Response to:
2.3 and 3.4 GHz spectrum award: Consultation on a 3.4 GHz band plan, varying UK Broadband Limited's licence and a call for inputs on other aspects of the award

Frank Kelly
University of Cambridge

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1. Introduction

I would like to respond to **Question 12**, in respect to the method of award. I would like to suggest that Ofcom consider an auction method other than the combinatorial clock auction (CCA); specifically, that Ofcom employ the PAUSE auction procedure [1 - 4].

PAUSE has the following five properties:

1. The procedure is combinatorial, *i.e.*, allows package bids
2. The procedure is iterative
3. The procedure is conducted as a first-price auction
4. The procedure is transparent for the bidders
5. In each round of the auction, the auctioneer is faced with a computationally simple problem.

In comparison, the combinatorial clock auction only possesses the first two of these five properties.

2. Overview of the PAUSE auction

PAUSE proceeds in stages comprised of discrete rounds over *m* stages, where *m* is the number of items in the auction. Stage 1 is conducted as a simultaneous multi-round auction (SMRA) for all the items, thus facilitating price discovery. In stages 2, 3, …., *m*, bidders can realise their synergies via package bidding. However, the bids on packages cannot be submitted in isolation. Rather, each bidder is required to submit his package bids as components of a *composite bid*, which is a set of non-overlapping package bids (including individual bids) that cover *all the items in the auction*. A bidder will generally be interested in bidding only on a subset of the items in the auction—and in any given round, perhaps only a subset of these. *A composite bid consists of the bidder's own bids, together with previously submitted bids by any of the bidders (including himself).*

In general, there are only two restrictions on composite bids:

- For a composite bid placed in stage *k*, the package bids that comprise it cannot contain more than *k* items each.
- Composite bids are restricted in how they use bids from other bidders. Specifically, when a bidder X submits a composite bid that includes previously submitted bids from a bidder Y, the bids included from Y must all have been submitted by Y in a single round of the auction.[1]

[1] Two suggested bidding algorithms for use in PAUSE are developed and analysed via numerical methods in [5].

The auctioneer's role in a PAUSE auction is straightforward. At each stage, the auctioneer checks the validity of the composite bids, and chooses the one yielding the highest revenue. Validating a composite bid is an easy procedure for the auctioneer, as it consists only of verifying that the two restrictions above are satisfied, and that any additional restrictions on the bidding, such as caps or activity rules, are satisfied by the composite bid. For example, the cap might be that no one operator could hold spectrum in excess of 36% of the total in the relevant bands, which can be easily verified by the auctioneer by examining the composite bid.

3. An example

Consider the following example based on a TDD band plan for both the 2.3 GHz band and the 3.4 GHz band with the proposed relocation of UK Broadband's spectrum allocation. (For simplicity of exposition, we will not include any caps or activity rules.) The 2.3 GHz band, between 2350 MHz and 2390 MHz, is partitioned into eight blocks of 5 MHz each, denoted A01, A02, …, A08, and the 3.4 GHZ band, between 3410 MHz and 3560 MHz, is partitioned into 15 blocks of 10 MHz each, denoted B01, B02, …, B15 (Fig. 1). There are eight bidders, denoted S, T, U, V, W, X, Y, Z. Stage 1 ended with the following bids (Fig. 2a):

On the 2.3 GHz band

- S has a bid 25 on A01 and a bid of 50 on A02
- T has a bid of 50 on A03
- U has bids of 50 each on A04 and A05
- V has a bid of 50 on A06
- W has a bid of 50 on A07 and a bid of 25 on A08

On the 3.4 GHz band

- X has bids of 50 each on B01 and B15, and a bid of 100 on B02
- Y has bids of 100 each on B03, B04, B13, B14
- Z has bids of 100 each on B05, B06, B11 B12
- T has a bids of 100 each on B07 and B10
- V has a bids of 50 each on B08 and B09.

Consequently, Stage 1 ended with revenue to the auctioneer from these 23 bids of 1650.

Stage 2 ended with a composite bid by Bidder Z consisting of the following component bids (Fig. 2b):

- 100 by S on (A01, A02)
- 50 by T on A03
- 150 by U on (A04, A05)
- 50 by V on A06
- 100 by W on (A07, A08)
- 200 by X on (B01, B02) and 50 by X on B15
- 250 each by Y on (B03, B04) and (B13, B14)
- 250 by Z on (B05, B06) and (B11, B12)
- 100 each by T on B07 and B10
- 50 each by V on B08 and B09.

Consequently, Stage 2 ended with revenue to the auctioneer from this composite bid of 2000.

Stage 3 ended with a composite bid by V consisting of the following component bids (Fig. 2c):

- 25 by S on A01
- 300 by T on (A02, A03, A04)
- 300 by V on (A05, A06, A07)
- 25 by W on A08
- 200 by X on (B01, B02)
- 250 by Y on (B03, B04) and 350 by Y on (B13, B14, B15)
- 250 each by Z on (B05, B06) and (B11, B12)
- 100 each by T on B07 and B10
- 50 each by V on B08 and B09.

Consequently, Stage 3 ended with revenue to the auctioneer from this composite bid of 2250. Stage 3 turned out to be the final stage of the auction, as there was no bidding past this stage. Consequently, the auction ended with revenue to the auctioneer of 2250.

Two observations may be in order. First, in this example, all the package bids were of contiguous blocks, and were located on one of the two bands, but certainly no such restrictions are required for the working of the auction procedure. Second, although in this example the auction ended at the end of stage 3, since there were 23 blocks it is possible that this auction could have gone on for 23 stages, but no more. As mentioned above, this example did not include caps or activity rules, but they could easily be incorporated into the procedure, by having the auctioneer check the composite bids at each stage.

4. Further reading

PAUSE was originally developed for use in assigning subsidies for universal service in the United States, *i.e.*, , for 'Carrier of Last Resort' (COLR). This prompted a U.S. public interest group to make an *ex parte* submission to the U.S. Federal Communications Commission of the first document describing PAUSE [4]. Subsequently, the PAUSE auction procedure was cited in the FCC's 'Further notice of proposed rule making' (see [6] and [7]). Although it had been developed originally for assigning universal service support, the procedure can be used wherever a combinatorial auction is required. In fact, the first published paper on PAUSE [1, p. 590] specifically pointed out that the procedure could be used as a combinatorial spectrum auction.

A detailed and up-to-date presentation of PAUSE can be found in [3].

Bibliography

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