

# **Price Distortions in Combinatorial Clock Auctions, a Theoretical Perspective**

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## **Abstract**

This paper shows that the use of Combinatorial Clock Auctions (CCAs) for spectrum auctions is often associated with strategic incentives for bidders to drive up the prices that rival bidders pay. These incentives may vary across bands, with the result not only that overall prices may be inflated but also that (implied) relative prices across spectrum bands may be distorted. The circumstances in which CCAs are most vulnerable to strategic bidding and resulting price distortion are similar to those in the SMRA, i.e. situations where there are only a small number of bidders, each having a predictable demand for spectrum in some or all available bands. Especially in auctions with few, asymmetric bidders where spectrum that is currently in use (so-called legacy bands) is re-auctioned and where the spectrum caps are so weak that one or two bidders can buy up (almost) all spectrum, bidders in a CCA have the ability and the incentive to bid up competitors' prices without engaging in relatively risky bidding strategies. The multi-band auction in Austria had all these features, and produced an exceptionally high price outcome, with prices exceptionally weighted towards the legacy 900 MHz and 1800 MHz bands, when compared to other European auctions. Therefore, it is most unlikely that the auction prices of the Austrian multi-band auction are good indicators of the market value of spectrum in other European countries, including the UK. Accordingly, Ofcom's designation of Austrian LRP prices as a Tier 1 benchmark for the UK is not justified.

## 1. Introduction

Ofcom is in the process of determining annual license fees (ALFs) for mobile frequencies in the 900 MHz and 1800 MHz bands. Ofcom has developed a methodology in which auction prices in other countries are used as a basis for the determination of annual license fees in the United Kingdom. The consultation document of February 19, 2015 informs that the main countries of comparison with the UK are Ireland and Austria: these two countries form the so-called Tier 1 evidence.<sup>2</sup> Ireland and Austria were among a group of countries where mobile frequencies in the 900 MHz and 1800 MHz bands were re-auctioned after 2010. As the relative value of frequencies may change over time, recent auction prices for these frequencies may provide a good reference point for determining the current market value.

The weight Ofcom attaches to the auction prices in Austria and Ireland is much larger than the weight attached to the auction prices in other countries. Both countries used a combinatorial clock auction (CCA) to allocate spectrum in different frequency bands (including 800 MHz, 900 MHz and 1800 MHz bands). Many other countries used the more traditional simultaneous multi-round auction (SMRA). One reason why Ofcom may favour CCAs is that they believe that “(t)he fundamental rationale for the CCA as an auction format is that it provides incentives for straightforward bidding by bidders” (see A8.122 of the February 2015 consultation document).

It is well-known that in the more traditional simultaneous multi-round auction (SMRA) bidders may be able to manipulate auction prices by engaging in strategic demand reduction (see, *e.g.*, Grim *et al.*, 2003). The CCA was originally presented to national regulators as an alternative auction format that, because of its second-price rule, eliminates the scope for strategic bidding or “gaming” (see, *e.g.*, Cramton, 2012). Despite the complexity of the

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<sup>2</sup> OFCOM, 2015 Annual licence fees for 900 MHz and 1800 MHz spectrum. Provisional decision and further consultation.

auction format, bidders were advised to “simply bid their valuations”.<sup>3</sup> The next two sections of this paper address the possibilities for gaming in both auction formats, SMRA and CCA. The conclusion is that when bidders can predict the demand of their competitors with a reasonable degree of accuracy, there exist ample opportunities for bidders to game the auction outcome in both SMRA and CCA. In a CCA, these opportunities arise because bidders pay “individualized prices” that are determined by how much their competitors bid on spectrum they do not acquire. In an auction with predictable demand in some bands, but not in others, this may lead to relative auction prices across bands that are skewed towards the band with the predictable demand.<sup>4</sup> Thus, without ascertaining that strategic bidding did not take place, there is no reason to believe that the auction prices in a CCA are good indicators of the absolute and relative market value of different spectrum bands.

I then consider the question under which circumstances strategic bidding in CCAs is more likely and when strategic bidding is unlikely. CCAs differ from one another in important supply and demand dimensions. On the supply side, Different CCAs differ as auctioneers have to decide on many important aspects of the auction rules. For example, the auctioneer has to decide on:

- (i) what spectrum to auction, including new spectrum bands, such as 800 MHz or legacy bands, such as 900 MHz and 1800 MHz owing to expiry of existing licences;
- (ii) whether or not to impose spectrum caps on bidders, and if so, at what level to set them and how to structure them across bands; and
- (iii) additional rules, such as the ratio of eligibility points of different bands (determining how easy it is for bidders to switch between different bands during the auction) and the information that is provided to bidders during the clock phase (whether to release in each round demands of individual bidders, total demand or nothing at all).

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<sup>3</sup> For example, in the abstract of his paper, Cramton (2012) says about CCA, “the pricing rule and information policy are carefully tailored to mitigate gaming behavior”. The Irish regulator Regcom (2012, p. 70) states that their consultancy firm DotEcon notes, “...the second price rule is utilized to disincentivise gaming behavior and encourage straightforward bidding”.

<sup>4</sup> While a CCA does not produce band-specific prices, final clock prices are sometimes used as a proxy. This issue will be discussed in Section 5 of the paper.

On the demand side, there are typically three, four or five incumbents and there may or may not be an interest from entrants to participate in the auction. The incumbents may be asymmetric in that there is a clear market leader, a number two and a weaker player in the market, or incumbents may be symmetric in that they all have more or less similar market shares. That is, the market conditions may significantly differ between countries, influencing the outcome of any auction, including ones using the CCA.

The outcome of a CCA depends on how these different demand and supply factors interact. Auction prices in one CCA (country) are only good indicators of the likely auction prices in another country if the auction rules as well as the market circumstances are roughly identical. Section 4 discusses these issues with respect to the question of whether the Austrian auction prices are likely to be good indicators of the market value of spectrum in the UK. I conclude that in the absence of public information after the auction on the bidding history, there are many indicators that strategic bidding was important in the Austrian auction and that the relative prices between the 800 MHz spectrum band on one hand and the 900 and 1800 MHz spectrum bands, on the other hand, were significantly distorted. Accordingly, it does not seem justified to consider the Austrian auction prices as a Tier 1 benchmark for the market value of spectrum in the UK.

In this paper, I consider that, *ceteris paribus*, bidders prefer outcomes where rivals pay more for their winning allocation. In the literature this is sometimes referred to as bidders having a spite motive (or raising rivals' cost), but as I will explain below, there are perfectly rational (defensive, instead of offensive) reasons why bidders may be interested in outcomes where rival bidders pay more. Following Janssen and Karamychev (2015), I model this raising rivals' cost motive in a lexicographic way, *i.e.*, a bidder always prefers an outcome with a larger intrinsic surplus (the value of the winning package minus the payment). Rival payments

only distinguish between outcomes with identical intrinsic surplus.<sup>5</sup> Thus, bidders' preferences only marginally depart from the preferences that are usually assumed in standard auction theory.

Having discussed these alternative preferences with many practitioners (members of bidding teams of telecom companies and consultants advising these firms), regulators and academics, I am of the opinion that these preferences are closer to the real preferences that govern bidding behaviour in telecom auctions than the intrinsic preferences that are conveniently assumed in most auction theory.<sup>6</sup> There are two arguments why, for the same intrinsic surplus, bidders may want to raise rivals' cost.

First, *after acquiring spectrum*, winners have to invest large sums of money in developing or upgrading a network. Given imperfect capital markets, the more bidders pay for their licenses, the more expensive it is to finance externally future investments.<sup>7</sup> Raising what rivals pay for their spectrum holdings may delay or otherwise obstruct investments of competitors in increasing the quality of their network. This may give the bidder who raises rivals' cost a competitive advantage in the market after the auction vis-à-vis its rivals. As a bidder in a CCA knows that the price he pays is determined by the rivals' bids (in the one-off supplementary round), a bidder who only wants to guarantee not paying more for similar spectrum than rival bidders may feel forced to raise rivals' prices. Incentives for such

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<sup>5</sup> As the value of spectrum also depends on how many winners the auction has, their identity (whether they are incumbents or entrants), and on the quality of the package of licenses the competitors get, bidders valuation may actually be endogenous to the auction outcome. The literature on this issue (see, *e.g.*, Goeree, 2003; Jehiel and Moldovanu, 2000, 2003; Jehiel, Moldovanu, and Stacchetti, 1996; Janssen and Karamychev, 2007, 2010; Klemperer, 2002a, 2002b) does not consider CCAs. I do not consider this issue in much detail in this paper. See also footnote 30.

