0.13)	Dimson+1: 1.32 (0.11)
24 Months	**Normal: 1.48 (0.09)**	**Normal: 1.75 (0.11)**	**Normal: 2 (0.11)**	**Normal: 1.76 (0.08)**	**Normal: 1.53 (0.06)**
	Dimson ±1: 1.52 (0.14)	Dimson ±1: 1.85 (0.18)	Dimson ±1: 1.88 (0.19)	Dimson ±1: 1.52 (0.14)*	Dimson ±1: 1.35 (0.11)*
	Dimson-1: 1.59 (0.11)	Dimson-1: 1.72 (0.15)	Dimson-1: 1.8 (0.15)*	Dimson-1: 1.54 (0.11)*	Dimson-1: 1.36 (0.09)*
	Dimson+1: 1.4 (0.11)	Dimson+1: 1.87 (0.15)	Dimson+1: 2.09 (0.15)	Dimson+1: 1.77 (0.11)	Dimson+1: 1.53 (0.09)
36 Months	**Normal: 1.42 (0.07)**	**Normal: 1.66 (0.08)**	**Normal: 1.83 (0.08)**	**Normal: 1.83 (0.07)**	**Normal: 1.65 (0.06)**
	Dimson ±1: 1.46 (0.12)	Dimson ±1: 1.7 (0.14)	Dimson ±1: 1.77 (0.14)	Dimson ±1: 1.69 (0.13)	Dimson ±1: 1.43 (0.1)*
	Dimson-1: 1.51 (0.1)	Dimson-1: 1.68 (0.11)	Dimson-1: 1.72 (0.11)	Dimson-1: 1.63 (0.1)*	Dimson-1: 1.46 (0.08)*
	Dimson+1: 1.37 (0.1)	Dimson+1: 1.68 (0.11)	Dimson+1: 1.88 (0.11)	Dimson+1: 1.91 (0.1)	Dimson+1: 1.63 (0.08)
48 Months		**Normal: 1.6 (0.07)**	**Normal: 1.75 (0.07)**	**Normal: 1.76 (0.06)**	**Normal: 1.71 (0.06)**
		Dimson ±1: 1.64 (0.12)	Dimson ±1: 1.69 (0.12)	Dimson ±1: 1.66 (0.11)	Dimson ±1: 1.57 (0.1)*
		Dimson-1: 1.61 (0.1)	Dimson-1: 1.69 (0.09)	Dimson-1: 1.62 (0.09)*	Dimson-1: 1.54 (0.08)*
		Dimson+1: 1.62 (0.1)	Dimson+1: 1.74 (0.09)	Dimson+1: 1.82 (0.09)	Dimson+1: 1.75 (0.08)
60 Months			**Normal: 1.7 (0.06)**	**Normal: 1.71 (0.06)**	**Normal: 1.68 (0.05)**
			Dimson ±1: 1.64 (0.11)	Dimson ±1: 1.63 (0.1)	Dimson ±1: 1.57 (0.09)
			Dimson-1: 1.64 (0.08)	Dimson-1: 1.62 (0.08)*	Dimson-1: 1.54 (0.07)*
			Dimson+1: 1.69 (0.08)	Dimson+1: 1.73 (0.08)	Dimson+1: 1.71 (0.07)

Note: This data is given for comparison purposes. Many of the estimates described above either directly violate the assumptions behind the CAPM model, or are out of date.

