

## Further Analysis of the Efficiency of BT's Network Operations.

A Report for BT

20 February 2009

This report has been prepared on the basis of the limitations set out in the engagement letter and the matters noted in the Important Notice From Deloitte on page 1.

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This report (the "Report") has been prepared by Deloitte LLP ("Deloitte") for BT in accordance with the contract dated 7<sup>th</sup> February 2008, the subsequent change order dated 12<sup>th</sup> December 2008 (together, "the Contract") and on the basis of the scope and limitations set out below. The cut off date for our analysis is 26<sup>th</sup> January 2009 and we have not considered any information that has come to our attention after this date.

The Report has been prepared solely for the purposes of assisting BT to respond to Ofcom's consultation on the appropriate efficiency factor to apply to BT in the wholesale price controls as set out in the Contract. It should not be used for any other purpose or in any other context, and Deloitte accepts no responsibility for its use in either regard. It should be read in conjunction with our previous efficiency report, which is appended as appendix I. Since the signing of the change order, you requested that we change the scope of our report to update our previous analysis, rather than summarise it. This was confirmed by BT in an email dated 12<sup>th</sup> February 2009.

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## Executive Summary

Ofcom is currently seeking to apply a set of price controls to products provided by BT Openreach and BT Wholesale. Typically, price controls in the telecommunications sector require the regulated company, in this case BT, to increase productivity to allow for three separate effects:

- Comparative efficiency: BT is expected to increase its relative efficiency to match that of an agreed benchmark representing an efficient comparator;
- Annual real cost change: The annual increase in productivity, assuming constant volumes, that the industry may be expected to experience during the price control period; and
- Economies of scale: The change in BT's unit costs that result from a change in volumes.

This report considers the first two of these effects.

In considering this question we focus on the efficiency of BT's entire network operations including both BT Openreach and BT's wholesale network business, as previous analysis conducted by each of Deloitte for BT and NERA for Ofcom has shown that it is not practical to disaggregate the efficiency effects of these two operations. This report updates our initial report on this issue, appended to this report, to reflect questions raised by NERA and examines a revised approach to measuring the productivity gain introduced in the latest Ofcom consultation on this issue.

The key findings from our work are that:

- BT is significantly ahead of the top decile of the US Local Exchange Carriers (LECs), the benchmark of efficiency imposed upon BT in previous charge controls. Our preferred econometric specification, using stochastic frontier analysis (SFA), finds BT to be 6.3% more efficient than this benchmark. This result is broadly consistent with the work produced by NERA.
- There are various econometric methods of measuring the annual real cost change, which were investigated in our initial report. We have now re-estimated these different methods following suggestions from NERA. Our results show a consistent estimated trend of between 0.0% and -2.2%:
  - SFA time trend analysis suggests the frontier is moving at -2.2%;
  - Total factor productivity (TFP) indexation suggests the frontier is moving at -0.5%; and
  - Direct TFP estimation suggests the frontier is moving in the range of between 0.0% and -1.9%.
- A re-estimation of Ofcom's approach of measuring the trend in productivity through bottom-up changes to network component costs also supports this range. Our adjustments to the Ofcom methodology improves the nature of indexation applied, includes distance-related elements that make up 35% of costs and includes the revised frontier moves reported

above. The result of these adjustments is to reduce and narrow the range for the annual real cost (productivity) trend from the 0% to -5% range reported by Ofcom to between -0.5% and -1.5%.

- In addition to the current productivity trend we believe that it is important to make an allowance for the finding that BT is significantly ahead of the benchmark for efficiency applied by Ofcom. It seems inappropriate for BT not to be rewarded for exceeding the efficiency target when in previous charge control periods it has been required to “catch up” to that benchmark. As such, it may be appropriate for BT to argue that allowance be made in the charge control for the benchmark to “catch up” to BT’s current performance. Implicitly this would result in a reduction in the X-factor applied within this RPI-X charge.

# 1 Introduction

Deloitte has been employed by BT to estimate and support an appropriate efficiency factor for 'BT Network', to be included within the upcoming charge control. Deloitte provided an initial report to BT on this issue, Deloitte (2008)<sup>1</sup>, appended to this report. This updated analysis should be read alongside our initial report. Subsequent to discussions with Ofcom and NERA<sup>2</sup> the findings of the original report have been updated. The results and methods behind these updates are discussed in this report.

## 1.1 Conventions

A number of differing estimates of the future target efficiency gains are discussed throughout this report. Despite fundamentally aiming to measure the same effect, the literature often associates differing names with these estimates. Given this, it should be noted that the following definitions are employed in this report.

The '**annual real cost change**' is the unit cost change resulting from productivity improvements, independent of volume effects. It is calculated as either:

- The estimated time trend from SFA minus the inflation rate (taken to be 2.2%, which is the average over the period studied and is consistent with NERA's studies), referred to as the cost frontier shift; or
- The future efficiency gains calculated from productivity models, referred to as productivity increases or total factor productivity (TFP) growth.

The term '**comparative efficiency**' refers to BT's efficiency compared to the top decile of US LECs ranked according to relative efficiency.

The '**future annual catch-up**' is the level of catch-up required so that BT's comparative efficiency is at the level of the top decile by the end of the charge control period.

The annual real efficiency adjustment for BT is referred to in this report as the '**BT efficiency assumption**'. This is the combination of the annual real cost change and the future annual catch-up.

## 1.2 Structure of this report

The report is structured as follows:

- Section 2 summarises our analysis on the appropriate efficiency assumption for BT Networks;

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<sup>1</sup> Deloitte, 2008: '*The Efficiency of BT's Network Operations*'

<sup>2</sup> NERA are providing technical support to Ofcom on the leased line charge control.



- Section 3 provides an update of our previous report and final results;
- Appendix A provides the detail behind our updated stochastic frontier analysis results and also provides a critique of the approach undertaken by Ofcom and NERA. This analysis provides both a measure of the comparative efficiency and annual cost change;
- Appendix B provides the TFP and provides a critique of the approach undertaken by Ofcom and NERA;
- Appendix C outlines our comments and suggested improvements to the Ofcom model which is used to estimate the annual real cost change;
- Appendix D sets out the future annual catch-up effect;
- Appendix E, Appendix F, and Appendix G provide additional details of the econometric and statistical outputs supporting this work;
- Appendix H provides a list of references and a glossary; and
- Appendix I contains our previous report on the efficiency of BT's network operations, dated 9<sup>th</sup> May 2008

## 2 Summary of Our Findings

Ofcom is currently consulting on applying RPI-X charge controls to six broad baskets of services provided by BT, as discussed in Ofcom (2008)<sup>3</sup>. The charge control will be applied to these products over a four-year period ending 30 September 2012<sup>4</sup>.

