Analysis of the Efficiency of BT's Regulated Operations.

19 September 2013

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Executive Summary

BT has commissioned Deloitte to consider the comparative efficiency and frontier shift of a sample of European and non-European telecom operators, including BT. This Final Report (hereafter the 2013 study) builds upon and improves the previous studies undertaken by Deloitte into BT’s comparative efficiency\(^1\), where the relative efficiency of BT has been compared to the US Local Exchange Carriers (LECs) (see the 2008, 2009 and 2010 studies) and European incumbent operators (see the 2012 study).

The 2012 study analysed the efficiency of a set of European (incumbent) companies, showing the relation between total costs and a mix of statistically significant input and output variables such as total switched lines, switched minutes, leased lines and ADSL lines times the relative bandwidth and a time trend.

The 2013 study seeks to improve upon the reliability of the analysis presented in the 2012 study by:

- Increasing the number of operators within the dataset and extending the time period for further year. By increasing the time series, for example, the nominal frontier shift of cost over time can be estimated based on greater use of information.

- Using the additional degrees of freedom to adopt a panel based stochastic frontier approach to the econometric estimation\(^2\) undertaken. The panel frontier approach allows for a company-specific efficiency in both a time varying and time invariant efficiency settings.

- Allowing the exploitation of the full explanatory power of the variables available for analysis and the application of further diagnostic tests to more fully consider issues of multicollinearity, time- or year-specific effects and heteroskedasticity.

An increase in the number of comparator operators has been necessary to extend the group to include non-European companies, e.g. Etisalat, Telkom South Africa. This has two implications for the study:

- As the free movement of goods, capital, service and people is not guaranteed, all costs need to be comparable in a unique currency metric. Consistently with previous studies, assets and depreciation figures first have been converted to a current cost accounting (CCA) basis in order to produce a homogeneous dataset, and then converted into UK

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\(^1\) Deloitte have conducted a number of previous studies for BT, namely: “The Efficiency of BT’s Network Operations - 2008” (hereafter the 2008 study), “Further Analysis of the Efficiency of BT’s Network Operations - 2009” (hereafter the 2009 study), “The Efficiency of BT’s Network Operation - 2010” (hereafter the 2010 study) and “Analysis of the Efficiency of BT’s Regulated Operation – 2012” (hereafter the 2012 study).

\(^2\) Specifically moving away from the pooled based methods implemented in the 2012 study.
Pounds Sterling by performing a Purchase Power Parity (PPP) adjustment, in order to ensure the exact comparability of costs.

- In order to address fundamental differences in the cost of labour and its impact upon the optimum production function with respect to labour and capital, a proxy variable explaining the labour differences is included in the explanatory variable set considered.

Turning to the econometric methodology adopted, the 2013 study improves upon the 2012 study by adopting a panel data Stochastic Frontier approach which has been chosen in a way to meet the new challenges of a broader cross-sectional and historical analysis. This modelling framework is considered to provide a more robust modelling structure as well as allowing a number of sensitivity tests to be conducted to validate the results and to test assumptions. In particular, variables have been tested for multicollinearity problems and heteroskedasticity characteristics, alongside a focus on the role of time- and year-specific features. In summary, the results of the analysis show this model specification is able to overcome data quality and availability problems and deliver coherent and robust results.

This report provides a multi-dimensional measure of comparative inefficiency by analysing the company-specific inefficiency both in a time invariant and time varying setting. Among the number of techniques and methods that have been developed to assess technical and/or cost inefficiency, the approach followed is the most appropriate as it allows the investigation to take into account company-specific inefficiency component both in a cross-sectional and a time series fashion.

The major findings of this enhanced analysis are that:

- The use of Stochastic Frontier Panel models provides an appropriate interpretation of the frontier composed by the individual performances of the operators and it estimates the impact of the explanatory variables, both in a time-varying and time invariant setting. Generally, the two specifications deliver similar parameter estimates and efficiency rankings, providing robust and consistent findings.

- BT is the most efficient operator within the sample, suggesting BT does not currently have a catch-up efficiency to the most efficient operators. This result is consistent to the 2012 study of European incumbents.

- The time trend variable is not statistically significant, implying that there is not a shift in the cost frontier through time in nominal terms. Again, this result is consistent with the 2012 study of European incumbents and the previous studies considering the US LECs.

Finally, the results show a high level of regularity in the inefficiency rankings found. Importantly, in both a time varying and time invariant setting, BT and Operator D rank respectively first and

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3 The importance of regional labour differences in analysing total costs has been highlighted also by a recent report prepared by Frontier Economics for OFGEM (see references).

4 Current more macro estimates of Total Factor Productivity (TFP) growth also indicate relatively shifts. For example, the EU KLEMS initiative estimates EU telecommunications and postal sector TFP growth of 1.9% for 2011.
second in any year and model specification adopted, while operators G and B rank always at the bottom.
1 Introduction

The aim of the study is to analyse the comparative efficiency of a sample of European and non-European incumbent telecom operators, including BT.

The 2013 Analysis of the Efficiency of BT’s Regulated Operations builds upon and improves the previous studies undertaken by Deloitte into BT’s comparative efficiency. Several similar studies have been conducted in previous years: “The Efficiency of BT’s Network Operations - 2008” (hereafter the 2008 study), “Further Analysis of the Efficiency of BT’s Network Operations - 2009” (hereafter the 2009 study), “The Efficiency of BT’s Network Operation - 2010” (hereafter the 2010 study) and “Analysis of the Efficiency of BT’s Regulated Operation - 2012” (hereafter the 2012 study).

In particular, the 2012 study analysed the relative efficiency of six European telecom operators, including BT, by means of Corrected Ordinary Least Squares and Stochastic Frontier Analysis on a pooled dataset of thirty observations (six companies over 5 years). Ofcom has expressed some concerns about the robustness of the results due to the limited size of the observations, which implies methodologies that pool the data as if they were observations of a unique company.

This study seeks to address the specific concerns raised by Ofcom and to improve the robustness of the analysis and, in doing so, consider the on-going validation the results.

For this purpose, as a first step, the dataset has been enhanced and extended both in cross-sectional dimension, by including further telecom companies, and in longitudinal dimension, by updating it to the latest possible record. Specifically, in order to increase the sample size, additional European and non-European companies have been included and available records for 2011 have been added. The limited availability of data provided by the companies forced the construction of an unbalanced panel, raising further challenges in comparing efficiency among firms observed over different time spans. In summary, a panel data Stochastic Frontier Analysis (henceforth SFA) approach was adopted which treats every company as a separate entity to be analysed over its specific period of observation.