Table 8: Beta estimates for comparisons, Vodafone vs. All World

	Ending 25th September				
	1999	2000	2001	2002	2003
12 Months	**Normal: 1.14 (0.15)**	**Normal: 1.3 (0.21)**	**Normal: 1.21 (0.16)**	**Normal: 1.45 (0.12)**	**Normal: 1.09 (0.1)**
	Dimson ±1: 1.29 (0.23)	Dimson ±1: 1.79 (0.34)*	Dimson ±1: 1.37 (0.27)	Dimson ±1: 1.44 (0.19)	Dimson ±1: 0.98 (0.18)
	Dimson-1: 1.38 (0.19)*	Dimson-1: 1.62 (0.28)*	Dimson-1: 1.26 (0.22)	Dimson-1: 1.56 (0.16)	Dimson-1: 1.07 (0.15)
	Dimson+1: 1.07 (0.19)	Dimson+1: 1.41 (0.28)	Dimson+1: 1.31 (0.22)	Dimson+1: 1.35 (0.16)	Dimson+1: 0.97 (0.15)
24 Months	**Normal: 0.96 (0.09)**	**Normal: 1.22 (0.13)**	**Normal: 1.24 (0.13)**	**Normal: 1.34 (0.1)**	**Normal: 1.28 (0.08)**
	Dimson ±1: 1.16 (0.15)*	Dimson ±1: 1.52 (0.2)*	Dimson ±1: 1.52 (0.21)	Dimson ±1: 1.41 (0.16)	Dimson ±1: 1.26 (0.13)
	Dimson-1: 1.17 (0.12)*	Dimson-1: 1.49 (0.16)*	Dimson-1: 1.4 (0.17)	Dimson-1: 1.43 (0.13)	Dimson-1: 1.36 (0.11)
	Dimson+1: 0.93 (0.12)	Dimson+1: 1.23 (0.17)	Dimson+1: 1.34 (0.17)	Dimson+1: 1.33 (0.13)	Dimson+1: 1.18 (0.11)
36 Months	**Normal: 0.88 (0.07)**	**Normal: 1.05 (0.09)**	**Normal: 1.22 (0.1)**	**Normal: 1.33 (0.09)**	**Normal: 1.26 (0.07)**
	Dimson ±1: 1.08 (0.12)*	Dimson ±1: 1.33 (0.14)*	Dimson ±1: 1.45 (0.16)*	Dimson ±1: 1.48 (0.15)	Dimson ±1: 1.29 (0.12)
	Dimson-1: 1.07 (0.1)*	Dimson-1: 1.28 (0.12)*	Dimson-1: 1.4 (0.13)*	Dimson-1: 1.47 (0.12)*	Dimson-1: 1.33 (0.1)
	Dimson+1: 0.87 (0.1)	Dimson+1: 1.06 (0.12)	Dimson+1: 1.26 (0.13)	Dimson+1: 1.34 (0.12)	Dimson+1: 1.22 (0.1)
48 Months		**Normal: 0.98 (0.08)**	**Normal: 1.1 (0.08)**	**Normal: 1.3 (0.08)**	**Normal: 1.27 (0.07)**
		Dimson ±1: 1.24 (0.12)*	Dimson ±1: 1.34 (0.13)*	Dimson ±1: 1.45 (0.12)	Dimson ±1: 1.36 (0.12)
		Dimson-1: 1.19 (0.1)*	Dimson-1: 1.28 (0.1)*	Dimson-1: 1.45 (0.1)*	Dimson-1: 1.38 (0.1)*
		Dimson+1: 1 (0.1)	Dimson+1: 1.13 (0.1)	Dimson+1: 1.29 (0.1)	Dimson+1: 1.25 (0.1)
60 Months			**Normal: 1.04 (0.07)**	**Normal: 1.19 (0.07)**	**Normal: 1.25 (0.07)**
			Dimson ±1: 1.27 (0.11)*	Dimson ±1: 1.36 (0.11)*	Dimson ±1: 1.36 (0.1)
			Dimson-1: 1.21 (0.09)*	Dimson-1: 1.36 (0.09)*	Dimson-1: 1.38 (0.09)*
			Dimson+1: 1.08 (0.09)	Dimson+1: 1.19 (0.09)	Dimson+1: 1.22 (0.09)

Note: This data is given for comparison purposes. Many of the estimates described above either directly violate the assumptions behind the CAPM model, or are out of date.

Table 9: Beta estimates for comparisons, mmO2

	Ending 25th September 2003	
	vs. All Share	vs. All World
12 Months	**Normal: 1.58 (0.11)**	**Normal: 1.33 (0.13)**
	Dimson ±1: 1.24 (0.2)*	Dimson ±1: 1.04 (0.22)
	Dimson-1: 1.31 (0.16)*	Dimson-1: 1.19 (0.19)
	Dimson+1: 1.5 (0.16)	Dimson+1: 1.16 (0.19)
22 Months	**Normal: 1.57 (0.09)**	**Normal: 1.43 (0.11)**
	Dimson ±1: 1.27 (0.16)*	Dimson ±1: 1.23 (0.18)
	Dimson-1: 1.32 (0.13)*	Dimson-1: 1.36 (0.15)
	Dimson+1: 1.51 (0.14)	Dimson+1: 1.29 (0.15)