<sup>6</sup> That bidders may be interested in raising rivals' cost is also confirmed in the consultation phase in the United Kingdom, where there was an active discussion on this point. See, for instance, Ofcom, 2012, page 122, paragraph 7.9. In the end, this was one of the reasons why the final cap rule was not adopted in the UK. The logic that OFCOM adopted was that with market clearing at the final clock price, the final cap rule implies that the final allocation is fully determined by the clock phase and that the supplementary phase only determines prices. In that case, bidders will fully raise rivals' cost (without running the risk of obtaining packages they would not like to acquire at too high a price) and Ofcom thought this undesirable. I show in this paper that bidders can also raise rivals' cost under the relative cap rule without running too much risk.

<sup>7</sup> There is a reasonably large literature on auctions with financial externalities, or auctions with a "spite motive" (see, *e.g.*, Cooper and Fang, 2008; Morgan *et al.*, 2003; Shandra and Sandholm, 2010; and Lu, 2012). This literature typically deals with a single object auction. Our paper is different in that in a multi-object auction, bidders can be winners and raise rivals' cost by placing bids on packages that are not winning themselves. This complicated gaming aspect is not present in single object auctions.

behaviour may also be asymmetric, as entrants and smaller incumbents are more likely to be vulnerable to capital market constraints than larger rivals.

Second, the only way to evaluate the success of a firm's bidding strategy after the auction is to compare the prices that different firms have paid for the packages they have obtained. Owing to governance issues between the bid team and senior management (or between senior management and shareholders), a bid strategy is often considered to be *unsuccessful* if another firm paid much less for objectively similar spectrum.<sup>8</sup> This is particularly important for bidders acquiring less spectrum than others as they must rationalise to shareholders why they won less.

For the CCA format, there was a very rapid transition from concept to practical reality. According to Ausubel and Baranov (2014) the format was proposed by Ausubel, Cramton, and Milgrom (2006), and first presented at the FCC's Wye River Conference in October 2003. The first practical implementations were the Trinidad and Tobago Spectrum Auction, in 2005, and the UK's 10–40 GHz and L-Band Auctions, in 2008.<sup>9</sup> There are two reasons for this quick adoption. First, the traditional SMRA was known to be prone to strategic bidding and the so-called exposure problem.<sup>10</sup> As package bidding is at the heart of a CCA, the exposure problem does not exist. Moreover, as the pricing rule is based on the second-price principle, it was initially thought that strategic bidding was also not an issue.<sup>11</sup> It is only recently that the opportunities for strategic bidding in a CCA have been explored in the academic literature. A first set of papers discusses the pricing rule. It is known that the VCG pricing rule produces prices that are not in the core and that as a consequence there may be bidders who are willing to pay more for the available spectrum than what the winners have to pay. This has led to implementation of core-adjusted pricing rules (see, e.g. Day and Raghavan (2007) and Day and Milgrom (2008)). Erdil and Klemperer (2009) and Goeree and

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<sup>8</sup> The Swiss auction outcome shows that payments for similar spectrum can be quite unequal creating quite a discussion at that time. See BAKOM (2010) for the Swiss auction design and BAKOM (2012) for the outcome.

<sup>9</sup> Ofcom was among the first regulators in 2007 to express an intention to adopt the CCA. It proposed the format for the 2.6 GHz auction, but this auction was time and again postponed, so the 10-40 GHz auction took place first.

<sup>10</sup> The exposure problem states that bidders may end up with a subset of spectrum they were bidding for and with complementarities between different units, they run the risk of acquiring spectrum at a price that is larger than its value.

<sup>11</sup> With standard preferences, it is well-known that a simple single-unit second-price auction has a weakly dominant strategy to bid value.

Lien (2012) show, however, that different core selection principles introduced in the pricing rule used in CCAs imply that bidding valuation is no longer an optimal bidding strategy if VCG prices are not in the core. Beck and Ott (2011) show that these principles may imply that bidding both above and below valuations can be optimal in CCAs.<sup>12</sup> Knappek and Wambach (2012) show in a series of examples that bidding in a CCA may be strategically complicated. They show that, in the supplementary phase, bidders can both increase and decrease the prices of their competitors without affecting their own intrinsic surplus, while in the clock phase, bidding truthfully is no longer a dominant strategy. Bichler *et al.* (2013) present experimental results on inefficient outcomes in CCAs. They attribute the inefficiency to the so-called missing bids problem.<sup>13</sup> In the Appendix, they provide an example of spiteful bidding in a CCA.<sup>14</sup> Two recent working papers provide a more substantial analysis of strategic bidding in a CCA. Janssen and Karamychev (2015) focus on the possibilities to raise rivals' cost, while Levin and Skrzypacz (2014) discuss both strategic demand reduction and strategic demand expansion strategies. The current paper uses some of the insights of Janssen and Karamychev (2015), while the results of Levin and Skrzypacz (2014) are discussed in more detail in footnote 21. Towards the end of their paper, Ausubel and Baranov (2014) also provide some considerations on strategic bidding due to the second-price principle adopted in a CCA. They argue that it may be desirable to move towards a first-price version of a CCA.

The rest of the paper is organized as follows. Section 2 provides an analysis of strategic bidding in the SMRA and in the CCA with the aim of providing some insights on the question whether a CCA is more likely to result in prices that reflect market value than an SMRA. Section 3 extends this analysis to multi-band auctions to ask whether a CCA produces relative prices for different spectrum bands that are in line with the relative market value for the different spectrum bands. Section 4 discusses the Austrian market circumstances and some aspects of the 2013 Austrian multi-band auction design with the aim of answering the question of whether the Austrian auction outcome should be considered important evidence of

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<sup>12</sup> Hence, while the core-adjusted pricing rule guarantees an outcome that is in the core with respect to bids, the outcome is not necessarily in the core with respect to actual valuations as intended.

<sup>13</sup> The missing bid problem is the issue (also addressed in Ausubel and Baranov (2014)) that bidders do not submit a full set of bids on all packages they potentially may value. In a multi-band, multi-unit auction, the number of possible packages a bidder may have a positive value for is typically very large, making it practically impossible for bidders to express a bid on each and every such package. Without a full set of bids, the CCA outcome may be inefficient.

<sup>14</sup> Salant (2013) provides an informed overview of some recently used auction design and a discussion on some outcomes of CCAs that have been held recently.



the market value of spectrum in Europe (and the UK in particular). Section 5 makes a brief aside to point out that a CCA calculates “individualized prices” that may greatly differ for the same spectrum between different bidders. Moreover, a CCA does not calculate prices per band, but only per package an individual bidder acquires. Section 6 concludes with the main findings of this paper.

## **2. Do auction outcomes of CCAs reflect market value?**

Economists widely agree that the market value of spectrum is based on its opportunity cost. The opportunity cost in an auction is the highest marginal value of any bidder for a spectrum block she did not acquire.<sup>15</sup> If bidders bid according to the value for the blocks that are auctioned (which is typically called straightforward or truthful bidding) and do not engage in strategic bidding, most auction formats result in prices that reflect market value. This fundamental insight is easily illustrated in an English auction for one object. In such an auction, each bidder drops out of the auction at a price that is equal to his value for the object. The bidder with the highest valuation wins the auction and pays the price of the bidder with the second-highest valuation. The main question I address in this section is whether one may expect CCAs to deliver auction prices that reflect market value.

As mentioned in the Introduction, Ofcom has argued that CCA encourages “straightforward bidding.” (see, e.g., A8.122 of the February 2015 consultation document). Straightforward bidding implies that, at the current clock round prices, bidders bid for the package that maximizes their surplus (the difference between value and package price calculated at the current prices). Thus, bidders will reduce their demand if the incremental price exceeds the marginal value for spectrum. If bidders bid straightforwardly, both CCA and SMRA formats typically lead to bidders paying market value for the spectrum they acquire. In an SMRA, bidders gradually drop demand and the auction stops when the bidder with the largest marginal value for a non-winning block drops out. In a CCA, the bidding phase in the clock

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<sup>15</sup> In order not to complicate the text too much, I use here the language of a multi-unit, single-band auction. In a multi-band auction, one has to compare the opportunity costs expressed in terms of alternative combinations of winning packages. As the same principles apply, I use the simpler language of a single-band auction.

stage is mechanically very similar to that in an SMRA, when bidders bid straightforwardly. This allows bidders in the supplementary phase to bid their value on all packages. The winner determination algorithm and the associated pricing rule then determine that all bidders pay the opportunity cost they impose upon others. Consider a single band CCA in which all bidders win exactly one lot, and further suppose that bidder 1 was the last bidder to drop from 2 lots to 1 lot. In this case, all bidders apart from bidder 1 pay the expressed marginal value of bidder 1 for a second lot, whereas bidder 1 himself pays the second-largest marginal value (as expressed by another bidder).<sup>16</sup>

The next example illustrates the scope for strategic bidding in both SMRA and CCA in a simplified case where there is only spectrum in one frequency band for sale.