Typically, price controls in the telecommunications sector require the regulated company, in this case BT, to increase productivity to allow for three separate effects:

- Comparative efficiency: Where BT is expected to increase its relative efficiency in the current time period to match that of a defined benchmark representing an efficient comparator. This is referred to as future catch-up.
- Annual real cost change: Referred to as the productivity gain, this is the annual increase in productivity, assuming constant volumes, that BT may be expected to experience during the price control period<sup>5</sup>.
- Economies of scale: The change in BT's unit costs that result from a change in volumes.

Efficiency analysis estimates parameters for the first two effects only. The efficiency assumption in Ofcom's cost modelling seeks to converge BT's costs with an efficient benchmark level by the end of the price control. The economies of scale effect is usually considered separately by the application of asset and cost volume elasticities within the price control cost model.

In this study we have estimated the first two elements focusing on estimating BT Network's<sup>6</sup> efficiency. This report updates our previous study, Deloitte (2008), based on comments made in NERA (May 2008)<sup>7</sup> and further discussions with Ofcom and NERA.

The methods we have deployed to estimate the two elements of the efficiency adjustment, are summarised in Table 1, and we subsequently consider the results of each of these in turn.

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<sup>3</sup> Ofcom, 2008: '*Leased Lines Charge Control*'

<sup>4</sup> We understand that the charge control will be backdated to 1 October 2008 given a delayed implementation as a result of the regulatory financial statement restatement.

<sup>5</sup> Productivity is measured for the telecommunications industry. BT's underlying productivity is assumed to change at the rate of the industry.

<sup>6</sup> Defined as BT Wholesale and BT Openreach together. This approach is followed due to the lack of robustness found by Deloitte and NERA in previous studies attempting to disaggregate Openreach's business from the network as a whole.

<sup>7</sup> NERA, 2008: '*Comments on the Deloitte paper on the efficiency of BT's network operations*'

**Table 1: Approaches to calculating comparative efficiency and annual real cost change**

Comparative efficiency	Annual real cost change
<b>Stochastic Frontier Analysis:</b> Econometric technique used to estimate BT's distance from the efficient frontier or comparative efficiency. The frontier is estimated based on data from US LECs.	<b>SFA time trends:</b> Rate of change of the efficient frontier over time, excluding volume effects <b>TFP rate of change:</b> Data from a selection of European Incumbents is used to calculate the change in total factor productivity compared to BT networks <b>Econometric TFP Model:</b> Econometric approach based upon growth economics to analyse the change in productivity of US LECs and BT networks <b>The Ofcom model:</b> Bottom up approach to analyse changing BT unit cost trends

## 2.1 BT's comparative efficiency

We calculated the comparative efficiency of BT networks using Stochastic Frontier Analysis (SFA) on a US LEC data set. This is the approach favoured by Ofcom and its consultant's NERA as well as other regulatory authorities<sup>8</sup>.

Our preferred specification involved regressing total costs against sheath per line, total switch minutes, leased lines and PSTN lines variables. Further account was also made for two identified structural breaks, a time trend component, two regions and stranded assets. The later two variables were included subsequent to comments made by NERA (December 2008)<sup>9</sup>. The stranded assets variable seeks to account for varying network utilisation. The models we estimated are robust and statistically good fits, with all results being relatively insensitive to changes in specification or assumptions. The econometric details of our preferred specification and sensitivities are discussed in Section 3 and provided in full in Appendix A.

On this basis of this specification, we estimate that BT is more efficient than the top decile by 6.3%. This finding is consistent with NERA (December 2008) who find BT to be 6.0% more efficient than the top decile<sup>10</sup>. This is not consistent with Ofcom's statement that BT that is "roughly on the decile"<sup>11</sup>.

This result has an important impact on the level of catch-up allowed for in future periods. This is covered in Section 2.3 and discussed in more detail in Appendix D.

<sup>8</sup> Regulators that have used this technique include Ofcom and Oftel for BT, Comreg and ODTR for Eircom, OPTA for KPN, the Communications Commission for Telecom New Zealand and the ACCC for Telstra.

<sup>9</sup> NERA, 2008: '*NERA's Analysis of the Efficiency of BT's Network Operations*'

<sup>10</sup> Based on NERA (December 2008), SFA Table 4.7. Using NERA's main model and their stated preferred sample from 1999 to 2006. Preference discussed on page 11.

<sup>11</sup> Paragraph A9.22, page 194 of Ofcom (2008).

## 2.2 Annual real cost change

The annual real cost change is the unit cost change resulting from productivity improvements independent of volume effects. We consider the historical productivity performance of the telecommunications sector to inform the likely future annual cost changes that might be experienced by the sector. We calculate this as:

- The time trend from the SFA model, set out above; and
- Total factor productivity analysis.

In addition we adjust the Ofcom model to calculate a BT specific productivity improvement from historic BT performance.

We set out our analysis behind each approach below, but find that under all approaches we arrive at consistent values for the annual real cost change.

### 2.2.1 SFA time trend

The first method of measuring the annual real cost change is derived as the estimated slope coefficient on the time trend in the SFA analysis. This estimate is, however, sensitive to pricing distortions and hence provides only a nominal estimate of cost changes. It is therefore standard practice, as used by Ofcom in previous network charge controls<sup>12</sup>, to subtract a measure of price inflation from the estimated coefficient to retrieve a pure measure of annual real cost change.

The time trend in the Deloitte preferred SFA regression discussed in Section 2.1 above is estimated to be 0.0%, implying a flat profile for nominal cost changes. Taking an inflation rate of 2.2%, which is the average over the period studied and is consistent with NERA's studies, we conclude that there is a real cost change estimate of -2.2% (calculated over the period 1996 to 2006, controlling for structural breaks). Our model indicated that cost changes in more recent periods have been slower than in earlier periods and therefore it is more appropriate to use the time trend from the last period (2003-2006) rather than for the whole estimated period (1996-2006). Our findings of relatively slowing cost changes were consistent across differing specifications; including data truncation and a reduction to a single structural break.

Despite retaining some differences in model specification, NERA (in their December 2008 paper) estimated the real cost change also to be -2.2% in their main model<sup>13</sup>. NERA bases this on the period 1999-2006, although notes that the level of unit cost improvement has declined in recent years. This contrasts with NERA (May 2008), in which they state their preferred range from the SFA analysis is between 2.5% to 3.0%<sup>14</sup>. This latter result relates to the entire period and does not allow for declines in later years; in addition, it does not take account of structural breaks (which is inconsistent with NERA (December 2008), in which structural breaks are investigated).

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<sup>12</sup> Including the 2005 and 2001 network charge controls.

<sup>13</sup> NERA (December 2008), table 4.2 and page 14

<sup>14</sup> NERA (May 2008), page 19

Further details of our SFA analysis are provided in Section 3.3.1.

## 2.2.2 Total Factor Productivity rate of change

In economic literature real cost changes have alternatively been measured by estimating TFP gains<sup>15</sup>. Such approaches are able to account for inflationary distortions in a more direct manner by utilising indices to specify variables in real terms.