This report is structured as follows:

- Section 2 introduces and analyses the panel data SFA, highlighting the features of this approach;
- Section 3 explains the construction process of the dataset; and
- Section 4 presents the econometric results obtained with further detailed econometric and statistical outputs presented as an annex.
The panel data Stochastic Frontier Analysis

Stochastic Frontier Analysis has been a significant contribution to the econometric modelling of production and the estimation of technical efficiency of firms. It can be used to assess the relative technical inefficiency of groups of similar firms. Following the duality of production and cost function, this method can fit both production and cost frontiers.

2.1.1 Theoretical background

Assume that a producer has a production function such that the outputs produced $Y_{it}$ are some function of the implemented inputs $X_{it}$ and a vector of corresponding parameters $\beta$. In an environment without error or inefficiency, at time $t$, the $i^{th}$ firm would produce $Y_{it} = f(X_{it}, \beta)$, where $f(\cdot)$ is a (production) function that transforms the inputs $X_{it}$ in output $Y_{it}$ in $\beta$ proportion.

Stochastic Frontier Analysis assumes that each firm's production is affected by some degree of inefficiency that can potentially make the firm produce a quantity lower than the optimal one. A more realistic production takes into account this inefficiency and has the following form:

$$Y_{it} = f(X_{it}, \beta)\xi_{it}$$

(Equation 1)

Where $\xi_{it}$ is the level of inefficiency for a firm $i$ at time $t$ and it lies in the interval $(0,1]$. If $\xi_{it} = 1$ the firm is achieving the optimal output with the technology represented by the production function (full efficiency, or zero inefficiency), while if $\xi_{it} < 1$ the firm is not optimally using the inputs $X_{it}$. Given that outputs are assumed to be strictly positive, the degree of technical inefficiency $\xi_{it}$ is strictly positive.5

A reasonable assumption is also that production is also subject to company-specific random shocks $v_{it}$ over time:

$$Y_{it} = f(X_{it}, \beta)\xi_{it}\exp(v_{it})$$

(Equation 2)

Taking the logarithmic transformation of the above expression6, the production function becomes:

$$\ln(Y_{it}) = \ln(f(X_{it}, \beta)) + \ln(\xi_{it}) + v_{it}$$

(Equation 3)

Assuming that there are $J$ inputs and that the production function is linear in logarithms, by defining $\ln(\xi_{it}) = -u_{it}$, the specification of a stochastic frontier is:

$$\ln(Y_{it}) = \beta_0 + \sum_{j=1}^{k}\beta_j\ln(X_{ijt}) + v_{it} - u_{it}$$

(Equation 4)

5 For $\xi_{it} = 0$ the whole expression in equation 1 is zero, therefore no firm can have full inefficiency/zero efficiency and $\xi_{it} \in (0, 1]$.

6 The logarithmic function is a monotonic transformation that preserves all the properties of the original function.
Finally, assuming that there are Q outputs, Kumbhakar and Lovell (2000) provide a general form for the dual cost function problem:

\[
\ln(C_{it}) = \beta_0 + \sum_{q=1}^{Q} \beta_q \ln(Y_{it}) + \sum_{j=1}^{J} \beta_j \ln(p_{jit}) + v_{it} - su_{it}
\]  
(Equation 5)

Where

- \( C_{it} \) is a variable representing the cost(s) of company \( i \) at time \( t \);
- \( Y_{it} \) are the output quantities produced by company \( i \) at time \( t \);
- \( p_{jit} \) are the input prices of input \( j \) implemented by company \( i \) at time \( t \);
- \( s \) is an indicator function that takes values equal to 1 for production functions, and to -1 for cost functions;
- \( u_{it} \) is a company-specific inefficiency component at time \( t \);
- \( v_{it} \) is a random error specific component.

### 2.1.2 The inefficiency component \( u_{it} \)

Henceforth, this report refers to inefficiency models without loss of generality with respect to the dual concept of efficiency. The estimators produced by a Stochastic Frontier Analysis vary depending on the specification of the company-specific inefficiency term \( u_{it} \). Two modelling choices are possible: a time invariant inefficiency component or a time-varying inefficiency term.

In a time invariant inefficiency model, the inefficiency term is assumed not to vary over time and it is computed in a way that represent the inefficiency level realized by the firm under the individual period of observation. This specification assumes that the time invariant inefficiency component \( (u_{it}) \) is distributed as a truncated normal variable\(^7\); further it assumes that \( u_{it} \) and the exogenous company-specific random shocks \( v_{it} \) are distributed independently of each other and of the covariates in the model. Therefore, this model delivers a unique measure of each company’s inefficiency.

In a time varying-decay inefficiency model, inefficiency is assumed to vary over time toward its base level. The company-specific inefficiency term is expressed as \( u_{it} = \exp(-\eta(t - T_i))u_i \) where \( T_i \) is the last period of the \( i^{th} \) panel, \( \eta \) is the decay parameter to be estimated and, as in the previous specification, \( u_{it} \) and \( v_{it} \) are distributed independently of each other and the covariates in the model where \( u_{i} \sim iid N^* (\mu, \sigma_u^2) \) and \( v_{it} \sim iid N(0, \sigma_v^2) \).

In the last period, when \( t = T_i \), the obtained inefficiency is the base level of inefficiency of firm \( i \). The time-decay parameter \( \eta \) provides the direction of the inefficiency through time: when \( \eta < 0 \) the level of inefficiency increases to the base level, while when \( \eta > 0 \) the level of inefficiency decays toward the base level. Finally, when \( \eta = 0, u_{it} = u_i \) and the model is specified as a time invariant inefficiency model.

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\(^7\) Specifically a time invariant specification assumes that \( u_{it} = u_i \) where \( u_i \sim iid N^* (\mu, \sigma_u^2) \) and \( v_{it} \sim iid N(0, \sigma_v^2) \).
2.1.3 The estimation method and the technical inefficiency $u_{it}$

The most accurate way to estimate a stochastic frontier is through Maximum Likelihood Estimation. From the previous equation, composite error can be expressed as the sum of the (in)efficient component and of the random shocks, i.e. $\varepsilon_{it} = u_{it} + v_{it}$, and transform the expression in:

$$\varepsilon_{it} = \ln(C_{it}) - \beta_0 - \sum_{q=1}^Q \beta_q \ln(Y_{it}) - \sum_{j=1}^I \beta_j \ln(p_{jit})$$  \hspace{1cm} (Equation 6)

Given the assumptions on the distribution of the inefficiency term and the random shocks, the log likelihood function $L$ has the following functional form:

$$\ln L = K - n \ln \sigma + \sum_{i=1}^n \ln \Phi\left(\frac{z_{it}}{\sigma}\right) - \frac{1}{2\sigma^2} \sum_{i=1}^n \varepsilon_{it}^2$$  \hspace{1cm} (Equation 7)

Where:

- $\varepsilon_{it} = u_{it} + v_{it}$ is the composite error;
- $\sigma^2 = \sigma_u^2 + \sigma_v^2$ is the variance of the composite error, obtained as sum the variances of the inefficiency component and of the random shocks;
- $\lambda$ is the ratio of the standard deviation of the components of the composite error, i.e. $\lambda = \frac{\sigma_u}{\sigma_v}$;
- $\Phi(\cdot)$ is the standard Normal cumulative distribution function;
- $n$ is the number of sampled observations;
- $K$ is a constant;

Following Battese and Corra (1977), it is possible to re-parameterise the function in order to capture the relation among the inefficient component and the random shocks, by considering $\gamma = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_v^2}$ as the proportion of variability of the composite error that is due to technical inefficiency.