**Example 1. Strategic Bidding in SMRA and CCA**

*Let there be two bidders and two identical blocks to be auctioned in a CCA. Bidder  $i$ 's valuation for one object is  $v_i(1)$ ,  $i = 1, 2$ , and for two blocks is  $v_i(2) = v_i(1) + \varepsilon_i$ , where it is known that  $v_i(1) > 10$ , and  $\varepsilon_i$  is (uniformly) distributed over  $[0, 10]$  and both  $v_i(1)$  and  $\varepsilon_i$  are private information to bidder  $i$ .*

*The key feature of the valuation structure is that both bidders should get one unit each, i.e. prices are set purely by the rival bidder's value for a second lot. Hence, it is clear that the market value of one lot is equal to  $\max(\varepsilon_1, \varepsilon_2)$  or, alternatively  $(\varepsilon_1 + \varepsilon_2)/2$  (see also footnote 16). At a price larger than  $\max(\varepsilon_1, \varepsilon_2)$  no bidder wants to get two units and  $\max(\varepsilon_1, \varepsilon_2)$  is effectively the highest price that any one of the two bidders is willing to pay for the second unit. The question then is whether SMRA and CCA result in an auction price equal to this value. If bidders would bid truthfully and reduce demand when price equals the marginal value of the second unit, the SMRA results in both bidders winning one unit and a price equal to  $\max(\varepsilon_1, \varepsilon_2)$ . Prices will be slightly different in a CCA. Here, bidder 1 reduces demand to one unit at a price equal to  $\varepsilon_1$ , while bidder 2 reduces demand in the clock phase to one unit at a price equal to  $\varepsilon_2$ . In the supplementary phase, both bidders bid value on one and two*

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<sup>16</sup> In the rest of this paper I do not make a distinction between these (small) differences in prices in an SMRA and a CCA. The pricing rule in a multi-band auction CCA is more difficult to explain, but essentially captures the same flavour.

units. Bidder 1 pays  $\varepsilon_2$  for one unit, while bidder 2 pays  $\varepsilon_1$ . Hence, the average price that a bidder pays equals  $(\varepsilon_1 + \varepsilon_2)/2$ .

The question then is whether bidders have the possibility to bid strategically in both auction formats. I first consider strategic bidding in the SMRA. It is well-known that, in an SMRA, bidders may engage in strategic demand reduction. Suppose that given their values and the expectations (or knowledge) they have about the values of the other bidder, they infer that whatever the detailed behaviour or valuation of the other the end result will be that both bidders get one unit. They will therefore not find it rational to compete to get the second unit and should reduce demand early on in the auction. Thus, the SMRA may result in prices  $p < \max(\varepsilon_1, \varepsilon_2)$  and  $p < (\varepsilon_1 + \varepsilon_2)/2$ .<sup>17</sup>

Now consider the possibility of strategic bidding in the CCA. Suppose without loss of generality that in the clock phase of the CCA, the price increases from  $p = 0$  (or reservation price) upwards and that (like in the first phase of the 2010 and 2013 Austrian auctions) bidders are only informed about whether there is excess demand at the current price level (to simplify the description of the strategies).<sup>18</sup>

The following strategies form (a perfect Bayesian) equilibrium in the CCA. In the clock phase, bidders demand:

- a) if  $p < 10$ , bidder  $i$  demands  $d_i(p, \varepsilon_i) = 2$  units;
- b) if  $p = 10$ , bidder  $i$  demands  $d_i(10, \varepsilon_i) = 1$  unit;
- c) if  $p > 10$ , bidder  $i$  demands  $d_i(p, \varepsilon_i) = 0$  or 1 unit depending on value.

Thus, the clock phase stops at the moment that the price reaches 10. Both bidders submit supplementary round bids  $b_i(1) = v_i(1)$  and  $b_i(2) = v_i(1) + 10$ , and win one object each at the auction price  $p(1) = 10$ . As with probability 1  $\max(\varepsilon_1, \varepsilon_2) < 10$ , the bidders are victims of the price driving strategy of the other bidder. As (unless he changes the allocation) each

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<sup>17</sup> Note that the raising rivals' cost motive does not affect the argument for strategic demand reduction in an SMRA. Due to the lexicographic nature of this motive, bidders find it much more important to pay less themselves than to raise rivals' cost. In an SMRA, bidders can only raise rivals' cost if they simultaneously also raise their own cost. Under lexicographic preferences, the only possibility to rationally raise rivals' cost in an SMRA is by dropping out of the spectrum band altogether at some point.

<sup>18</sup> For the purpose of keeping the example simple, I assume the following tie-breaking rule: If after the supplementary round there are multiple combinations of winning bids, then if the last-clock round allocation is among the value-maximizing allocations it is chosen as the final auction allocation, otherwise the allocation with the largest number of winners is chosen.

*bidder cannot affect the price he pays himself, but can only affect the price paid by the rival bidder (by bidding above value on the second unit), the auction puts the bidders in a kind of Prisoner Dilemma situation and pay a price much above market value.*

*To see that this is an equilibrium, first note that deviating in the supplementary round is not optimal as, given the bids of the competitor, no bidder wants to change the allocation and with these supplementary bids, each bidder maximally raises the price the competitor. We next consider three types of deviations in the clock phase. Each of these deviations is associated with a corresponding change in the supplementary round bids. First, consider bidder  $i$  deviates and demands  $d_i(p, \varepsilon_i) = 1$  units at prices  $p \geq p_1$ , where  $0 \leq p_1 < 10$ , then the supplementary round bids of this bidder  $i$  are capped by  $b_i(2) \leq b_i(1) + p_1$  so that the other bidder  $j \neq i$  wins his unit at a price of  $p_1 < 10$ , whereas the surplus of bidder  $i$  remains the same (and equal to  $v_i(1) - 10$ ). Thus, this deviation does not affect the final allocation, the price that bidder  $i$  pays, but only lowers the price that bidder  $j \neq i$  pays. Given that bidders have a lexicographic preference for raising rivals' cost, this deviation is strictly worse than the proposed equilibrium strategy. (A similar argument shows that bidding truthfully is not an equilibrium strategy when bidders also value to raise rivals' cost. If one bidder would reduce demand to one block in the clock phase, this bidder is capped to raise rivals' cost, which benefits the rival, but the rival can safely continue to demand two units as the price he pays for one unit is not determined by the price when the clock phase ends, but by the marginal bid of the first bidder on the second unit and this bid is capped).*

*Second, if bidder  $i$  deviates and demands  $d_i(p, \varepsilon_i) = 1$  units at prices  $p \in [p_1, p_0]$ , where  $0 \leq p_1 \leq p_0 < 10$ , and bids for  $d_i(p, \varepsilon_i) = 0$  units at prices  $p \geq p_0$ , then the supplementary round bids of this bidder  $i$  are capped by  $b_i(1) \leq p_0$   $b_i(2) \leq p_0 + p_1$  so that the other bidder  $j \neq i$  wins both units, and bidder  $i$  gets zero surplus. Thus, this deviation is also not profitable.*

*Third, if bidder  $i$  deviates and demands  $d_i(p, \varepsilon_i) = 2$  units at all prices  $p \in [0, p_1]$ , where  $p_1 > 10$ , then there is a chance this bidder wins both units at price  $p_i(2) = v_j(1) + 10$ , and his surplus is  $v_i(2) - p_i(2) \leq v_i(1) + \varepsilon_i - 20$ , which for all types  $\varepsilon_i \in [0, 10]$  is smaller than the surplus  $v_i(1) - 10$  of obtaining one unit. As this bidder does not want to risk*

*this outcome (raising rivals' cost is only of secondary importance), this deviation is also not profitable.<sup>19</sup>*

*Thus, both auction formats have an equilibrium where the price bidders pay is different from the market value. The SMRA results in demand reduction and lower prices, while the CCA results in demand expansion and higher prices.*

It is important to understand the crucial assumptions behind the results presented in the example. Strategic demand reduction and strategic demand expansion are not always equilibrium behaviours in SMRA or CCA. What is important for all bidders to engage in strategic demand reduction in this example of an SMRA is that they have a common understanding of the final allocation (and that they get a positive amount of spectrum in all frequency bands) if they bid straightforwardly according to value in the auction. If this is the case, they realize that they only increase their own price (as well as the prices for others) if they demand more than their share in the final allocation. In the above example, both bidders have a common understanding that their value for one unit is much larger than the value of the second unit. If this was not the case, and bidders would believe that they would be able to get both units at a price such that their surplus from obtaining both units may be larger than the surplus from obtaining one unit at a price close to the reserve price then a bidder may not engage in strategic demand reduction.<sup>20</sup>

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<sup>19</sup> If the auction rules are such that bidders observe aggregate demand (which is equivalent in this example to let them observe the demand of the competitor), the equilibrium outcome remains intact. For example, if the other bidder  $j \neq i$  switches to  $d_j = 1$  at price  $p \in (0, 10)$ , bidder  $i$  keeps bidding for 2 units.

<sup>20</sup> This may, for example be the case when a bidder believes the other bidder may also have a low value for the first unit.