Our first measure of TFP growth is an indexed approach. We calculated the rate of change of input and output indices, using parent level data for the US LECs and 10 European incumbent operators. The indices were calculated using a number of different techniques including the Törnqvist, Laspayers, Paasche and chain methods although the Törnqvist is our preferred method of indexation. The annual difference in each index is calculated to provide a growth rate for outputs and inputs. The subtraction of the annual input growth rate from the annual output growth rate, in theory, provides the growth in TFP. However, many of these companies have relatively unstable volumes and hence input utilisation. This is likely to bias the TFP growth estimate. In order to mitigate for this bias we estimated the relationship between volumes and our calculated TFP growth measure. After applying this remedial measure, TFP growth is estimated to be 0.5% per annum. This implies a real annual cost change of -0.5% per year.

It should be noted that this model does not account for movements of firms towards or away from the frontier. However, since the frontier itself is defined by the average of firms in any year, we would expect these movements relative to the frontier to average out at any time.

Further details of our analysis are provided in Section 3.3.2.

## 2.2.3 Econometric Total Factor Productivity model

We have also estimated TFP growth using a full econometric growth model, run over the LEC data set constructed for the standard TFP analysis. We have omitted EU operators from this analysis due to a lack of disaggregation of output data.

The model is derived from a standard economic production function and a specification of technology commonly used in the economic growth literature. As the LECs produce several outputs, we additionally used an adjusted indices approach to combine these into one output measure.

Based on this approach we estimated that TFP growth is in the range of 0% to 1.9%. This is equivalent to an annual real cost change of between 0% and -1.9%. This range is driven by whether the break is entertained over just the time trend or all the variables, or when it is specified<sup>16</sup>.

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<sup>15</sup> For discussion of TFP and the Solow residual, see chapter 1 in Romer (2000).

<sup>16</sup> This structural break was originally identified in NERA (May 2008).

**Table 2: TFP estimates across various specifications**

TFP growth after the structural break	Structural break: 2001/02	Structural break: 2002/03
Structural break applied to time trend only	1.9% (-137.6)	1.6% (-143.3)
Structural break applied to whole model	0.0% (-151.6)	0.0% (-147.2)

*Schwartz Bayesian information criterion are provided in brackets*

*Source: Deloitte analysis*

To assess the relative merits of each model we calculated the Schwartz Bayesian information criterion which provides a measure of the model fit. Across the models we found this statistic varied only marginally suggesting no particular model is preferable.

NERA were given access to our dataset and stated a preferred point estimate of 2.0% TFP growth<sup>17</sup>. This slightly higher estimate is attributable to the differing estimation techniques they have deployed.

Further details of our analysis are provided in Section 3.3.3.

## 2.2.4 The Ofcom model

Ofcom's approach, outlined in Annex 9 of Ofcom (2008), measures the average real unit cost change for individual network components, holding volumes constant and controlling for BT's historical catch-up to the frontier. This provides a range of 0% to 5% for the efficiency assumption<sup>18</sup>. This approach is unusual and does not accord with the econometric SFA approach that is more usually used to calculate the frontier shift. We have concerns over the rigour of this technique. However, we understand that Ofcom intends to continue to rely upon this approach alongside more traditional econometric measures and so we focus on providing a series of adjustments to increase the robustness of outputs.

The Ofcom model is based a number of key assumptions. There are a number of assumptions that we have either not tested or adjusted as they are fundamental to the underlying model. However we propose three adjustments which we believe will increase the robustness of the estimate. These are:

- Including distance-related elements in the analysis: The Ofcom model does not include costs and volumes of distance-related components, due to inconsistency in the volume measures for the component. This implies only capacity and copper related costs were considered, covering only 65% of the cost base relevant for the leased line charge control

<sup>17</sup> Page 8, NERA (May 2008).

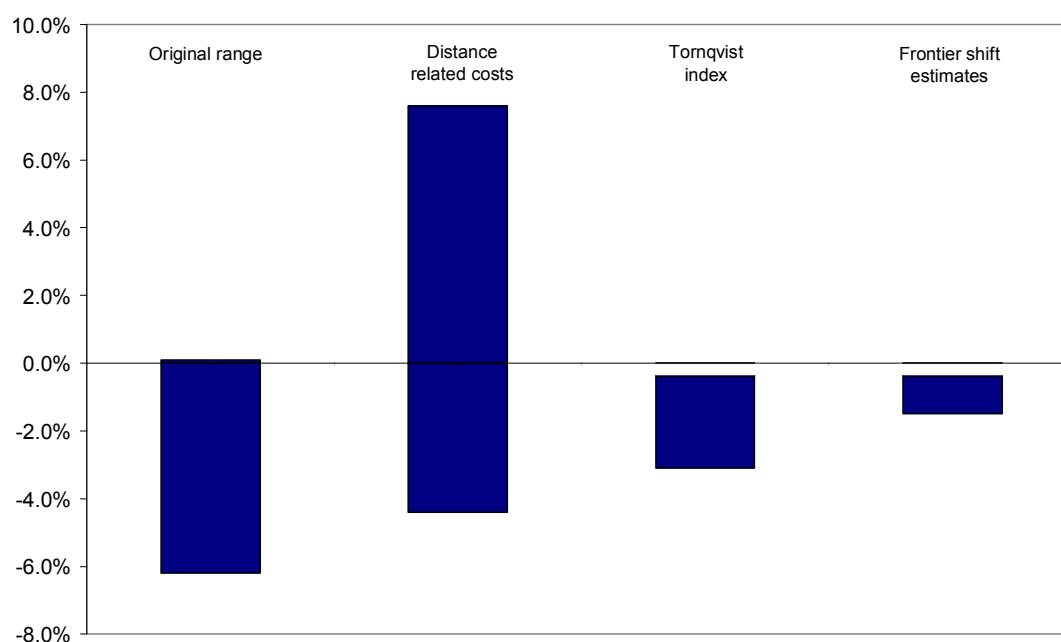
<sup>18</sup> Paragraph A9.17, Ofcom (2008).

(LLCC). The results could therefore be biased if cost trends of components included in the analysis differ from the omitted components.

- Using a Törnqvist index, instead of the alternative base year weights: the Törnqvist index is a standard measure used in productivity analysis. The particular attraction of its use in this case is that it is designed for continuously weighted multi-component analysis, using both the current and the base year cost shares in weighting each component in the index.
- Updating and refining the estimates of BT efficiency relative to the top decile: NERA (2005)<sup>19</sup> estimates BT's relative inefficiency in 2003; the measure from the preferred specification is 1.05%.<sup>20</sup> The end point of the catch-up adjustment used in the Ofcom analysis comes from a report focusing on the relative efficiency of BT Openreach. The estimate from NERA's main model is that BT is 6.0% above the decile.