The log-likelihood final functional form to be estimated is then:

$$\ln L = K - n \ln \sigma + \sum_{i=1}^n \ln\left[1 - \Phi\left(\frac{z_{it}}{\sigma}\right)\right] - \frac{1}{2\sigma^2} \sum_{i=1}^n \varepsilon_{it}^2$$  \hspace{1cm} (Equation 8)

Where $z_{it} = \frac{\varepsilon_{it}}{\sigma} \sqrt{\frac{Y_{it}}{1-\gamma}}$.

Under this form, the log-likelihood function allows for testing the appropriateness of the stochastic frontier estimated. As $\gamma \to 0$ (because either $\sigma_u^2 \to 0$ or $\sigma_v^2 \to \infty$) the overall inefficiency level of the sample under consideration is very low and the model could be also estimated by means of Ordinary Least Squares, because every firm approaches the frontier. Conversely, when $\gamma \to 1$ (because either $\sigma_u^2 \to \infty$ or $\sigma_v^2 \to 0$) the impact of random shocks is very limited and the frontier tends to be almost deterministic with no random noise. The estimation of the parameters of the log-likelihood function varies depending on the assumption on inefficiency concept considered (time varying or invariant inefficiency).

The estimate of the technical inefficiency $TE$ of a stochastic cost frontier is expressed as the expected value of the technical inefficiency given the composite error of the regression:
\[ TE = E\{u_{it}|\epsilon_{it}\} \]  
(Equation 9)

Where \( u_{it} \) and \( v_{it} \) have distributional properties corresponding to the time specification that is estimated and \( \epsilon_{it} = u_{it} + v_{it} \).
3 Data

This section describes the data collected and any methods implemented to build up a complete and reliable dataset that allows a consistent comparative analysis between the European and non-European operators and BT.

3.1 Comparator data

Building upon the data collected for the 2012 study, comparator data collection was based on a clearly defined template, whilst a number of additional clarifications and discussions were held with operators to ensure comparability of data. Nine companies provided data over a period of seven years, from 2005 to 2011. Unfortunately, not all the information sought was available from all operators, meaning that the dataset constructed has an unbalanced panel dataset form. However, given the panel data econometric techniques implemented the econometric results are not affected.

The companies that make part of the sample analysed together with BT are:

- BTC;
- Eircom;
- Etisalat;
- France Telecom;
- Telecom Italia;
- Telefonica Spain;
- Telkom South Africa; and
- Vivacom.

The collected data includes:

- The Gross book value (GBV), Net book value (NBV) and annual depreciation of assets, on an historical cost accounting (HCA) basis and in national currencies;
- Average useful life of assets;
- Operating expenditures;
- Weighted average cost of capital;
- Number of employees;
• Number of switched access lines, defined as aggregate of PSTN lines, ISDN lines measured in channels, and number of unbundled local loop lines (LLU lines);

• Number of leased lines and average leased line bandwidth;

• Percentage of switched access lines that are xDSL enabled;

• Switched minutes;

• Revenues related to core output variables;

• Network data (including sheath and cable length, percentage of total cable which is fibre and number of switches); and

• Fault reports per line.

This data was provided to us under strict confidentiality terms. Deloitte are not able to share this information with BT or third parties, nor show analysis or results that may identify individual companies to an informed reader. In a few cases missing data was extracted from operators’ annual reports, where this was required the data was discussed with the operators concerned to ensure that it was being used appropriately.

Other data was collected for the countries of the comparators:

• Inflation data⁸;

• GDP and GDP per capita⁹; and

• Average xDSL line bandwidth¹⁰.

### 3.2 Cost data

Most of the companies provided historic cost accounting data (HCA), while BT’s network business is regulated on the current cost accounting (CCA) basis. Consistently with previous studies, assets and depreciation figures have been converted to a CCA basis in order to have a homogeneous dataset¹¹. In fact, while under HCA, the reported GBV for similar assets bought at different points in time can differ considerably, depending on the prices of the asset at the time it was bought, under CCA the reported value of an asset depends on the current replacement price of the asset, irrespective of the price at which it was purchased.

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⁸ Source: Eurostat’s Harmonised Price Index;


¹¹ All previous Deloitte studies have undertaken the analysis based on CCA costs.
The performed adjustment consists of the following steps:

- **Calculation of asset age:** For each of the asset categories above and for each operator separately the average age of the asset has been computed according to the following formula:

\[
\text{Average age of asset} = \text{Useful asset life} \times \left(1 - \frac{\text{NBV}}{\text{GBV}}\right)
\]  

(Equation 10)

where NBV is the Net Book Value of the asset, defined as GBV minus accumulated depreciation.

- **Asset price indices:** Have been collected for each asset category, i.e., poles, duct, cable, switches, transmission, power, vehicles, buildings, land and ‘other assets’ (e.g., office supplies and IT equipment). For poles, cable, switches, transmission, vehicles and ‘others’, asset-specific price indices (common for all countries) have been used. For duct and buildings, a country-specific manufacturing labour price index was employed. For power a price index was not readily available, so a country-specific general inflation index was used. Finally, for land, a price index was extracted from the Financial Times.

- **Calculation of net replacement cost (NRC):** The final step in the CCA adjustment of the comparator asset value figures was to adjust the NBV figures for the price changes which had occurred for each asset between the year in which the asset was bought (given by the average asset age figure calculated in the first step) and the current year. Effectively, this adjustment converts NBV to NRC. For example, the NRC figure for duct for company \(i\) in 2007 was calculated as:

\[
NRC_{\text{Duct},i,2007} = NBV_{\text{Duct},i,2007} \times \left(\frac{\text{Price}_{\text{Duct},2007}}{\text{Price}_{\text{Duct},2007-\text{average age}}}\right)
\]  

(Equation 11)

- **A measure of the cost of capital related to each asset category:** Was obtained by multiplying the NRC associated with each category by the weighted average cost of capital (WACC) supplied by the comparators:

\[
\text{CoC}_{\text{Duct},i,2007} = NRC_{\text{Duct},i,2007} \times WACC_{i,2007}
\]  

(Equation 12)

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12 Source: US Bureau of Labor Statistics


15 Source: http://www.ft.com/cms/s/0/4fda2cc8-525a-11dd-9ba7-000077b07658,dwp_uuid=e70ca99e-a4b0-11db-b0ef-0000779e2340.html#axzz1SkcHuWX5
Finally, the total cost of capital for each operator was obtained by summing the costs of capital related to each asset category. The relevant asset categories were those directly related to the network part of the comparator operations. All comparators have been requested to exclude also mobile network costs, costs associated with international call services and costs related to broadcasting.