A similar condition is necessary to get the strategic demand expansion result in a single-band CCA.<sup>21</sup> If bidders believe that by continuing to bid on two units at prices above their marginal value, they may actually win both units, they will not be keen to do so: if bidders only have a lexicographic preference for raising rivals' cost, they are not willing to sacrifice their own intrinsic surplus (the risk of getting a unit at a price above marginal value) in an attempt to make others pay more for their spectrum. If the rival bidder has low values for both units, a bidder will certainly run the risk of winning spectrum at too high a price. Thus, the condition to get strategic demand expansion is that rival bidders are (reasonably) certain to have valuations for some units that are larger than the marginal valuations of the bidder under consideration. The crucial points in the CCA part of the above example are: (i) bidders know that the rival bidder will not want to drop out of the clock phase as this is likely to result in not winning any spectrum, (ii) bidders know that the price the rival pays solely depends on their bids on large packages, and (iii) given the other bidder stays active in the clock phase, there is no risk of winning both units. If a bidder knows that the rival bidder has a high enough value for some package, it may engage in price driving strategies, without incurring much risk.<sup>22</sup>

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<sup>21</sup> Levin and Skrzypacz (2014) mention that bidders may also have an incentive to engage in strategic demand reduction in a CCA. There are, however, several points why it is difficult to implement strategic demand reduction in real world CCAs. First, as explained before, bidders in a CCA pay "individualised prices" that follow from the opportunity cost expressed by rival bidders. As the supplementary round involves simultaneous bidding, bidders cannot retaliate if another bidder bids in such a way to effectively raise rivals' cost. The only thing they could try to do is to bid in the clock phase in such a way that (through the constraints imposed by the relative cap) the opportunities to raise rivals' cost in the supplementary phase are limited. Bidders may try to achieve this by reducing demand in steps, where each new demand reduction step is taken if the others are doing their part. However, even this "strategic demand reduction by moving in small steps" is difficult to implement in practice for the following reasons. First, in most auctions, bidders are not informed about individual demands of rival bidders, but only about total demand, or sometimes (like in the Austrian auction) not even that. In that case, bidders cannot observe whether other bidders follow suit and (like in the literature on collusive behaviour in markets) collusion – here, strategic demand reduction – breaks down if one cannot monitor others' behaviour. Second, as bidders demand discrete blocks, there will always be (at least) one bidder that is supposed to make the last reduction in demand to stop the clock phase. Once the others have reduced their demand, and are probably not willing to reduce further, this last bidder to reduce demand in the clock benefits from the fact that others are already restricted to raise rivals' cost in the supplementary round, whereas he can postpone finalising the clock phase to relax his own supplementary round bidding allowing to raise rivals' cost further. The analysis of Levin and Skrzypacz (2014) does not, however, address these practical complexities. They do not provide a full equilibrium analysis where strategic demand reduction arises. Moreover, they restrict a bidder's strategy to be linear and they do not consider both bidders having a preference for raising rivals' cost. Finally, they do not consider discrete demand.

<sup>22</sup> The theoretical examples show when bidders can engage in demand expansion at zero risk. In practice there is some risk involved, but on the other hand the incentive to raise rivals' cost is also more than just a lexicographic one. The question is how bidders make the trade-off between raising rivals' cost and playing safe.

To conclude this section, I summarize as follows. Both SMRA and CCA are susceptible to strategic bidding and there is no reason to believe that a CCA results in prices that are closer to the true market value than an SMRA. Whether or not SMRA or CCA results in prices that reflect market values depends on the specific details of the auction design and the market circumstances. Without looking at these specifics, one cannot make a general claim that favours one or the other auction design.

### **3. Multi-band considerations and relative prices across bands**

The above discussion on the merits of SMRA versus CCA is in the context of single-band auctions. Most recent European telecom auctions have been multi-band auctions, however, and in multi-band auctions additional concerns are relevant. First, I will argue that the strategic demand reduction strategies relevant in an SMRA are more difficult to implement, whereas the strategic demand expansion strategies relevant in a CCA are less sensitive to complexities that arise in a multi-band context. More importantly, strategic demand expansion in a CCA may lead to clock prices being driven up in some bands, but not in others. Specifically, it may be that the combination of predictable and irreducible demand by incumbents in certain bands, especially legacy bands, and unpredictable demand in other spectrum bands, especially new ones, leads to relative (clock) prices that are not in line with the underlying marginal values in the different bands; hence relative auction prices may not reflect relative market value.

In the previous section, I showed that strategic demand reduction in an SMRA may arise in situations when all participating bidders have identical views on the outcome of the competitive auction process if they bid straightforwardly according to value. The final outcome is a focal point and all bidders individually can infer that competition will only drive prices up without affecting the allocation. Without prior communication, it may be more difficult in multi-band auctions, to have, in advance of the auction, a common understanding

(focal point) of what the final allocation will be in case of head-on competition. Accordingly, it may be more difficult to determine which bidder should reduce demand in each band and by how much. The combination of blocks that a bidder wants to acquire through strategic demand reduction should not be too large as other bidders should be willing to accept that the demand reducing bidder acquires that combination of blocks. The demand over all bands should also not be too small as this would clearly be suboptimal from the bidder's perspective. Thus, an implicit agreement to strategically reduce demand is much more difficult to establish in a multi-band SMRA than in a single band SMRA. The 2011 German LTE auction is an example of an SMRA where bidders were not able to reach an early agreement.

The success of a demand *expansion* strategy in a CCA is less sensitive to the multi-band complexity as it does not require guessing what one's share in the final allocation will be. It only requires the bidder to bid above value (and possibly to bid for the maximum amount of spectrum that is allowed by the auction rules) in bands where other bidders have predictable demands at moments in the clock phase where it is unlikely that the clock phase will stop.

The more important point in the context of Ofcom's proposal to determine the amount of annual license fees for 900 MHz and 1800 MHz frequencies is whether a CCA is likely to produce relative prices between spectrum bands in line with the real relative values of these spectrum bands, based on bidder valuations. As I have argued before, the market value of spectrum depends on the opportunity cost (the marginal value) of the spectrum. Thus, the relative market value between say 800 MHz and 900 MHz spectrum depends on the relative marginal values of spectrum in the 800 MHz and 900 MHz band. It is essential for Ofcom's proposal that the relative prices for 800 MHz and 900 MHz spectrum from the CCA (however they are determined – see also section 5) are in line with the relative marginal values of spectrum in the 800 MHz and 900 MHz band (even if absolute prices may have been distorted). If there is no reason to believe that a CCA gets the relative prices right in general, then one should not rely to a large extent on CCAs to determine the relative value of 900



MHz spectrum relative to 800 MHz spectrum without going into the details of the specific auction designs and the specific market circumstances arguing that, in that particular context, there was limited incentives for or evidence of strategic bidding.

The example below, based on a two-band structure, illustrates that a CCA may produce relative (clock) prices that are not proportional to bidders' relative values of the two spectrum bands. The example combines the observations of the previous section as to when strategic demand expansion is likely to arise in a CCA and when it is not. If there is predictable and irreducible demand in one band and unpredictable demand in another band, prices may be driven up only in the first band, but not in the second.

**Example 2.** *How relative prices in a CCA may be distorted by rational strategic bidding*

*Let there be two bidders and two spectrum bands with two identical units in each band. For simplicity, let us assume that the valuations for packages are additive across bands so that I only have to consider the bidders' valuations per band.<sup>23</sup> In the first band, valuations are (more) predictable as in the example of the previous section: bidder  $i$ 's valuation for one unit is  $v_i(1)$ ,  $i = 1, 2$ , and for two blocks is  $v_i(2) = v_i(1) + \varepsilon_i$ , where it is known that  $v_i(1) > 10$ , and  $\varepsilon_i$  is (uniformly) distributed over  $[0, 10]$  and both  $v_i(1)$  and  $\varepsilon_i$  are private information to bidder  $i$ . In the second band, valuations are unpredictable: bidder  $i$ 's marginal valuation for the first unit is  $\mu_i(1)$ ,  $i = 1, 2$ , while for the second unit it is  $\mu_i(2)$ . To make values across bands comparable,  $\mu_i(1)$  is (uniformly) distributed over  $[0, 20]$ , while  $\mu_i(2)$  is uniformly distributed over  $[0, \mu_i(1)]$ .<sup>24</sup>*

*I analyse this auction for the case where it so happens that  $\max \mu_i(2) < \min \mu_i(1)$  so that if bidders bid truthfully in the second band, both bidders obtain one unit. Consider first bidding in these bands as if there were separate auctions for each band. In the previous Section, I considered the case of a CCA for the first band only and concluded that an*

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<sup>23</sup> Additional types of price driving strategies may arise if there are synergies across bands. The core message of the example does not depend on the assumption of additive valuations.