The unit cost reduction estimated by Ofcom varies from 0.1% to -6.2%. Including the distance-related components produces a wider range, however using the Törnqvist index produces a reasonable range of unit cost constant annual growth rate (CAGR) for the time period. The update to the start and end points of BT relative efficiency leads to a further narrowing of the range and frontier shift estimates between -0.5% and -1.5%. This is consistent with the efficiency assumptions that we calculate using the various econometric approaches.

**Figure 1: Cumulative effect of the adjustments to the Ofcom model**



Source: Deloitte analysis

<sup>19</sup> NERA, 2005: 'The Comparative Efficiency of BT in 2003'.

<sup>20</sup> Page 35, NERA (2005).

Further details of our analysis are provided in Section 3.3.4.

## 2.2.5 Summary of the annual real cost change analysis

In the table below, we summarise the various estimates of annual real unit cost change. This demonstrates a range of annual real cost change from -2.2% to 0.0% per year.

**Table 3: Real annual cost change**

Model: BT relative to top decile in 2006/2007	Estimated time trend	Annual real cost change
Deloitte SFA <sup>2</sup> : -6.3%	0%	-2.2%
Deloitte FE TFP <sup>3</sup> : N/A	N/A	0% to -1.9%
Deloitte indexation <sup>4</sup> : N/A	N/A	-0.5%
Ofcom model revised by Deloitte	N/A	-0.5% to -1.5%
NERA SFA (table 4.7, model giving -6%) <sup>5</sup>	0%	-2.2%
NERA FE (table 5.2) <sup>5</sup>	N/A	-1.9%

1) Source: SFA analysis in NERA (December 2008) and Appendix A

2) Source: revised Deloitte analysis in Appendix A of this report

3) Source: revised Deloitte analysis in Appendix B of this report

4) Source: Deloitte (2008)

5) Source: NERA (December 2008)

## 2.3 Future catch-up

Ofcom has traditionally computed a single measure for BT's efficiency assumption by combining the catch-up requirement with the annual real cost change. Through the application of the catch-up requirement, Ofcom has taken a stance that BT should be at the efficiency level of the top decile by the end of the price control period. This is accepted by Ofcom as a means for encouraging efficiency improvements and has been consistently applied within past price controls.

We believe that such a reward and penalty structure in a price control should be applied in a symmetrical manner, incentivising BT to outperform. Therefore the catch-up element should also be applied where BT is found to be above the efficiency decile. Not applying the catch-up element symmetrically may:

- Provide reduced incentives for BT to continue to outperform efficiency targets.
- Be viewed as inconsistent with Ofcom's previous approach to catch-up in instances where BT was found to be comparatively inefficient and would imply a more stringent definition of efficiency than that applied to BT in the past.
- Increase the risk faced by BT, due to more stringent efficiency standard, as well as an increase in regulatory risk. These could have the effect of increasing the required return by investors, and hence should potentially be reflected in the allowed weighted average cost of capital in the price control model.



Ofcom has not clearly set out a view on the applicability of future catch-up. However, the Ofcom model adjusts the measured unit cost trends for historical catch-up before drawing the frontier shift estimate. Where BT has overtaken the top decile, Ofcom's analysis requires BT to maintain that level of efficiency instead of allowing the decile to catch up to BT.

The asymmetrical treatment of the top decile as a benchmark imposes a substantially stricter efficiency standard on BT than Ofcom has applied previously.

NERA (March 2008)<sup>21</sup> estimates that BT's relative efficiency was 6.0% above the top efficiency decile in 2006/2007. The updated Deloitte analysis finds that BT's relative efficiency was 6.3% above the top efficiency decile in 2006/2007. If we assume that a constant glide path approach to catch-up is applied and that the top decile catches up to BT by the end of the price control, this would result in a future annual catch-up of 1% per year<sup>22</sup>.

Further discussion is provided in Appendix D.

## 2.4 Efficiency adjustment for BT

Our analysis indicates an annual real cost change for BT of 0.0% to -2.2% per year. The BT efficiency assumption then depends on the level of catch-up that is permitted by Ofcom. If, for example, we assume that a 1% per year catch-up is allowed (as discussed above), this would imply a BT efficiency assumption of 0% to -1.2% per year<sup>23</sup>.

These estimates are consistent with the outputs of the revised Ofcom model and with those produced in the preferred specifications of NERA's analysis.

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<sup>21</sup> NERA, 2008: '*The Comparative Efficiency of BT Openreach*'

<sup>22</sup> This is based upon the calculation of BT's comparative efficiency of 6% above the decile in 2006 and requiring BT to be at the decile by the end of the price control in 2012, assuming a constant glide path approach. Other methodologies for incorporating future catch-up could be employed by Ofcom which may result in an alternative value for catch-up.

<sup>23</sup> This assumes that BT would not be permitted to raise annual unit costs and therefore the maximum value of the BT efficiency assumption is capped at 0%.

### 3 Update to our Previous Analysis

In this section of the report we provide a summary of our approach to efficiency analysis. This is an update of our Deloitte (2008) report to reflect those areas of NERA's critique of our analysis, in NERA (May 2008), which we agree to be an improvement to our previous model specifications. The values in this chapter are based upon our updated analysis. Any differences between the approaches used in Deloitte (2008) and this report are clearly noted.

Our revised study finds BT to be comparatively efficient when compared to the top decile of US LECs ranked according to their efficiency. We find BT to be 6.3% more efficient than the top decile. This updated estimate, which reflects Ofcom's critique, finds BT relatively more efficient than our original estimates of between 1.5% and 2.6%<sup>24</sup>.

We have recalculated the annual real cost change from both the SFA model and, separately, from TFP analysis. We calculate that the efficiency (or productivity) frontier of telecommunications network operations is shifting at between 0% and -1.9% per year in real terms when calculated using econometric TFP models. This result is consistent to NERA's point estimate of -2.0% estimated across the same dataset<sup>25</sup>. In addition, our SFA time trend analysis implies an efficiency shift of -2.2% in real terms. This also is consistent with NERA's SFA time trend estimate of -2.2% from their main model and estimated range of -2.2 to -2.9%<sup>26</sup>.

As BT is above the top decile by 6%, this implies that BT should not be subject to a future annual catch-up component that requires it to move towards the frontier. In setting its price control, Ofcom should allow for the decile to catch-up with BT. The future catch-up effect, and its justification, is discussed more in Appendix D.

#### 3.1 Summary of our approach

This study focuses on computing two measures of efficiency:

- Comparative efficient: The efficiency of BT relative to the top decile. Where BT is not aligned to the decile, a catch-up component may be applied so that BT is expected to increase its relative efficiency in the current time period to match that of a predefined benchmark representing an efficient comparator or, when BT is above the decile, to allow the comparators to increase their relative efficiency until BT is on the decile.
- Annual real cost change: this is the annual increase in productivity, assuming constant volumes, that BT may be expected to experience during the price control period.

It does not seek to consider the impact of volume changes on economies of scale and both of the above approaches are calculated assuming constant volumes.