CCA figures for annual depreciation related to each asset have been consistently computed as follows:

\[
CCA_{Duct,2007} = HC_{Duct,2007} \times \left( \frac{Price_{Duct,2007}}{Price_{Duct,2007-average}} \right) \tag{Equation 13}
\]

Consistent with the approach taken for the calculation of each comparator’s total cost of capital, the total CCA annual depreciation for each comparator was calculated as the sum of the CCA annual depreciation figures over the relevant asset categories defined above.

This approach is consistent with previous efficiency studies that have used the LECs’ data as a benchmark, particularly those previously commissioned by Ofcom and Oftel\(^\text{16}\).

Finally, operating expenditures have been collected from each of the comparators. The relevant cost categories identified in this case were:

- Network-related operating expenditures (e.g. installation, maintenance and repair costs);
- Wages for employees directly related to the Network operations;
- Allocation of wages relating to overheads that are associated with Network activities (e.g. an allocation of wages paid to finance, HR, CEO’s offices etc.); and
- Any other operating expenditure (including a portion of overheads related to the Network operations and not captured by the previous categories).

### 3.3 PPP adjustment

In order to ensure the exact comparison of costs, all comparators’ costs provided in historical national currencies have been converted into Pounds Sterling by performing a Purchase Power Parity (PPP) adjustment as follows:\(^\text{17}\)

\[
Cost_i^\text{currency} = \frac{Cost_i^\text{currency}}{PPP_{\text{index}}^{t-as-a-base}} \tag{Equation 14}
\]

\(^{16}\) Including NERA (2005)

\(^{17}\) PPP conversion rates have been extracted from: [http://stats.oecd.org/Index.aspx?DataSetCode=PPPGDP](http://stats.oecd.org/Index.aspx?DataSetCode=PPPGDP)
This PPP adjustment results in all costs for each operator being expressed on comparable CCA basis denominated in Pounds Sterling.

### 3.4 DSL line and bandwidth data

Deloitte requested all operators provide both the percentage of total switched access lines that are xDSL enabled and a measure of the average xDSL bandwidth\(^{18}\). All operators provided information on the former, but due to confidentiality issues it was not possible to collect information on xDSL bandwidth directly from the operators.

As such, a measure of the average xDSL bandwidth for each operator from publicly available data has been constructed. Determining a robust and consistent source for this information was challenging. Operator-specific data on actual broadband speed for the period 2009-2012 was collected and extrapolated backwards to 2005, using country-specific growth rates of xDSL bandwidth.\(^{19}\) This leads to some uncertainty surrounding the xDSL bandwidth data series, which is taken into account in the model specifications and results discussed in Section 4.2.1.

### 3.5 BT data

Data covering BT’s costs and output data has been collected from BT covering 2005/6 to 2011/12 inclusive.

In common with the comparators, cost, output and network data has been collected and adjustments have been made to facilitate comparability. In the following the next sections provide a breakdown of data sources and any adjustments made.

### 3.6 Cost data

#### 3.6.1 Operating costs

The starting point for the cost data was the total CCA operating costs, defined as the sum of Operating Expenditure (OPEX) and depreciation, in BT’s Financial Statements from 2005/6 to 2010/11. However, several items included required adjustment to ensure comparability to the European operators’ data:

- Payments to other fixed line operators for international calls and to mobile operators: costs related to these activities have been removed from BT data.

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\(^{19}\) Source: [http://www.google.com/publicdata/explore?ds=z8li06k9csels2_&ctyle=1&strail=false&nsele=h&met_y=avg_download_speed&scale_y=lin&ind_y=false&rdim=country&tdim=true&hl=en&dl=en&iconSize=0.5&unisize=0.035#ctyle=l&strail=false&bcs=d&nsele=h&met_y=avg_download_speed&scale_y=lin&ind_y=false&rdim=country&tdim=country:GB&ldim=country&hl=en&dl=en]
- Exceptional costs related to changes in depreciation lives have been excluded. Given that these are exceptional, they should not be included when estimating the actual efficiency of BT.

- Costs related to business rates have been removed. These rates are excluded because of differences in their treatment between the different European countries and UK.

- Adjustments for costs relating to duct were made. From 2009/10 forward, BT implemented a change in the revaluation basis applied to its duct assets. Given the specificity of this change to BT’s CCA revaluation methods, Deloitte replaced BT’s CCA duct valuations with a simple and consistent CCA revaluation for duct. This employed the same CCA revaluation method applied to the comparators and discussed above. BT provided information on the annual HCA capital expenditure associated with duct and also provided the corresponding annual HCA depreciation. Deloitte then performed the conversion into a CCA basis (i.e. from annual HCA depreciation to annual CCA depreciation) using the same indexation procedure as applied to the comparators’ asset costs. This can be summarised as:

\[
C_{\text{CCAOpCosts}}^{\text{new}}_{\text{Total},\text{year}} = C_{\text{CCAOpCosts}}^{\text{old}}_{\text{Total},\text{year}} - C_{\text{CCAdep}}^{\text{BT}}_{\text{Duct},\text{year}} + C_{\text{CCAdep}}^{\text{Deloitte}}_{\text{Duct},\text{year}}
\]

where

\[
C_{\text{CCAdep}}^{\text{Deloitte}}_{\text{Duct},\text{year}} = H_{\text{CCAdep}}^{\text{BT}}_{\text{Duct},\text{year}} \times \left( \frac{\text{Price}_{\text{Duct},\text{year}}}{\text{Price}_{\text{Duct},\text{year} - \text{average age}}} \right)
\]

The above adjustments yield a total CCA operating cost of £4,750 million in 2010/11.

3.6.2 Cost of capital

In addition to CCA operating costs, Deloitte collected data on BT’s cost of capital. To calculate the cost of capital BT provided a breakdown of its assets at NRC. This asset base however corresponded to BT Group thus necessitating the following adjustments:

- As BT Group includes BT’s retail and international operations, all components and activity groups associated with these areas were excluded.

- In common with the CCA operating costs, the standard adjustments for comparability outlined above were made, such as exclusion of costs for interconnection and so on.