<sup>24</sup> This last assumption guarantees that there are decreasing marginal values for each of the bidders, which makes the bidding process easier.

equilibrium exists where each bidder gets one unit for a price of 10, which is greater than the market value, based on opportunity cost pricing, which equals  $\max(\varepsilon_1, \varepsilon_2)$ .<sup>25</sup>

If the second band would be auctioned separately, bidders would not engage in strategic demand expansion and will not bid above their marginal values.<sup>26</sup> The reason is that if bidder  $i$  bids for two units up to a price  $p > \mu_i(2)$  it may well be that the marginal value for the first unit of the other bidder is such that  $p > \mu_j(1) > \mu_i(2)$ . If bidder  $i$  believes that there is some chance that this will be the case, then he will have to pay more for the second unit than his marginal valuation (if bidder  $j$  bids truthfully) and it would be better not to overbid. Thus, if bidders care much more about their own intrinsic surplus than about raising rivals' cost, they will not engage in strategic demand expansion in the auction for this band. Bidding truthfully, both bidders win one unit and bidder 1 pays  $\mu_2(2)$ , while bidder 2 pays  $\mu_1(2)$ . The CCA price in this case equals the market value.

Given the above, it is easy to see that the ratio of prices between band 1 and band 2 is given by  $\frac{10}{\frac{\mu_1(2)+\mu_2(2)}{2}}$ , while the ratio of relevant marginal values (market values) equals  $\frac{\max(\varepsilon_1, \varepsilon_2)}{\frac{\mu_1(2)+\mu_2(2)}{2}}$ . It follows that if  $\max(\varepsilon_1, \varepsilon_2) < 10$ , there is no reason to believe that the relative CCA prices are in line with the relative market values in the two bands.

To complete the argument, I demonstrate that the bidding behaviour in a multi-band auction will be such that the asymmetric bidding behaviour in two separate auctions materializes. Below I set out a simple but realistic process for the development of the bidding process in the clock phase and the supplementary rounds from which no bidder has any incentive for a unilateral deviation. I do this under the assumption that the eligibility points for the two bands are identical and the individual bids are not disclosed during the clock phase (as was the case in a large part of the Austrian auction).<sup>27</sup>

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<sup>25</sup> One could also argue that one should take  $(\varepsilon_1 + \varepsilon_2)/2$  as market value as, when bidders bid truthfully in a CCA, one bidder would have to pay  $\varepsilon_1$ , while the other has to pay  $\varepsilon_2$ .

<sup>26</sup> This follows from the assumption of lexicographic preferences. In practice, most bidders would be willing to take on a small risk of overpaying if it means that they could make rivals pay a lot more. This trade-off is absent under lexicographic preferences.

<sup>27</sup> If players can infer individual demands, the argument is slightly different, but the substance of the argument is not affected.

*Consider a situation where  $\mu_2(1)$  and  $\mu_2(2)$  are relatively small. When prices in the two bands are initially small, both bidders may bid for two units in each band, and prices in both bands will rise. When their respective marginal valuations for the second units in the second band are reached, they reduce their demand to one unit and price increases will stop in this band. At this point, the total demand in the first band still equals four units and prices in this band will continue to increase until the price reaches 10 and both bidders reduce their demand to one unit and the clock phase stops. In the supplementary round, bidders bid on all possible packages as follows: they bid  $\mu_i(1)$  for one unit in the second band,  $\mu_i(1) + \mu_i(2)$  for two units in the second band only, they bid  $v_i(1)$  for one unit in the first band and  $v_i(1) + \varepsilon_i$  for two units in the first band, and in addition any combination of these bids for packages consisting of units in both bands.*

*It is not difficult to see that the above behaviour forms an equilibrium in the CCA. The described behaviour is based on bidding marginal value, but it drives up prices in bands where there is no perceived risk of acquiring more spectrum than desired at the relevant clock price. As the bids in the clock phase remain valid bids in the winner determination algorithm, it is risky to raise rivals' bids in the second band as it may be that the competitor has low values, in which case the price driving bidder acquires both units and has to pay more than marginal value. Thus, each bidder only raises prices in one band and relative prices emerge that do not reflect relative market value for the spectrum.*

To conclude this Section, there is no good reason to believe that in a multi-band auction, a CCA is more likely than other auction formats to get the relative prices right in the sense that the relative (final clock round) prices between different bands reflect the relative market value, measured by the ratio of marginal values of spectrum in the different bands. The example uses the idea that demand in certain bands can be much more predictable than in other bands (for example when one band is a legacy band and the other is spectrum that has become available for telecommunications purposes for the first time in history). In addition, this Section has made an argument that there are conditions in a multi-band auction where strategic demand reduction strategies in an SMRA are more difficult to be successfully

implemented, whereas strategic demand expansion strategies in a CCA are not much affected by the multi-band context.

#### **4. Aspects of the 2013 Austrian multi-band auction**

In 2013, the Austrian regulator RTR ran a multi-band CCA for the award of spectrum in three bands: 800 MHz, 900 MHz and 1800 MHz. The auction resulted in very high prices on a price per MHz pop basis, as demonstrated in the statistical analysis produced by NERA, which shows the Austrian prices are a clear outlier in comparison to other European telecom auctions.<sup>28</sup> Using a linear reference pricing<sup>29</sup> (LRP) methodology, the price ratios for both the 900 MHz and 1800 MHz bands relative to the 800 MHz band, turned out to be also exceptionally high in comparison with other European countries (see Tables A8.1 and A8.2 of the Ofcom February 2015 consultation). Ofcom, however, asserts that the Austrian (relative) prices are a good benchmark for UK prices and includes Austria, together with Ireland, in the top tier of countries for best and most reliable comparisons. Based on the arguments in the previous sections, I conclude in this Section that, in the absence of knowledge of the full bid history of the Austrian auction, the conclusion that Austria is a Tier 1 benchmark is unsound. This is because there is clear evidence that bidders had strong incentives for strategic bidding behaviour, that these incentives varied across bands, and that the final price outcome is consistent with bidders having acted on these incentives.

The analysis in this section addresses the following supply and demand issues, specific to the Austrian auction:

- (i) the choice of very light spectrum caps, in combination with the fact that only three incumbents participated in the auction;

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<sup>28</sup> Falk, J., Marsden R., and Pike, J., NERA Economic Consulting (2014), Review of country benchmarks used for setting lump sum values for UK 900 MHz and 1800 MHz, submitted to Ofcom by Telefonica UK and available at: [http://stakeholders.ofcom.org.uk/binaries/consultations/annual-licence-fees-900-MHz-1800-MHz/responses/Telefonica\\_Annex\\_-\\_Review\\_of\\_country\\_benchmarks.pdf](http://stakeholders.ofcom.org.uk/binaries/consultations/annual-licence-fees-900-MHz-1800-MHz/responses/Telefonica_Annex_-_Review_of_country_benchmarks.pdf)

<sup>29</sup> LRP is a methodology developed by DotEcon for Ofcom which uses bid data and price outcomes to impute a proxy for the market clearing price for each band in a multi-band CCA. Although the RTR did not release bid data for the Austrian auction, it did compute LRP in a response to a request for such information from Ofcom.

- (ii) the choice of not providing bidders with any information regarding total demand by rival bidders during the initial phase of the clock round;
- (iii) the auctioning of the new 800 MHz spectrum band together with two legacy bands (900 MHz and 1800 MHz spectrum) in combination with the presence of incumbent bidders with predictable asymmetric valuations; and
- (iv) the presence of excess supply in the last clock round.

Before proceeding to these four points, it is important to note that after raising revenues of 2 billion Euros for a country of 8 million inhabitants, the Austrian regulator RTR felt compelled to reveal a qualitative overview of the bidding behaviour of the three participating bidders in a press release (see, <https://www.rtr.at/en/pr/PI28102013TK>).<sup>30</sup> In that press statement, the Austrian regulator seems to argue that, in the supplementary round, bidders made many bids on very large packages that they knew they were unlikely to win, and that these bids were effective in raising the prices that other bidders had to pay:

*“The three bidders actually submitted a total of more than 4,000 supplementary bids. More than 65% of these supplementary bids were submitted for the largest permissible combinations of frequency blocks, with a share of some 50% of available frequencies. In addition, the bidders utilized almost to the full the price limits that had applied to these large packages during the sealed-bid stage. ... These supplementary bids submitted on large frequency packages had a significant effect on the prices offered by the other bidders. At the same time, such bids generally only have a marginal likelihood of winning out in the end. If these bids for very large numbers of frequencies had been ignored when determining the winners and prices, the revenue from the auction would have settled at a level of about EUR 1 billion”.*

Thus, the press release states that there are clear indicators of strategic bidding. These comments of the Austrian regulator RTR (who has access to all bidding information) seem to

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<sup>30</sup> Because of legal procedures that are still running in different countries, other regulators have been reluctant to provide much information on the bidding behavior during the auction.

be at odds with what Ofcom says in A8.48, namely that there are “*suggestions that there were some bids that bidders knew would not win. However, the evidential basis for bidders having such certainty was unclear to us.*” This section discusses in more detail some aspects of the Austrian design, what is known about the bidding behaviour, and the market circumstances in Austria that point to strategic bidding as a key driver of the high prices and unusual price ratios seen in the Austrian auction outcome.