<sup>24</sup> Table 8 and Table 10 in Deloitte (2008)

<sup>25</sup> See NERA (May 2008) page 8

<sup>26</sup> NERA (December 2008) Table 4.1 and 4.2, and page 14 for the inflation adjustment

We focus on the efficiency of BT's entire network operations including both BT Openreach and BT's wholesale network business, as previous analysis has shown that it is not practical to disaggregate the efficiency effects of these two operations. We calculate both comparative efficiency and the annual real cost change under a number of approaches, as shown in Table 4.

**Table 4: Approaches to the calculation of efficiency**

Comparative Efficiency Approaches: To calculate catch-up	Productivity frontier shift approaches: To calculate the real annual cost change
<p><b>SFA:</b> Econometric calculation of the efficient frontier using data from US LECs and BT's distance from the frontier and the top decile.</p>	<p><b>SFA time trends:</b> Rate of change of the efficient frontier over time, excluding volume effects minus the inflation rate.</p> <p><b>TFP fixed effects model:</b> Econometric approach based upon growth economics to analyse the change in productivity of US LECs and BT networks.</p> <p><b>TFP rate of change:</b> Data from European Incumbents is used to calculate the change in total factor productivity compared to BT networks.</p> <p><b>Ofcom model:</b> Weighted component unit cost changes. Considered in Section Appendix C of this report.</p>

Source: Deloitte

### 3.1.1 Comparators

Accurate data is a key part of the techniques utilised for the empirical estimation of efficiency. The data set used in this update is the same as that used in our previous analysis, in which BT data is compared to that of the US Local Exchange Carriers (LECs), obtained from the ARMIS database maintained by the Federal Communications Commission (FCC). This dataset is regarded by regulators, including Ofcom, as being robust and detailed.

In addition, we used publicly available information from European incumbent operators to calculate adjusted benchmarks for TFP. This information was not sufficiently disaggregated to be used in the SFA or econometric TFP, and information on European incumbents should therefore be considered to be a check on the results calculated from the US LECs.

A number of adjustments were made to the BT data to provide for better comparability and these are reconciled to published accounts in the report. All adjustments were made to ensure comparability between datasets, and included a removal of all payphone costs, checks on the definition of leased lines and special access lines, and calculations of depreciation and asset values in current cost accounting terms.

However, it should be noted that our adjusted SFA data set did not seem totally comparable to the adjusted dataset produced by NERA. Whilst both parties have agreed that the differences do not appear to be large, they remain unreconciled and as such NERA analysis on Deloitte data may not produce identical results to Deloitte analysis on Deloitte data, and we are unable to replicate

NERA's results on our dataset<sup>27</sup>. This results in differences in the SFA outputs, such that we are unable to replicate the output of the regressions that NERA has undertaken on their version of the Deloitte dataset. We therefore do not consider any NERA results on the Deloitte dataset in this report. This problem did not extend to the TFP data set as NERA undertook their critique based only upon the data set we provided to them.

Our datasets are described in full in Deloitte (2008) which is appended to this report.

## 3.2 Calculation of comparative efficiency

We have calculated the comparative efficiency of BT networks using SFA analysis on the US LEC panel data set described above, using data from 1996 to 2006. This is the approach favoured by Ofcom and its consultant's NERA as well as by other regulatory authorities<sup>28</sup>.

Our preferred specification uses a panel data stochastic frontier model, with sheath per line, total switch minutes, leased lines and PSTN lines as explanatory variables. Following a critique commissioned by Ofcom, we have also included a variable reflecting the degree of network utilisation, and two additional regional dummy variables. The model includes two structural breaks and a time trend. The full result of the model can be seen in Appendix F and a full discussion of this model is found in our previous report, Deloitte (2008), as amended in Section A.4 below. These results show that:

- BT lies inside the top decile by 6.3% meaning that it cannot be considered inefficient. Assuming a six year period for catch-up, this implies a 1% per year future catch-up component which reduces BT's efficiency adjustment by 1% per year as the decile catches up to BT; and
- Our models are robust and statistically good fits, with all results being relatively insensitive to changes in specification or assumptions.

The econometric output from the above analysis is provided in Appendix E.

## 3.3 Calculation of the annual real cost change

In previous studies, Ofcom has set the expected change in real costs each year as the movement in the efficient frontier, calculated simply as the time trend resulting from panel data SFA studies minus the inflation rate. We believe that in the current climate, with rapidly changing volumes (both increasing and decreasing), this reliance on one method may give an inaccurate description of productivity movements. We therefore used three separate measures to calculate the annual real cost change.

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<sup>27</sup> We consider this further in Appendix A as it affects the comparability of SFA analysis.

<sup>28</sup> Regulators that have used this technique include Ofcom and Ofel for BT, Comreg and ODTR for Eircom, OPTA for KPN, the Communications Commission for Telecom New Zealand and the ACCC for Telstra.

### 3.3.1 Stochastic Frontier Analysis: Time trends

In previous network charge controls, Ofcom have used the time trend from comparative efficiency analysis to predict the trend in productivity gains. The time trend which we find over the entire period in our SFA analysis is generally consistent with those found in previous studies of BT's efficiency. In our original analysis we found a time trend between 0% and 1%, showing that costs in nominal terms are only slightly changing over time, increasing by just less than one percent in nominal terms. Our updated analysis finds generally no significant increase in nominal costs through time for main sample period<sup>29</sup>.

However, this method of estimating the movement in the efficiency frontier does not measure the change in costs directly, and it is quite possible that the time trend could be picking up other external factors which have changed over time. This is beneficial for our SFA regression, as we would not want these external effects being included in our measure of inefficiency. However, this is not desirable when looking to estimate the true time trend.

Furthermore, this method looks at costs rather than productivity movements, which may not be directly related. These costs are in nominal terms, so we are introducing inflation fluctuations into the general trend. In addition, estimating movements in productivity in this way does not take into account the capacity utilization effects. Applying an inflation rate of 2.2%, which is consistent with NERA (December 2008), results in an annual real cost trend of -2.2%.

Finally, the frontier as calculated by SFA is based on the assumption that firms' inefficiency is identically distributed in each time period (therefore, with an identical mean and variance). The frontier itself moves over time only with the inclusion of the time variable in the regression model. However, it is possible that over time firms may become generally more or less efficient, meaning that all firms should move towards or away from the actual efficiency frontier. However, such a movement could lead to SFA incorrectly shifting the estimated frontier, therefore overestimating or underestimating all firms' inefficiency.

We would not expect such a uniform shifting in efficiency, but it is not possible to definitively state that it does not exist. Certainly, the ranges of the LEC inefficiency estimates found in previous studies carried out by NERA fluctuate considerably.

We therefore believe that there exist better methods to estimate the movement in productivity over time in order to calculate an accurate estimate of the real unit cost change. We look to directly compare outputs to inputs through total factor productivity methods. Estimates from such methods can also be compared against the SFA results to consider whether the results from our previous analysis are reasonable.