- Adjustments for costs relating to duct were made. Consistent with the adjustment for duct performed on BT’s total CCA operating costs, Deloitte adjusted also BT’s NRCs. BT provided information on the portion of its total NRCs and net book value (NBV) associated with duct. Deloitte then performed the conversion into a CCA basis (i.e. from NBV to NRC) using the same procedure applied to the comparators’ asset costs. This can be summarised as:

\[
N_{\text{R}}^{\text{new}}_{\text{Total},\text{year}} = N_{\text{R}}^{\text{old}}_{\text{Total},\text{year}} - N_{\text{R}}^{\text{BT}}_{\text{Duct},\text{year}} + N_{\text{R}}^{\text{Deloitte}}_{\text{Duct},\text{year}}
\]

(Equation 16)
where \[ NRC_{Duct, year}^{Deloitte} = NBV_{Duct, year}^{BT} \times \left( \frac{Price_{Duct, year}^{NBV}}{Price_{Duct, year - average age}} \right) \]

To calculate the total cost of capital from this, BT assigned the remaining components to either being owned by BT Openreach or by BT Wholesale, since these have different regulated WACCs\(^{20}\). Allocating NRC in this way and multiplying by the appropriate WACC yields a total cost of capital of £1,857 million in 2010/11.

### 3.7 Output and network data

Volume data for each of the products was provided within the financial statements and key performance indicators, which were supplemented with additional information from BT.

BT also supplied output and network data, including:

- Length of duct;
- Length of sheath;
- Number of switches;
- Switched minutes;
- Number of switched lines;
- Number of active leased lines; and
- Number of xDSL lines.

Outputs and network data were deemed to be comparable to the comparators’ data, due to the design of the data request.

It has been noted in previous SFA studies, including Deloitte (2010) that the measure of leased lines in 64k bps equivalents is outdated, as higher bandwidth lines become common. Ideally, the leased line variable would take into account the number of lines and bandwidth, since cost is not expected to vary linearly with bandwidth. This would also overcome the problem of leased lines appearing have grown by a very large amount in the last few years, as BT has started to provide some very high bandwidth products. For this reason, leased line bandwidth is included in the data request and in the econometric estimation, rather than a number of leased lines at a defined capacity (historically 64k) as the leased lines variable.

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4 Results

This section presents the main model specification proposed and discusses the results of the econometric comparative efficiency analysis of BT's network operations. The analysis proposed is a panel data stochastic frontier analysis. This econometric approach allows the assessment of company-specific efficiency both in a time varying and time invariant efficiency setting.

4.1 Summary of findings

The model proposed delivers robust and consistent results and the findings include the following:

- The two specifications deliver highly similar parameter estimates and efficiency rankings, providing robust and consistent findings.
- BT is ranked at the top of the sampled companies both in a time varying and time invariant efficiency specification.
- The time trend is not statistically different from zero:
  - In the time invariant efficiency model, the time trend variable capturing cost trends in nominal terms is not statistically significant and it lies in a tight confidence interval close to zero, meaning that there is a 0% nominal frontier shift.
  - In the time-varying efficiency model the time-varying decay parameter is also not statistically significant, strengthening confidence in the time invariant inefficient model specification.

4.2 Model Specification

This section presents the results of several regressions with varying specifications based on a Cobb-Douglas cost function, following the general form of:

\[
\ln(C_{it}) = \beta_0 + \sum_{q=1}^{Q} \beta_q \ln(Y_{it}) + \sum_{j=1}^{L} \beta_j \ln(p_{jt}) + \delta_t + v_{it} + u_{it} \quad \text{(Equation 17)}
\]

Where:
- \( \ln \) is the natural logarithm
- \( i \) represent the individual company observation;
- \( t \) represents the year of the observation;
- \( C_{it} \) is a variable representing the cost(s) of company \( i \) at time \( t \);
- \( Y_{it} \) are the output quantities produced by company \( i \) at time \( t \);
- \( p_{jt} \) are the input prices of input \( j \) implemented by company \( i \) at time \( t \);
- \( u_{it} \) is a company-specific inefficiency component at time \( t \);
- \( v_{it} \) is a purely random error component.

The company-specific inefficiency component plays a crucial role in the model specification.
• In a time invariant inefficiency model it is assumed that that \( u_{it} = u_i \) where \( u_i \sim iid N^*(\mu, \sigma_u^2) \) and \( v_{it} \sim iid N(0, \sigma_v^2) \), i.e. that the company-specific inefficiency component is normally distributed and does not vary over time.

• In a time-varying inefficiency model it is assumed that the company-specific inefficiency component is normally distributed and varies over time with the following distribution:

\[
\begin{align*}
\log u_{it} &= -\eta (t - T_i) u_i \\
\end{align*}
\]

Where:

- \( u_i \sim iid N^*(\mu, \sigma_u^2) \) and \( v_{it} \sim iid N(0, \sigma_v^2) \) as in the time invariant inefficiency specification;
- \( \eta \) is the time decay parameter to be estimated;
- \( t \) is the time period of the observation under consideration;
- \( T_i \) is the last period in which company \( i \) is observed.

Therefore, the company-specific inefficiency component is assumed to be normally distributed and to vary over time. Furthermore, the time trend \( \delta t \) is dropped in order to allow orthogonality between the explanatory variables and the inefficiency component.

4.2.1 Output Variables

The main measures of output are the following:

• number of switched minutes;
• number of switched access lines\(^{21}\);
• number of leased lines aggregate bandwidth;
• number of ADSL lines;

Of these, the number of switched access lines and leased lines appears to be the most significant driver of costs, as shown in Table 1.

\(^{21}\) Where switched access is defined as the aggregate of PSTN lines, ISDN lines measured in channels, and the number of local loop unbundled lines (LLU lines).
### 4.2.2 Network Variables

Two potential variables were considered to reflect the differences in network capacity investments by the operators:

- Logarithm of total leased lines bandwidth, provided by the comparators via a data request; and
- Logarithm of total ADSL bandwidth.

Where the total ADSL bandwidth is variable is calculated as:

\[
ADSL \text{ bandwidth} = (\% \text{ of lines that are xDSL enabled}) \cdot (\text{Lines}) \cdot (\text{Average broadband speed})
\]

The logarithmic transformations of these two variables are highly correlated with each other, with a correlation coefficient of 0.90, and with the output variables above.

Due to the above concerns and the collinearity between the two variables, the variable total ADSL bandwidth has been given preference in the specifications below.

### 4.2.3 Labour Variables

In contrast to the 2012 study, the sample of operators considered in this study is not limited to Europe. This poses an additional challenge since the differences in the level of economic development implies significant differences in labour costs. As such, based on the underlying resource costs, the optimal capital-labour mix for the different providers may vary significantly. This is controlled for by including employees’ average wage as an explanatory variable.