#### *Light spectrum caps in auction with only three incumbents*

In total, there was a supply of 2x30 MHz spectrum in the 800 MHz band, 2x35 MHz in the 900 MHz band and 2x75 MHz spectrum in the 1800 MHz band. The Austrian regulator RTR chose to impose relatively light spectrum caps. The overall package of each bidder could not exceed 2x70 MHz, while in the Sub-1GHz frequencies no bidder could acquire more than 2x35 MHz. There were also caps in individual bands, of 2x20 MHz at 800 MHz and 2x30 MHz at 900 MHz. Thus, it was possible that two bidders could acquire all available spectrum.

As discussed above, in a CCA each bidder pays the opportunity cost expressed in the losing package bids of the other bidders. With three bidders (and all spectrum being sold), each bidder essentially pays the price that the other two bidders are willing to pay for together acquiring all spectrum, less the bid they expressed for the packages they actually won.

In Section 2, we have seen that in a CCA bidders can raise rivals’ cost without too much risk involved if they have reasonable certainty about the valuations of the other bidder(s). In the Austrian context, the risk of overbidding was considerably mitigated for the following reason: If bidders bid high on large packages of (almost) half the total available spectrum and if these packages became winning packages, they would effectively reduce the market to a duopoly with much higher subsequent revenues in the medium term, and thus, a much higher value of the spectrum.<sup>31,32</sup> Thus, by bidding high on (almost) half the spectrum, bidders either acquire this spectrum (in which case they effectively reduce the market to a duopoly) or they do

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<sup>31</sup> Here is another aspect where the traditional analysis of CCAs is of limited value in understanding bidding in real-world CCAs: the traditional analysis assumes bidders have a value for the spectrum they acquire that is independent of how the rest of the available spectrum is allocated among the other bidders. In real-world auctions, bidders may have a much higher valuation for a certain package if the remaining spectrum is bought by one other bidder than when the remaining spectrum is allocated to two or more other bidders.

<sup>32</sup> In Table A8.1.7, Ofcom tries to show that even if a bidder did not acquire spectrum, it still would have at least 17% of the available paired spectrum in the years up to 2019. However, it is clear that for any operator it would be difficult to build a viable network after 2019 without having any block in the 800, 900 or 1800 MHz band.

acquire it, but then this bid is still effective in raising the price of their competitors. Either way, this is a good outcome from the individual bidder's perspective.<sup>33</sup>

In A8.45 of its February 2015 consultation document, Ofcom states that “*(f)or the allegations of **strategic investment**, we recognised that the overall cap allowed as few as two operators to acquire all spectrum in the auction, and this could potentially raise the prospect of strategic investment for foreclosure. However, unless bidders were able to coordinate successfully, any firm pursuing such a strategy would have to rely on one of its other rivals pursuing the same strategy in order to exclude the third bidder. Otherwise, the first bidder would risk paying more than its intrinsic valuation for spectrum without achieving its strategic objective. We noted that, in practice, the available spectrum, including the sub-1 GHz spectrum, was not won by two bidders, so such a foreclosure strategy either was not attempted, or did not succeed.*”

To better understand how strategic investment or foreclosure interacts with price driving strategies and to see why, contrary to what Ofcom claims, without being successful in foreclosing the market, the strategy may still have been pursued and raised rivals' cost, consider the following simple one band example. Suppose there are three bidders and six units to be auctioned with a spectrum cap of three units for each bidder. Bidders only have an intrinsic value for two units, and do not have an intrinsic value for one or three units; however, each bidder has a high strategic value of blocking one bidder from acquiring 2 lots. Bidders expect the outcome to be that all win two units. They know, however, that what others pay is determined by their marginal bid for the third unit. Bidding for one unit does not make much sense in this example: bidders do not have an intrinsic value for it and if they indeed will acquire two units, a bid on one unit will not help to raise the price rivals' pay. Consider then a bid on three units. Each bidder knows that without someone winning one unit or less, they will not win three units. This bid is thus mainly made to raise rivals' cost. If each bidder makes a marginal bid for the third unit equal to half of the valuation for two units, then

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<sup>33</sup> Later on in this Section, I show in the context of an example how, in a CCA with three bidders, bidders can bid relatively high on a very large package (that will only be winning if the market reduces to a duopoly) without bidding too high on a somewhat smaller, but still large, package (that is still compatible with a triopoly). In this way bidders can guard themselves against winning a larger package they do not want to acquire without restricting the market to a duopoly.

each of them maximally raises the price of the other bidders. However, doing so is somewhat risky as the precise valuation for two units of the other bidders is unknown: if one tries to raise the price of the other bidder too much, a bidder may win three units rather than two. It is clear in this case, however, that (given the low likelihood that bidders make bids on one unit) if one bidder wins three units, one other bidder also wins three units thereby foreclosing the third bidder. The additional value these two winning bidders obtain from foreclosure compensates for the risk of raising the bid on three units and obtaining something they otherwise do not value. Thus, even if unsuccessful in foreclosing the market, the possibility of doing so makes it much more attractive to raise rivals' cost. Among other things, it is this important strategic consideration that is lacking in Ofcom's analysis of the Austrian auction.<sup>34</sup>

Thus, the combination of three active bidders in the auction and light spectrum caps, allowing two bidders to acquire all the spectrum, means that bidders can considerably raise the prices of the two rival bidders without much risk of winning packages they do not value.

*No information regarding total demand by rival bidders in in initial clock phase*

The Austrian regulator decided not to provide information on aggregate demand in the initial phase of the clock stage of the auction. This followed the 2010 Austrian 2.6 GHz auction design when a dominant objective of the regulator was to prevent collusion amongst the bidders.<sup>35</sup> I discussed above (see footnote 21) the possibility that if bidders in a CCA are informed about each other's demands in the clock phase, they may attempt to engage in strategic demand reduction by reducing demands in small steps. By not providing this information, it is impossible for firms to engage into this form of tacit collusion in a CCA.

Given that total demand was not observed, and given that bidders knew that their bids on large packages that (almost) use the maximum spectrum caps determine the prices rivals pay, they knew that the only way to avoid ending up to be the perceived loser of the Austrian

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<sup>34</sup> This also explain why half of the bids in the supplementary round where on very large packages, as the Austrian regulator RTR revealed in its press release.

<sup>35</sup> Klemperer (2002a,b) writes that there were indications that the UMTS auction outcome in Austria was the result of collusion between bidders



auction was to continue to bid for the maximum spectrum caps for many clock rounds.<sup>36</sup> This behaviour remained unnoticed as no information about demand was revealed. By the time RTR changed the rules and revealed information about aggregate demand, it probably was already clear to the bidders that their competitors were not much restricted in the supplementary round in their ability to raise rivals' cost.

Thus, the fact that no information on aggregate demand was revealed in the first phase of the auction likely increased the incentive for bidders to engage in price driving strategies.

*The combination of new and legacy spectrum bands with clearly asymmetric bidders*

In the previous section, I argued that relative prices for different spectrum bands can be significantly distorted if bidders have predictable demand in one band, but not in another. It is clear that the Austrian multi-band auction satisfies this condition. RTR had decided to auction new 800 MHz spectrum together with the 900 MHz and 1800 MHz bands that were already in use. Moreover, some blocks in these 900 MHz and 1800 MHz bands had different starting dates as existing licenses had different expiration dates. To serve their current customers, it was essential that incumbent spectrum holders would re-acquire most of their holdings in the 900 MHz band and would acquire at least 2x20 MHz in the 1800 MHz band.

In terms of our previous analysis, the 900 MHz and 1800 MHz frequency bands are legacy bands where competitors' demand is much better predictable than in the new 800 MHz band. Thus, these two bands served the purpose of raising rivals' cost much better than the 800 MHz band.

In its February 2015 consultation band, Ofcom comments as follows: "*An effect on the relative values was, however, being suggested in the allegation of price driving specifically in 900 MHz and 1800 MHz. In practice each of the three operators gained, and lost, some 900 MHz or 1800 MHz spectrum compared to their holdings before the auction (H3G and T-Mobile each lost some 1800 MHz spectrum, while Telekom Austria lost some 900 MHz). This*

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<sup>36</sup> Given the outcome of the 2012 Swiss auction, at the time of the Austrian auction all bidders knew the risk of ending the auction with paying much more than rival bidders.

*meant that an expectation, before the auction, that an operator would outbid rivals for all of the spectrum it previously held would not have been borne out by the results of the auction, and a bidder who followed a price driving strategy based on such an expectation would have risked winning spectrum at prices above its value of that spectrum. “*

This Ofcom statement does not, however, take the following important observations into account. First, in the final allocation, Telekom Austria acquired the maximum amount of spectrum in the sub 1 GHz frequencies that was allowed by the spectrum caps. The fact that Telekom Austria lost a small amount of 900 MHz is easily explained by the fact they were hitting the spectrum cap and were willing to relinquish one block of 900 MHz in return for getting the particularly attractive bundle of 2x20 MHz in the 800 MHz band, which would also secure that only one rival could acquire 2x10 MHz in that band.