### 3.3.2 Total Factor Productivity: Rate of change

The annual real cost change can be computed directly based upon TFP analysis. In order to calculate the TFP using the rate of change of input and output indices, we used data as follows:

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<sup>29</sup> This result is estimated using data from 1996 to 2006, with structural breaks applied in 1998 and 2004.

- 26 US LECs, using regulatory accounting data from 1996 to 2006. This is a lower number of LECs than for the SFA, as data for calculating TFP was only available at the parent level, for example from Verizon South as opposed to Verizon South Illinois; and
- 10 European telecommunication incumbent operators using financial accounting data from 2002 to 2006, where available, to obtain fixed line business data.

We ran our standard TFP analysis across a panel including European operators, so as to ensure that any results are directly applicable to BT. While we would not intuitively expect there to be a large difference in productivity improvements between operators in Europe and the US, without testing for this it would not be possible to draw robust conclusions. We have therefore run standard TFP models both including and excluding the European operators, to see what effect, if any, they have on our results.

The calculation of TFP growth here looks within a company over time; therefore it is not necessary to adjust the data for comparability. However, for consistency we use the same set of output and input measures across the companies although the reporting standards may differ between them.

The dataset used in our revised TFP analysis was consistent with that used in our original analysis. This is set out fully in Deloitte (2008), appended to this report.

We calculate output and input indices for each of the LECs and European incumbents, using a number of different techniques including the Törnqvist, Laspayers, Paasche and chain methods. The annual difference in each index is calculated to provide a growth rate for outputs and inputs. The subtraction of the annual input growth rate from the annual output growth rate, in theory, provides the TFP. However, many of these companies have relatively unstable volumes. Therefore we estimate the relationship between volumes and our calculated TFP measure to allow for the separation of the TFP measure independent of volume effects.

Once we have removed capacity utilisation effects (using an econometric model as specified below), we estimate an annual productivity increase of 0.5%, using our preferred index calculated using the Törnqvist indexing method for outputs and inputs<sup>30</sup>, based on quantity changes, excluding any price effects. If prices were in fact increasing significantly, we would find a positive average change in *nominal* costs.

Generally, individual TFP estimates are relatively volatile both within and between firms. This is caused directly by some rapidly varying volumes. However, the econometric model which we run to take out capacity utilisation effects finds a clear correlation between TFP and volume changes for all outputs. The econometrics is therefore able to strip out these effects to leave the true TFP of 0.5%, equivalent to a change in real costs of -0.5%. Our final specification is shown in Table 5.

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<sup>30</sup> Other indexing methods were tried but produced worse-fitting models, or had difficulty specifying the constant term.

**Table 5: Output for standard TFP analysis removing capacity utilisation effects**

Explanatory Variables	Coefficient
Change in leased lines	0.188
Change in switched lines	-0.293
Change in local and internet minutes	0.544
Change in long distance minutes	0.096
Constant (TFP estimate excluding capacity utilisation effects)	0.005
<b>Overall R<sup>2</sup></b>	<b>0.54</b>

Source: Deloitte analysis

It should be noted that our model does not take into account movements of firms towards and away from the frontier. However, since the frontier itself is defined by the average of firms in any year, we would expect these movements relative to the frontier to average out at any time, and so these will not affect our results.

### 3.3.3 Total Factor Productivity: Fixed effects growth model

In addition to the 'standard' TFP analysis, we estimated the TFP using a fixed effects econometric growth model, run over the LEC data set constructed for the standard TFP analysis. We have omitted EU operators from this analysis due to a lack of disaggregation of output data.

The model is derived from a Cobb-Douglas production function and a specification of technology commonly used in the classical growth literature. As the LECs produce several outputs we additionally used an adjusted Törnqvist index to combine these into one output measure. Our model is of the form:

#### Equation 1: Production function

$$\ln \bar{y}_{it}^{\text{Adjusted Törnqvist}} = a + w_i + \alpha \ln K_{it} + \beta \ln L_{it} + \lambda \ln M_{it} + g_t + v_{it}$$

where:

- $a$  is a constant;
- $K$  the stock of capital measured in real terms;
- $L$  labour input measured in real terms;
- $M$  materials measured in real terms;
- $w_i$  time invariant firm heterogeneity; and
- $v_{it}$  an error term.

This specification allows the general technology trend to be estimated,  $g$ , whilst accounting for invariant heterogeneity across firms, idiosyncratic technology effects and other movements in output from changes input factors. The former are captured within  $w_i$  whilst the latter are subsumed into the residual  $v_{it}$ .

The results of our estimations from Deloitte (2008) are reported in Table 6. The coefficient on the time trend estimates the growth in TFP to be 1.1% per annum. This is equivalent to a real annual cost change of -1.1%.

**Table 6: Regression output for econometric TFP – Deloitte (2008)**

Explanatory Variables	Coefficient	T-value <sup>31</sup>	Significance
Log of materials	0.023	0.22	0.828
Log of capital	0.506	5.01	0.000
Log of staff compensation	0.201	2.19	0.024
Time	0.011	2.26	0.024
Constant	-12.604	-5.15	0.000
<b>Overall R<sup>2</sup></b>	<b>0.8768</b>		

Source: Deloitte analysis

Following NERA's comments, we have amended our original specification to account for a structural break. The structural break however, was difficult to precisely specify leading us to estimate TFP growth in the range of 0.0% to 1.9%, as outlined in Table 7. This range is driven by whether the break is entertained over just the time trend or all the variables, or when it is specified.

**Table 7: Revised TFP estimates across various specifications – Deloitte (2009)**

TFP growth after the structural break	Structural break: 2001/2002	Structural break: 2002/2003
Structural break applied to time trend only	1.9% (-137.6)	1.6% (-143.3)
Structural break applied to whole model	0.0% (-151.6)	0.0% (-147.2)

Schwartz Bayesian information criterion provided in brackets

Source: Deloitte analysis

To assess the relative merits of each model we calculated the Schwartz Bayesian information criterion which provides a measure of the model fit. The information criterion does not differ substantially between models and therefore we suggest TFP is within the range of 0.0% to 1.9%. This translates into a real annual cost change of between 0.0% and -1.9%.

### 3.3.4 The Ofcom model

Since we undertook our initial efficiency analysis, Ofcom has developed a bottom-up model to measure the average real unit cost change for individual network components, holding volumes

<sup>31</sup> All standard errors were calculated using methods robust to the error term being heteroscedasticity or serially correlated.



constant and controlling for BT's historical catch-up to the frontier. This produces an efficiency assumption for BT of 0% to 5% per year. We believe that this approach is an unusual method and certainly less common than the econometric approaches that we employed in our initial analysis. However, at BT's request, we have taken the Ofcom model, critiqued this, and undertaken a number of adjustments. A full examination of Ofcom's model can be found in Appendix C.