Employees’ average wage can be considered to be a proxy for labour cost differences. The variable is computed by dividing the total wages and salaries paid by the number of employees in PSTN service. As noted in Section 3.3, the nominal variable reflecting wages and salaries has been converted into Pounds Sterling by performing a PPP adjustment. The variable is therefore measured in thousands of Pounds Sterling per employee. This represents a proxy of the labour input price.
4.2.4 Correlation among explanatory variables

The list of variables included in the final specification therefore includes: logarithm of switched lines, logarithm of minutes, logarithm of ADSL lines times the relative bandwidth, logarithm of average wage plus a time trend, when applicable.

Table 2 demonstrates the correlations among explanatory variables. While output variables are highly correlated among themselves, the log of average wage has a low correlation with the other explanatory variables.

Table 2: Correlations between the explanatory variables (40 observations).

<table>
<thead>
<tr>
<th></th>
<th>Log of switched lines</th>
<th>Log of minutes</th>
<th>Log of ADSL lines times bandwidth</th>
<th>Log of average wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of switched lines</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log of minutes</td>
<td>0.90</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log of ADSL lines</td>
<td>0.83</td>
<td>0.82</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>times bandwidth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log of average wage</td>
<td>0.31</td>
<td>0.48</td>
<td>0.36</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Source: Deloitte analysis

Finally, the pairwise correlation among the explanatory variables and the dependent one are presented in Table 3. While the logarithmic transformation of the number of switched lines, of minutes and of ADSL lines times the relative bandwidth are positively and highly correlated with total costs, the average wage is again lowly correlated, and the time trend lowly and negatively correlated.

Table 3: Pairwise correlations between the dependent variable and the explanatory variables (40 observations).

<table>
<thead>
<tr>
<th></th>
<th>Log of total costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of total costs</td>
<td>1.00</td>
</tr>
<tr>
<td>Log of switched lines</td>
<td>0.97</td>
</tr>
<tr>
<td>Log of minutes</td>
<td>0.90</td>
</tr>
<tr>
<td>Log of ADSL lines</td>
<td>0.84</td>
</tr>
<tr>
<td>times bandwidth</td>
<td></td>
</tr>
<tr>
<td>Log of average wage</td>
<td>0.49</td>
</tr>
</tbody>
</table>

According to figures in Table 3, the logarithm of employees’ average wage can help as a proxy variable of the labour cost differences applying in each country, without affecting the model’s estimates.
4.3 Estimation and Results

This section presents the results from the preferred model specifications and the relative efficiency estimations. The results are also tested in order to consider any:

- Multicollinearity problems;
- Time- or year-specific features; and
- Heteroskedasticity characteristics.

4.3.1 Time invariant inefficiency model

The list of variables included in the final specification includes: logarithm of switched lines, logarithm of minutes, logarithm of ADSL lines times the relative bandwidth, logarithm of average wage and a time trend, applicable to this setting.

Table 4: Time invariant inefficiency model (40 observations, 8 companies).

<table>
<thead>
<tr>
<th>Log of Total costs as a function of:</th>
<th>Coefficient</th>
<th>Z Value</th>
<th>Significance</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of total switched lines</td>
<td>0.850</td>
<td>19.84</td>
<td>0.000</td>
<td>0.766 - 0.934</td>
</tr>
<tr>
<td>Log of minutes</td>
<td>-0.112</td>
<td>-2.39</td>
<td>0.017</td>
<td>-0.204 - -0.020</td>
</tr>
<tr>
<td>Log of ADSL lines times bandwidth</td>
<td>0.060</td>
<td>1.75</td>
<td>0.080</td>
<td>-0.007 - 0.127</td>
</tr>
<tr>
<td>Log of average wage</td>
<td>0.423</td>
<td>10.27</td>
<td>0.000</td>
<td>0.342 - 0.504</td>
</tr>
<tr>
<td>Time trend</td>
<td>-0.031</td>
<td>-1.47</td>
<td>0.140</td>
<td>-0.074 - 0.010</td>
</tr>
<tr>
<td>Constant term</td>
<td>3.455</td>
<td>6.23</td>
<td>0.000</td>
<td>2.368 - 4.542</td>
</tr>
</tbody>
</table>

Source: Deloitte analysis

The model correctly estimates the impact of logarithm of total switched lines and logarithm of average wage. Among the statistically significant variables, logarithm of minutes has a negative impact, while logarithm of ADSL lines times the relative bandwidth and the time trend are not statistically significant. The significance level of the variable minutes can be suspicious at a first sight. In fact, there is some possibility that the estimates could be affected by the unbalanced nature of the dataset due to the differences in the number of observations provided by each company.

By dropping the data of the two operators that provided us with a low number of observations, an almost balanced dataset can be obtained22.

22 Under this setting the sample is composed by 6 companies with an average of 6.2 observations.
Table 5: Time invariant inefficiency model (37 observations, 6 companies).

<table>
<thead>
<tr>
<th>Log of Total costs as a function of:</th>
<th>Coefficient</th>
<th>Z Value</th>
<th>Significance</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of total switched lines</td>
<td>0.846</td>
<td>17.62</td>
<td>0.000</td>
<td>0.752 0.940</td>
</tr>
<tr>
<td>Log of minutes</td>
<td>-0.106</td>
<td>-1.88</td>
<td>0.059</td>
<td>-0.216 0.004</td>
</tr>
<tr>
<td>Log of ADSL lines times bandwidth</td>
<td>0.058</td>
<td>1.33</td>
<td>0.184</td>
<td>-0.027 0.143</td>
</tr>
<tr>
<td>Log of average wage</td>
<td>0.410</td>
<td>6.00</td>
<td>0.000</td>
<td>0.276 0.544</td>
</tr>
<tr>
<td>Time trend</td>
<td>-0.029</td>
<td>-1.11</td>
<td>0.266</td>
<td>-0.081 0.022</td>
</tr>
<tr>
<td>Constant term</td>
<td>3.623</td>
<td>4.87</td>
<td>0.000</td>
<td>2.164 5.083</td>
</tr>
</tbody>
</table>

Source: Deloitte analysis

The model applied to this refined sample delivers robust estimates of the statistically significant variables and distributional parameters and highlight the non-statistically significant nature of logarithm of minutes. The time trend variable is not statistically significant and it is centered in a tight interval around zero.

Table 6: Efficiency ranking of comparator operators (8 Companies)

<table>
<thead>
<tr>
<th>Operator</th>
<th>Panel SFA time invariant inefficiency (overall time period)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BT</td>
<td>1</td>
</tr>
<tr>
<td>Operator D</td>
<td>2</td>
</tr>
<tr>
<td>Operator E</td>
<td>3</td>
</tr>
<tr>
<td>Operator C</td>
<td>4</td>
</tr>
<tr>
<td>Operator H</td>
<td>5</td>
</tr>
<tr>
<td>Operator F</td>
<td>6</td>
</tr>
<tr>
<td>Operator G</td>
<td>7</td>
</tr>
<tr>
<td>Operator B</td>
<td>8</td>
</tr>
</tbody>
</table>

Source: Deloitte analysis

The results clearly show BT to have the most efficient (least inefficient) ranking. The distance from comparators is quite tight because the inefficiency score represents the individual inefficiency observed over the company-specific time span of available data. This is peculiar feature of this model due to the structure of the individual inefficiency component, assumed to be Normally distributed.