Second, given that Telekom Austria hit the sub 1 GHz spectrum cap, one of the most plausible ways to make sure others had to pay not less in comparative terms for their spectrum, was to bid high on more than 2x20 MHz in the 1800 MHz band. Whether Telekom Austria won “spectrum at a price above its value of that spectrum” is difficult to say, but it is likely that the price in the 1800 MHz band is partly determined by Telekom’s (defensive) desire not to let other bidders acquire their spectrum at a much lower price. In addition, all bidders could foresee that their competitors would ensure to get at least 2x40 MHz in aggregate in the 1800 MHz band so that given the availability of 2x75, any bidder could safely submit bids for packages including 2x40 MHz in the 1800 MHz band.

Third, given the asymmetric spectrum holdings in the 900 MHz band before the auction, it was predictable that Telekom Austria and T-Mobile Austria each would need to acquire at least two blocks in the 900 MHz band to service legacy customers. Given the supply of seven blocks in this band, H3G could reasonably infer that it would not acquire four 900 MHz blocks even if it bid high on packages including four such blocks. It also knew that high bids on such packages would likely be used in the price determination algorithm to determine the prices their competitors had to pay for their winning packages. Therefore, the combination of

available supply in the 900 MHz band and the predictable demand by Telekom Austria and T-Mobile Austria for at least two blocks each is such that H3G had the possibility and the incentive to raise prices in the 900 MHz band.

Thus, I conclude that the results of the auction and the relatively high prices in the 900 MHz and 1800 MHz bands, in comparison with the price in the 800 MHz band, is likely the result of predictable demand in the 900 MHz and 1800 MHz band, in combination with the binding spectrum caps in the sub 1 GHz bands and the desire of bidders not to let competitors acquire spectrum at a lower relative price.

#### *Excess supply in the final clock round*

Telekom Austria has released some information in the public domain from which it is clear that the clock round ended with excess supply.<sup>37</sup> In A8.51, Ofcom considers this as evidence for its claim that bidders may have considered price driving strategies to be too risky to entertain. A similar statement is made in A8.52b. The example below gives a very different interpretation of excess supply. Although Ofcom is correct in stating that the bids in the supplementary round usually determine the final prices to be paid, it is usually the bidding behaviour in the clock phase that lays down the foundation for the possibility of raising rivals' cost. The example shows how strategic behaviour in the clock phase leads to the clock ending with excess supply and how bidders can still raise rivals' cost in the supplementary round without excessive risk taking.

#### **Example 3.** *Excess supply in the clock phase of a CCA and raising rivals' cost*

*Suppose that there are three bidders (like in the 2013 multi-band Austrian auction) and four identical blocks to be auctioned.<sup>38</sup> The bidders' valuations (and their knowledge about other bidders' valuations) are similar to the examples in previous sections: bidder  $i$ 's valuation for*

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<sup>37</sup> See, [http://cdn1.telekomaustria.com/final/de/media/pdf/TKA\\_acquires\\_austrian\\_spectrum\\_Presentation.pdf](http://cdn1.telekomaustria.com/final/de/media/pdf/TKA_acquires_austrian_spectrum_Presentation.pdf).

<sup>38</sup> One could also create examples with six or seven blocks, but that would only add to the complexity of the example (as more cases need to be distinguished) without adding insight into the possibility of raising rivals' cost at minimal risk, even if the clock phase ended with excess supply.

one unit is  $v_i(1)$ , and for two blocks it is  $v_i(2) = v_i(1) + \varepsilon_i$ , where it is common knowledge that  $v_i(1) > 10$ , and  $\varepsilon_i$  is (uniformly) distributed over  $[0, 10]$  and both  $v_i(1)$  and  $\varepsilon_i$  are private information to bidder  $i$ . Unlike the previous sections, each bidder individually knows that their value for one unit is larger than 20, but this is unknown to the others. Also, with three bidders, there are three marginal valuations for the second unit:  $\varepsilon_1, \varepsilon_2$  and  $\varepsilon_3$ . Bidders may also express bids for three or four units, even though they do not have any intrinsic value for them. Without loss of generality, let us assume that the highest value bidder is bidder 3

It is clear that if bidders would bid straightforwardly, then the allocation would be that bidder 3 wins 2 units, while the other two bidders win one unit each and bidder 3 would have to pay  $\varepsilon_1 + \varepsilon_2$ , while bidders 1 and 2 have to pay  $\varepsilon_2$  and  $\varepsilon_1$ , respectively. Thus, the market value of one block of spectrum equals  $(\varepsilon_1 + \varepsilon_2)/2$ .

Consider the following bidding behaviour. In the clock phase, bidders demand:

a) if  $p < 10$ , bidder  $i$  demands  $d_i(p, \varepsilon_i) = 3$  units;

b) if  $p = 10$ , bidder  $i$  demands  $d_i(10, \varepsilon_i) = 1$  unit;

cc) if  $p > 10$ , bidder  $i$  demands  $d_i(p, \varepsilon_i) = 0$  or 1 units, depending on value.

Thus, the clock phase stops at the moment the clock price equals 10. Importantly, bidders demand three units at lower clock prices. As bidders do not value the third unit, this implies that bidders engage in an excessive form of strategic demand expansion. However, given that each bidder knows that the others want at least one unit (and that they are ready to pay significantly for this), they will never be able to acquire three units if they reduce demand at a price that is not higher than 10.<sup>39</sup> In addition, there is a sudden drop in demand at a price of 10: total demand falls from 9 to 3. Finally, the clock phase ends with excess supply, as at the final clock price total demand equals 3, whereas supply equals 4.

In the supplementary round, consider bidders bid  $b_i(1) = v_i(1)$ ,  $b_i(2) = v_i(1) + \varepsilon_i$ , and  $b_i(3) = v_i(1) + \varepsilon_i + 10$ . As each bidder has a valuation  $v_i(1) > 20$ , this supplementary bid is larger than the highest bid of almost 30 expressed on three units in the clock phase. Note that given the development of the clock phase, the relative cap allows these

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<sup>39</sup> Note that this bid strategy is very similar to what I have described above H3G may possibly have chosen in the 900 MHz band in the Austrian auction: excessively expand demand in the clock phase up until prices at which you know it is very unlikely you will acquire that package.

supplementary bids. All bidders are active for one unit in the last clock round and are allowed to bid value for one unit in the supplementary round. The only constraint the relative cap imposes on the marginal bids on the second and third unit is that they are not larger than 10. This is clearly satisfied by these supplementary round bids.<sup>40</sup>

If bidders submit these clock round and supplementary bids, the outcome is such that the bidder with the highest  $\varepsilon_i$  wins two units, while the others win one unit each. Bidders 1 and 2 that win one unit pay 10 (as this is the highest marginal non-winning bid, namely what the bidder 3 expressed to be willing to pay for his third unit). Bidder 3 pays  $10 + \max(\varepsilon_1, \varepsilon_2)$ . It is clear that these prices can be much higher than the prices under straightforward bidding.

To see that the proposed behaviour can be the realization of bidders choosing equilibrium strategies, I make the following observations.<sup>41</sup> First, no bidder wants to deviate in the supplementary phase. By expressing a marginal bid of more than 10 on the third unit, or by expressing a marginal bid above marginal value on the second unit, a bidder risks changing the allocation and winning additional units at a marginal price above marginal value. As the incentive to raise rivals' cost is only of secondary importance to bidders, with these supplementary bids bidders maximally drive up competitors' prices without risking to change the allocation. Second, no bidder wants to change behaviour in the clock phase. This is easiest to see in case bidders are not informed about total demand (as was the case in the first part of the clock phase in the Austrian auction). Given the behaviour of the others, an individual bidder cannot stop the clock earlier and by reducing demand earlier in the clock phase, he only achieves not to be able to drive up prices of competitors and potentially paying more than a rival for the same outcome. By expanding demand even further, he risks to win a second unit or even two more units at too high a price.

Thus, the bids for the third unit are effective in raising rivals' cost and by bidding for three units in the clock phase until the final clock round price, bidders are maximally able to raise their supplementary bid on three units. In order not to win a second unit at a price

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<sup>40</sup> For different clock round developments, the supplementary bids may need to be adjusted. For example, if a bidder would reduce demand to two units at a clock price of  $\hat{p} < 10$ , then the marginal bid that can be expressed on the third unit cannot be larger than  $\hat{p}$ . If a bidder would demand two units in the last clock round, then he is free to increase the bid on two units, but is restricted to bid for one unit in the supplementary round.