The Ofcom model is based on a number of key assumptions. Whilst we have made adjustments to many of these, there are a number of assumptions that we have either not tested or adjusted. These are set out in Appendix C of this report.

We suggest three adjustments to the Ofcom model which we believe are specifically needed to increase the robustness of the estimate. These are:

- Including distance-related elements in the analysis;
- Using a Törnqvist index, instead of the alternative base year weights; and
- Updating and refining the estimates of BT efficiency relative to the efficiency benchmark of the top decile, based on NERA estimates for 2003/2004 and 2006/2007.

### **Impact of including distance-related components**

The Ofcom model does not include costs and volumes of distance-related components. Only capacity and copper related costs were considered, covering only 65% of the cost base relevant for the LLCC. The results may therefore be biased if cost trends of components included in the analysis differ from the omitted components.

The volume data on the distance-related components in the original Ofcom model were affected by a change in the basis of measurement from a route distance to a radial distance measure. This affected the 2005/2006 and the 2006/2007 volume measures. The inclusion of distance-related elements leads to a range of estimated values from -4.4% to 7.6%. This is considerably wider than Ofcom's initial range and highlights the limitations of the static weighting approach employed by Ofcom. Instead of the static approach, the overall unit cost trend estimate should be based on a continuously weighted index, such as the Törnqvist index which reflects the changing importance of individual components through time.

### **Impact of using the Törnqvist index**

The Törnqvist index is a standard measure used in productivity analysis. It is designed for continuously weighted multi-component analysis, using both the current and the base year cost shares in weighting each component in the index.<sup>32</sup> We used the cost and volume data in the Ofcom model to calculate a Törnqvist input index (cost) and output index (volumes). Törnqvist unit cost indices are computed by dividing the input index by the output index. We measure the CAGR

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<sup>32</sup> Revenue shares would be preferable to cost shares as the weights used in the output index. However, the data on revenue weights of the components considered here does not exist, so cost weights have been used instead for both the input and output indices.

of the Törnqvist unit cost index to form a single starting point for the factor price and catch-up adjustments.

Applying a pure Törnqvist index, which carries the implicit assumption that the CVE is one, reduces the adjusted efficiency range from between -4.4% and 7.6% to between -0.4% and -3.2%.

### Updating BT relative efficiency estimates

Ofcom includes a correction for BT's catch-up to the top efficiency decile between 2003/2004 and 2006/2007. The adjustment acknowledges efficiency improvement by BT over and above the efficiency improvement driven by technological progress (frontier shift). Ofcom uses a range of NERA values to provide start and end points for this catch-up and we have sought to refine these estimates based on NERA's preferred specifications in up-to-date studies. These are:

- Starting estimate: The range for the starting point of BT's relative efficiency used by Ofcom, between 0.5% and 3.8% less efficient than the decile in 2003/04, is sourced from NERA estimates in 2005<sup>33</sup>. To achieve a narrow range based on best estimates, we have identified the estimate of the inefficiency from the preferred specification, as identified in NERA (2005). This indicates BT is 1.05% less efficient than the top decile.<sup>34</sup>
- End point: Ofcom takes its estimate from a report focusing on the relative efficiency of BT Openreach. NERA has since provided updated estimates based on analysis focusing on efficiency of BT's network operations.<sup>35</sup> This analysis finds that the efficiency of BT's network operations is between 3.8% and 6.8% higher than the top decile. The estimate from NERA's main model is that BT is 6.0% above the decile.<sup>36</sup>

In cost terms, we have used a single starting point of 1.05% and a range from -3.8% to -6.8% for the catch up adjustment. Applying the catch-up adjustment to the efficiency range calculated in the previous section (of -0.4% to -3.2%) reduces the range further to -0.5% to -1.5%.

### Summary of adjustments to the Ofcom model

The original range for unit cost reduction estimated by Ofcom varies from 0.1% to -6.2%. Including the distance-related components produces an even wider range due to the static weighting approach discussed above. Using the Törnqvist index to address the issues with the static weighting approach produces a reasonable range of unit cost CAGR for the time period. The update to the start and end points of BT relative efficiency leads to a further narrowing of the range, towards the lower end, resulting in a range of real unit cost change estimates between -0.5% and -1.5%.

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<sup>33</sup> NERA (2005), table 5.1

<sup>34</sup> NERA (2005), page 35

<sup>35</sup> NERA (December 2008)

<sup>36</sup> NERA (December 2008), table 4.6 and table 4.7

### 3.3.5 Summary of the annual real cost change analysis

In the table below, we summarise the various estimates of annual real unit cost change. This demonstrates a range of annual real cost change from -2.2% to 0.0% per year.

**Table 8: Real annual cost change**

Model: BT relative to top decile in 2006/2007	Estimated time trend	Annual real cost change
Deloitte SFA <sup>2</sup> : -6.3%	0%	-2.2%
Deloitte FE TFP <sup>3</sup> : N/A	N/A	0% to -1.9%
Deloitte indexation <sup>4</sup> : N/A	N/A	-0.5%
Ofcom model revised by Deloitte	N/A	-0.5% to -1.5%
NERA SFA (table 4.7 -6%) <sup>5</sup>	0%	-2.2%
NERA FE (table 5.2) <sup>5</sup>	N/A	-1.9%

1) Source: NERA SFA analysis in December 2008 and revised Deloitte SFA analysis in Section 3 below

2) Source: revised Deloitte analysis in Appendix A of this report

3) Source: revised Deloitte analysis in Appendix B of this report

4) Source: original Deloitte analysis in June 2008 paper

5) Source: NERA analysis in the December 2008 paper

## 3.4 Application of future catch-up

In our initial report, we stated that BT's comparative efficiency was above the top decile and therefore no efficiency assumption was required to catch BT up to the decile over the price control period. Our updated analysis, described in Appendix A, finds BT's comparative efficiency in 2006 to be 6.3% above the decile. BT has requested that we update our analysis to consider the appropriate catch-up factor in cases where BT is currently exceeding the decile.

Ofcom has traditionally computed a single measure for BT's efficiency assumption by combining the catch-up requirement with the annual real cost change. Through the application of the catch-up requirement, Ofcom has taken a stance that BT should be at the efficiency level of the top decile by the end of the price control period. This is accepted by Ofcom as a means for encouraging efficiency improvements and has been consistently applied within past price controls.

Such reward and penalty structure in a price control should be applied in a symmetric manner, incentivising BT to outperform. Therefore the catch-up element should also be applied where BT is found to be above the efficiency decile. Not applying the catch-up element symmetrically may:

- Provide reduced incentives for BT to continue to outperform efficiency targets.
- Be viewed as inconsistent with Ofcom's previous approach to catch-up in instances where BT was found to be comparatively inefficient and would imply a more stringent definition of efficiency than that applied to BT than in the past.
- Increase the risk faced by BT, due to more stringent efficiency standard, as well as due to an increase in regulatory risk. These could have the effect of increasing the required

return by investors, and hence should potentially be reflected in the allowed weighted average cost of capital in the price control model.