In order to formally test for collinearity between the explanatory variables, the variance inflation factors (VIFs) were calculated. A model with no collinearity would have an average VIF of around 1. Centering the VIF removes any unnecessary multicollinearity due to the intercept and highlights all the multicollinearity of the explanatory variables. A good rule of thumb is that VIFs above 10 for any of the variables are a symptom of high collinearity. For small samples, a stricter threshold of 5
can be applied through a cross-sectional demeaning of the variables. The results in Table 7 show a low level of multicollinearity among the explanatory variables.

**Table 7: Variance inflation factor test for collinearity (40 observations, 8 companies).**

<table>
<thead>
<tr>
<th>Explanatory variable</th>
<th>Centered Variance inflation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of total switched lines</td>
<td>4.37</td>
</tr>
<tr>
<td>Log of switched minutes</td>
<td>3.76</td>
</tr>
<tr>
<td>Log of ADSL lines bandwidth</td>
<td>4.20</td>
</tr>
<tr>
<td>Log of average wage</td>
<td>1.65</td>
</tr>
<tr>
<td>Time</td>
<td>3.07</td>
</tr>
<tr>
<td><strong>Average VIF</strong></td>
<td><strong>3.41</strong></td>
</tr>
</tbody>
</table>

*Source: Deloitte analysis*

Possible year-specific effects were tested for by adding six time dummy variables to the specification, and the results of this estimation are presented in Table 8.

**Table 8: Time invariant inefficiency model with year-specific dummies (40 observations, 8 companies).**

<table>
<thead>
<tr>
<th>Log of Total costs as a function of:</th>
<th>Coefficient</th>
<th>Z Value</th>
<th>Significance</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of total switched lines</td>
<td>0.923</td>
<td>7.34</td>
<td>0.000</td>
<td>0.676 1.169</td>
</tr>
<tr>
<td>Log of minutes</td>
<td>-0.095</td>
<td>-0.95</td>
<td>0.341</td>
<td>-0.291 0.100</td>
</tr>
<tr>
<td>Log of ADSL lines times bandwidth</td>
<td>0.003</td>
<td>0.10</td>
<td>0.919</td>
<td>-0.061 0.068</td>
</tr>
<tr>
<td>Log of average wage</td>
<td>0.400</td>
<td>5.38</td>
<td>0.000</td>
<td>0.254 0.546</td>
</tr>
<tr>
<td>Time 1: 2005</td>
<td>0.008</td>
<td>0.14</td>
<td>0.891</td>
<td>-0.109 0.126</td>
</tr>
<tr>
<td>Time 2: 2006</td>
<td>0.081</td>
<td>1.21</td>
<td>0.227</td>
<td>-0.050 0.212</td>
</tr>
<tr>
<td>Time 3: 2007</td>
<td>-0.045</td>
<td>-0.61</td>
<td>0.541</td>
<td>-0.188 0.098</td>
</tr>
<tr>
<td>Time 4: 2008</td>
<td>-0.053</td>
<td>-0.66</td>
<td>0.512</td>
<td>-0.210 0.105</td>
</tr>
<tr>
<td>Time 5: 2009</td>
<td>-0.034</td>
<td>-0.32</td>
<td>0.747</td>
<td>-0.238 0.171</td>
</tr>
<tr>
<td>Time 6: 2010</td>
<td>0.153</td>
<td>1.19</td>
<td>0.234</td>
<td>-0.099 0.406</td>
</tr>
<tr>
<td>Constant term</td>
<td>3.525</td>
<td>2.93</td>
<td>0.003</td>
<td>1.164 5.882</td>
</tr>
</tbody>
</table>

All the year-specific variables’ coefficients are sufficiently close to zero and lying in a confidence interval that includes the zero point. By jointly testing the hypothesis that such coefficients are all equal to zero, the value of the corresponding Chi-square with 6 degrees of freedom is 15.70 and the probability is 0.0155, meaning that at 5% confidence level, all the time dummy variables’ coefficients are effectively equal to zero. This finding implies that there are no year-specific effects, thereby strengthening confidence in the time invariant characteristic of individual inefficiency.
In order to detect a possible variance heteroskedasticity among individual operators’ data, Levene’s robust test statistic was used to test for the equality of variances between groups (W0) and the two statistics proposed by Brown and Forsythe that replace the mean with median (W50) or with the 10% trimmed mean in Levene’s formula with alternative location estimators (W10). The null hypothesis that variables have the same variance per operator, apart for the median, is rejected. The presence of heteroskedasticity among operators validates the panel data structure chosen for the Stochastic Frontier.

Table 9: Variance inflation factor test for collinearity (40 observations, 8 companies)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value for 7 degree of freedom, 32 observations</th>
<th>Probability &gt; F(7,32)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W₀</td>
<td>6.015</td>
<td>0.000</td>
</tr>
<tr>
<td>W₅₀</td>
<td>1.551</td>
<td>0.186</td>
</tr>
<tr>
<td>W₁₀</td>
<td>6.015</td>
<td>0.000</td>
</tr>
</tbody>
</table>

4.3.2 Time varying-decay inefficiency model

The list of variables included in the final specification includes: logarithm of switched lines, logarithm of minutes, logarithm of ADSL lines times the relative bandwidth, logarithm of average wage. The time trend variable is dropped in order to avoid orthogonality with the company-specific inefficiency term, that is a function decreasing in the time index.

Table 10: Time varying-decay inefficiency model (40 observations, 8 companies).

<table>
<thead>
<tr>
<th>Log of total costs as a function of:</th>
<th>Coefficient</th>
<th>Z Value</th>
<th>Significance</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of total switched lines</td>
<td>0.897</td>
<td>9.53</td>
<td>0.000</td>
<td>0.713 1.082</td>
</tr>
<tr>
<td>Log of minutes</td>
<td>-0.085</td>
<td>-1.11</td>
<td>0.268</td>
<td>-0.236 0.066</td>
</tr>
<tr>
<td>Log of ADSL lines times bandwidth</td>
<td>0.017</td>
<td>0.74</td>
<td>0.459</td>
<td>-0.029 0.063</td>
</tr>
<tr>
<td>Log of average wage</td>
<td>0.391</td>
<td>9.32</td>
<td>0.000</td>
<td>0.308 0.473</td>
</tr>
<tr>
<td>Constant term</td>
<td>3.635</td>
<td>4.20</td>
<td>0.000</td>
<td>1.939 5.331</td>
</tr>
<tr>
<td>Time decay η</td>
<td>0.058</td>
<td>0.34</td>
<td>0.731</td>
<td>-0.272 0.387</td>
</tr>
</tbody>
</table>

Source: Deloitte analysis

With respect to the time invariant inefficiency model, logarithm of minutes and logarithm of ADSL lines times the relative bandwidth are clearly and highly not statistically significant.