<sup>41</sup> Note that formally, the proposed behaviour does not fully specify the full strategy of each bidder. It is clearly beyond the scope of the current paper to specify a full equilibrium strategy and provide a formal equilibrium analysis.

*above the marginal value for the second unit, in the supplementary round bidders bid marginal value for the second unit. For the third unit they can bid, however, much above their marginal value as they know they will never win three units at these prices. They also do know, however, that their bid for the third unit does determine the price paid by their rivals.<sup>42</sup> Thus, even if there is excess demand at the end of the clock phase, bidders can drive up competitors' prices bidding strategically in the supplementary round without running the risk of winning too large packages at too high prices.*

The example shows how a bidder can bid for very large packages in the clock phase at prices much above their marginal value of the largest unit, without running a large risk to win that package. A bidder may wish to do so to soften the constraints on their supplementary bids imposed by the relative cap rule. In the supplementary round, bidders express a marginal bid equal to marginal value on units they may possibly acquire given their beliefs on other bidders' values. However, they fully raise their bids (until what is allowed by the relative cap rule) on packages they are very unlikely to acquire.

Significantly reducing demand in the last clock round is the only way a bidder can combine a relatively safe bid on a package it could potentially acquire and a very high bid on packages it is very unlikely to acquire. Thus, significant drops in the clock round demands are perfectly in line with strategic bidding.<sup>43</sup> In this way, unlike what Ofcom claims in its February 2015

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<sup>42</sup> The example is extreme in that all bidders drop demand in the clock phase at the same price. This is because the example assumes symmetric expectations of rival values. In real auctions, this is, however, not the case. It is easy to see, however, that the example can easily be adapted to asymmetric expectations. Total demand may then drop more gradually in the clock phase from 9 to 5 units. However, the crucial point is that when one bidder continues to demand three units, the auction price will continue to rise and this bidder may then drop demand to one unit so that again, the clock phase would end with excess supply. Typically, it may be the weakest bidder that continues the clock phase as he will expect the others to have the largest values (and therefore the lowest risk he is bidding above the value of the others). As the example shows, it is not this bidder that wins the units that are in excess supply, but the bidder with the largest marginal value. Applying this logic to the Austrian auction, it is not surprising that Telekom Austria was winning the units in excess supply in the final clock round, but it is likely that one of the other bidders was driving up the prices.

<sup>43</sup> An alternative reason may be that bidders value certain blocks as complements and either want to have all or nothing. Complementarities may well have arisen in this auction, for example in the 800 MHz band where the intrinsic value of two blocks may well be larger than two times the intrinsic value of one block. Similarly, in the 1800 MHz band, it may well be that all bidders had a high value for 2x20 MHz. Given the final outcome, auction prices are driven by bidders' demand for larger packages than what they acquired in the final allocation. It is less likely that complementarities also played a major role for these very large packages.

consultation document, they can raise rivals' cost at little risk even if the clock phase ended with excess supply.

### *Conclusion*

In comparison with other European countries, the 2013 Austrian multi-band auction resulted in high absolute prices and in high relative prices for the 900 MHz and the 1800 MHz bands, in comparison with the price for the 800 MHz band. In this Section, I have reviewed a combination of supply and demand factors that were present in the Austrian auction and that explain these high absolute and relative prices as the result of strategic bidding:

- (i) There were very light spectrum caps, which in combination with the fact that only three incumbents participated in the auction, meant that two of the three bidders could acquire all spectrum. This significantly reduced the risk of bidding high on large packages as when these large packages would be winning, the market could potentially be reduced in the longer run to a duopoly market with higher profits for the remaining two players;
- (ii) The choice of not providing bidders during the initial phase of the clock round with information regarding total demand meant that the only way to avoid ending up being perceived as the loser of the Austrian auction was to continue to bid at or close to the maximum spectrum caps for many clock rounds. This enabled bidders to bid high on large packages in the supplementary round.
- (iii) The auctioning of the new 800 MHz spectrum band together with two legacy bands (900 MHz and 1800 MHz spectrum) in combination with predictable demand in the legacy bands resulted in price driving strategies especially in the legacy bands, resulting in high relative prices in these bands. In particular, in the 900 MHz band, H3G could predict that the other two incumbents would want to acquire at least two blocks each, implying that bidding high on packages with four 900 MHz blocks would be a riskless strategy to drive up prices in this band. In addition, the predictable demand by all bidders of 2x20 MHz in the 1800 MHz band, was a likely cause of high prices in this band as well.
- (iv) The presence of excess supply in the last clock round also indicates the wish of some of the bidders to raise rivals' cost without running the risk of acquiring large packages. As Example 3 shows, significantly reducing demand late in the auction enables bidders to make a combination of bids in the supplementary

round with very high bids on very large packages and bids that reflect marginal valuations on packages that a bidder potentially could acquire.

## **5. CCAs only produce per bidder prices, not per unit or per band prices**

A further relevant issue is that even in a single band auction, a CCA does not result in a clear price per unit of spectrum. This is implicit in example 3 of the previous section, but it is good to make that point more explicit. In that example, the highest value bidder (which I termed bidder 3) pays  $10 + \max(\varepsilon_1, \varepsilon_2)$  for two units, while the other two bidders pay 10 for one unit. If  $\max(\varepsilon_1, \varepsilon_2)$  is very small, which it well may be, then bidder 3 pays a price per unit that is almost half of the price paid by the other two bidders. The reason for this is that, in a CCA, each bidder pays an individualised price that is based on the opportunity cost expressed in the bids of other bidders. As these expressed opportunity costs may well differ (as was also observed in the 2012 Swiss auction<sup>44</sup>), prices per unit may differ widely across bidders.

This is unlikely to be the case in an SMRA, where bidders typically overbid others with the minimal bid increment (or make only small deviations from this) so that when the auction stops, the prices bidders pay for different units in the same band typically do not differ more than the minimum bid increment.

This point gets exacerbated when one considers multi-band auctions. In a multi-band auction, a CCA does not produce prices per band, but only tells bidders the package they have won and the price they have to pay for that package. An SMRA is different in that bidders know exactly the price they have paid for each and every frequency block they have acquired. As, in order to determine annual service fees per frequency band, Ofcom needs to establish auction prices per band, it determines for each CCA in Europe hypothetical prices per frequency band using the LRP methodology. It is important to note that these are imputed prices, not realized prices, and it is unclear whether the bidders would have liked to acquire more or less

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<sup>44</sup> See BAKOM (2011, 2012).



spectrum if they had to express their true demand for each band at these prices. Thus, even if bidders would bid according to their true valuations and do not engage in strategic bidding, there is no guarantee that LRPs are equal to bidders' opportunity costs and are the appropriate measure of market value. Under the same condition on bidders' behaviour, SMRA prices do reflect opportunity cost.

## **6. Conclusion**

A general claim that the CCA format encourages straightforward bidding is flawed. Just like other multi-unit, multi-round formats, such as the SMRA, the conditions under which bidders in a CCA have good incentives to bid straightforwardly may be absent. It is questionable whether bidders in real world auctions bid straightforwardly as they may well have incentives to raise rivals' cost (if they can do so without raising the price they have to pay themselves for the spectrum they acquire).

The circumstances in which CCAs are most vulnerable to strategic bidding and resulting price distortion are similar to those in the SMRA, i.e. situations where there are only a small number of bidders, each having a predictable demand for spectrum. Such situations are common in spectrum auctions, where European markets typically only have 3-4 incumbents and entry costs are high, especially when legacy bands are auctioned and the minimum demand of certain incumbents is predictable.

Whereas the SMRA is particularly vulnerable to demand reduction, the CCA is more vulnerable to demand expansion. This is because prices in a CCA differ across bidders based on the opportunity cost that is expressed through the bids of rival bidders for packages that are larger than what they acquire after the supplementary round. Successful price driving strategies have no impact on the perpetrator. In contrast, price driving strategies in an SMRA typically will also hurt the bidder himself, at least when the bidder also acquires frequencies in spectrum blocks where he has driven up the auction prices.

In a multi-band context, incentives for over-bidding do not apply uniformly across bands, but rather are highly sensitive to bidder assumptions about rival demand. Gross distortions in the price ratios across bands are possible if demand in some bands is more predictable than others, a situation that is particularly important in multi-band auctions that include a combination of new spectrum and legacy bands.

I also considered the 2013 Austrian multi-band auction. Although no bid data is publicly available, it is not difficult to see that prices were unusually high and the price ratio exceptionally weighted towards the legacy 900 MHz and 1800 MHz frequency bands, versus the new 800 MHz spectrum band, when compared to other European auctions. At the same time, I also observed that the factors identified in this paper as being associated with over-bidding and price distortion across bands were present in this auction. It is very likely that relative prices were distorted by strategic bidding. Accordingly, Ofcom's designation of Austrian LRP prices as a Tier 1 benchmark for the UK is not justified as there is no good reason to believe that these prices reflect the relative market value of spectrum in the different spectrum blocks.

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