The interpretation of the parameter η is crucial to assess the explanatory features of this model, since it is the decay parameter of the inefficiency over time. When η is positive, companies increased their efficiency over time. In this case η is close to zero and positive, despite not statistically significant. This implies that the efficiency of companies is correctly measured as invariant over time, as in the previous model.
With respect to the time invariant inefficiency model, the level of aggregate inefficiency $\gamma$ is now much greater. This tendency is confirmed by the 2011 efficiency ranking shown in Table 11, which includes only companies that provided data for this specific year. With respect to Table 6, the number of companies reduces to five because of data availability. However, consistently with the time invariant inefficiency analysis, also under this specification BT ranks first every year in the panel regardless of the comparators available.

Table 11: Efficiency ranking of comparator operators 2011

<table>
<thead>
<tr>
<th>Operator</th>
<th>Panel SFA time varying decay inefficiency ranking 2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>BT</td>
<td>1</td>
</tr>
<tr>
<td>Operator D</td>
<td>2</td>
</tr>
<tr>
<td>Operator C</td>
<td>3</td>
</tr>
<tr>
<td>Operator E</td>
<td>4</td>
</tr>
<tr>
<td>Operator F</td>
<td>5</td>
</tr>
</tbody>
</table>

Source: Deloitte analysis

This specification is lowly affected by multicollinearity movements among the explanatory variables, as shown in Table 12.

Table 12: Variance Inflation Factors test for collinearity (40 observations, 8 companies)

<table>
<thead>
<tr>
<th>Explanatory variable</th>
<th>Centered Variance inflation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of total switched lines</td>
<td>4.07</td>
</tr>
<tr>
<td>Log of switched minutes</td>
<td>3.42</td>
</tr>
<tr>
<td>Log of ADSL lines bandwidth</td>
<td>4.05</td>
</tr>
<tr>
<td>Log of average wage</td>
<td>1.63</td>
</tr>
<tr>
<td>Mean VIF</td>
<td>3.29</td>
</tr>
</tbody>
</table>

Source: Deloitte analysis

In order to compare the time invariant inefficiency and the time varying-decay inefficiency models, the significance of the time inefficiency component can be tested by setting the correspondent parameter equal to zero ($\eta = 0$) and then computing a Likelihood ratio test.

This method has three steps:

- Re-estimate the preferred SFA model with $\eta = 0$;
- Compute (twice) the ratio between the likelihood values of the unrestricted and restricted model. This statistic is distributed as a chi-square with one degree of freedom (representing the constrained parameter); and
- Compare the computed likelihood ratio with the value of the corresponding theoretical distribution.
Table 13 presents the estimates of the time varying-decay inefficiency model constrained by $\eta = 0$. The impact and the significance levels of the variables are similar to the ones estimated in previous models.

**Table 13: Constrained Time varying-decay inefficiency model (40 observations, 8 companies)**

<table>
<thead>
<tr>
<th>Log of Total costs as a function of:</th>
<th>Coefficient</th>
<th>Z Value</th>
<th>Significance</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of total switched lines</td>
<td>0.887</td>
<td>12.02</td>
<td>0.000</td>
<td>0.743 – 1.032</td>
</tr>
<tr>
<td>Log of minutes</td>
<td>-0.072</td>
<td>-1.23</td>
<td>0.218</td>
<td>-0.186 – 0.042</td>
</tr>
<tr>
<td>Log of ADSL lines times bandwidth</td>
<td>0.013</td>
<td>0.72</td>
<td>0.474</td>
<td>-0.023 – 0.050</td>
</tr>
<tr>
<td>Log of average wage</td>
<td>0.384</td>
<td>10.19</td>
<td>0.000</td>
<td>0.311 – 0.458</td>
</tr>
<tr>
<td>Constant term</td>
<td>3.792</td>
<td>5.61</td>
<td>0.000</td>
<td>2.467 – 5.116</td>
</tr>
<tr>
<td>Time decay $\eta = 0$ (set by assumption)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 14 shows the results of the Likelihood ratio test. The null hypothesis of $\eta = 0$ is strongly accepted with $p=0.7297$, meaning that the restricted model is nested within the unrestricted one. This implies that the time-varying parameter $\eta$ in the time-varying decay inefficiency term $u_{it} = \exp(-\eta(t - T_i))u_i$ is likely to be zero and that this model's feature can be easily embodied within a time invariant inefficiency setting, as analysed in the previous section.

**Table 14: Likelihood-ratio Statistic test (40 observations, 8 companies).**

<table>
<thead>
<tr>
<th>Model</th>
<th>Number of Observations</th>
<th>Log-likelihood Value</th>
<th>Model’s Degrees of Freedom</th>
<th>AIC Value</th>
<th>BIC Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Restricted by $\eta = 0$</td>
<td>40</td>
<td>28.76</td>
<td>8</td>
<td>-41.53</td>
<td>-28.02</td>
</tr>
<tr>
<td>Unrestricted</td>
<td>40</td>
<td>28.82</td>
<td>9</td>
<td>-39.65</td>
<td>-24.45</td>
</tr>
</tbody>
</table>

**Likelihood Ratio and Statistics**

|                          |                         |                      |
|--------------------------|-------------------------|
| LR Chi-square            | 0.12                    |
| P-Value                  | 0.7297                  |
4.4 Final overview of inefficiency rankings

Finally, we can compare the inefficiency rankings of the two different methodologies. Table 15 presents the ranking of each company per method an year.

Table 15: Inefficiency ranking overview per model and year.

<table>
<thead>
<tr>
<th>Operator</th>
<th>Panel SFA time invariant inefficiency ranking</th>
<th>Panel SFA time varying decay inefficiency ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>BT</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Operator D</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Operator E</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Operator C</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Operator H</td>
<td>5</td>
<td>.</td>
</tr>
<tr>
<td>Operator F</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Operator G</td>
<td>7</td>
<td>.</td>
</tr>
<tr>
<td>Operator B</td>
<td>8</td>
<td>.</td>
</tr>
<tr>
<td>Operator A</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>

Missing positions are due to insufficient information to determine the inefficiency scores. The comparison among the models points out some interesting regularities. BT and operator D are clearly ranking first and second in any possible frontier. Operators E and F have provided limited information and therefore, as already explained in Table 5, their performance cannot be considered as effectively representative of the effective level of inefficiency. Operators C and H move together within the rankings. Same reasoning applies for operators G and B, which perform poorly in any specification.
References