Ofcom has not clearly set out a view on the applicability of future catch-up. However, the Ofcom model adjusts the measured unit cost trends for historical catch-up before drawing the frontier shift estimate. Where BT has overtaken the top decile, Ofcom's analysis requires BT to maintain that level of efficiency instead of allowing the decile to catch up to BT. The asymmetrical treatment of the top decile as a benchmark imposes a substantial stricter efficiency standard on BT than Ofcom has applied previously.

Our revised SFA analysis summarised in Section 2.1 showed that BT's comparative efficiency in 2006 is better than that of the top decile by 6.3%. This aligns with NERA's analysis which places BT at 6% above the decile. These comparative efficiency estimates are computed based on BT's position in 2006 and are to be used in price controls which end in 2012.

As stated above, in previous price controls, Ofcom has required BT to be at the decile at the end of the price control and has applied a constant glide path over the regulatory period to achieve this. A key question must therefore be where BT's efficiency will lie at the start of the price control. Given the analysis outlined above and in detail in Appendix A, we believe it is probable that BT will lie above the decile at the start of the regulatory period. Therefore, we believe that BT should be allowed some form of future annual catch-up in the calculation of the efficiency assumption. This catch-up amount should be subtracted from the annual real cost change to give the final BT efficiency assumption.

### 3.5 Conclusions

This report has detailed our analysis into BT's comparative efficiency, and the rate at which this can be expected to change over time. Our results are summarised in table 9. A positive figure implies a cost increase.

**Table 9: Summary of economic findings**

Comparative Efficiency Approaches: The catch-up effect*	Productivity frontier shift approaches: The annual real cost trend
<b>SFA:</b> BT is 6.3% more efficient than top decile. Assuming a 6 year catch-up period, the catch-up is 1% per year	<b>SFA time trends:</b> -2.2%
	<b>TFP fixed effects model:</b> 0.0% to -1.9%
	<b>TFP rate of change:</b> -0.5%

Source: Deloitte analysis. \* A positive figure implies a cost increase

For the annual real cost trend, we give greater weight to the SFA time trends and TFP fixed effects model since these are econometrically more robust (although note the discussion about structural change). However, the TFP rate of change analysis which incorporates European as well as US data supports our two favoured approaches.

Through these analyses, we conclude that:

- When compared to the US LECs, BT's network operations are 6.3% more efficient than the top decile;
- Using the most theoretically appropriate indexation technique, our standard TFP model gives a real annual cost change of 0.5% per year; and
- Our 'econometric TFP' model estimates a real annual cost change of 0% to -1.9% per year.

These results are summarised in table 10, below.

**Table 10: Revised estimates of BT efficiency**

Model: BT relative to top decile in 2006/2007	Estimated time trend	Annual real cost change
Deloitte SFA: -6.3%	0%	-2.2%
Deloitte FE TFP: N/A	N/A	0% to -1.9%
Deloitte indexation: N/A	N/A	-0.5%

*Source: Deloitte analysis*

As BT is above the top decile by 6.3%, a future catch-up component allowing the decile to catch-up to BT may be appropriate. For example, assuming a 6 year constant glide path for the decile and BT to equalise leads to the calculation of around a 1% per year catch-up. If this assumed catch-up were applied to our results above, the BT efficiency assumption would be reduced to between 0.0% and -1.2%, assuming that a positive change in costs is unrealistic.

## Appendix A Efficiency Estimates Using SFA

This section analyses, and comments on, NERA's stochastic frontier analysis, which was set out in NERA (December 2008).

SFA analysis estimates the relative efficiency of different companies in the sample, and the efficiency change due to movement in the efficiency frontier. In summary, analysing US LEC's data under various specifications from 1996-2006, NERA's SFA estimation finds that:

- Annual real cost change: Cost change due to frontier shift has slowed down since the late 1990s, and is now between -2.2% and -2.9% in real terms; and
- Comparative efficiency: BT is significantly above the efficiency benchmark applied to it, with relative efficiency of 6% above the top efficiency decile.

Amendments to Deloitte SFA specifications suggested by NERA lead to results that are in line with the above.

### A.1 Background

Ofcom engaged NERA to critique the SFA and econometric TFP analysis by Deloitte. This critique was published by Ofcom in May 2008<sup>37</sup>. Following this, NERA updated SFA and econometric TFP analysis and results in NERA (December 2008). NERA provided us the model specifications and data used in the SFA analysis of their updated study.

NERA have estimated the frontier shift as the coefficient on a time trend included in the estimation of the cost frontier. In the following model it is therefore taken as the coefficient  $\delta$ .

#### Equation 2: Estimation of cost frontier

$$\ln C_{it} = a + \sum_{k=1}^K \beta^k \ln Y_{it}^k + \sum_{l=1}^L \alpha^l \ln N_{it}^l + \delta t + u_{it} + v_{it}$$

where:

$\ln$  is the natural logarithm;

$i$  represents the individual company observation;

$t$  represents the year of the observation;

$C$  is the total cost;

$a$  represents the fixed costs;

$Y^k$  are the output variables;

$N^l$  are network (environmental) variables;

$\beta^k$  and  $\alpha^l$  are the coefficients on the output and network variables respectively;

$u$  is the inefficiency component; and

$v$  is the random error component.

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<sup>37</sup> NERA (May 2008)

This method has previously been adopted in charge controls by Ofcom and allows the shift in the frontier to be identified whilst controlling for:

- Output changes;
- Network variables;
- Idiosyncratic improvements in productivity; and
- Economies of scale.

To estimate this model NERA has used data for the US Local Exchange Carriers (LEC) taken from the ARMIS database, as maintained by the Federal Communications Commission (FCC). They have estimated the model using data over the period 1996-2006. In addition, they have estimated the model truncating the sample for the first three years, hence for the period 1999-2006.

NERA estimates the nominal annual cost time trend to be between -0.7% and 0.0%, using the 1996-2006 and 1999-2006 samples respectively<sup>38</sup>. All variables in the model are measured in nominal terms meaning the estimated frontier shift will be vulnerable to bias from any systematic movement in input prices. To correct for this, NERA net off the US general inflation of 2.2% per year<sup>39</sup>. The annual real cost change, estimated from the frontier shift, is therefore estimated to be between -2.9% and -2.2% in real terms, with the -2.2% corresponding to NERA's preferred sample period from 1999 to 2006.<sup>40</sup>

Using the data and specifications provided to us by NERA, we have been able to replicate the precise estimation results of NERA's preferred specifications. These are summarised in Table 11 and provided in full detail in Appendix E.1<sup>41</sup>.

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<sup>38</sup> NERA (December 2008), Table 4.1, Table 4.2 and page 14

<sup>39</sup> To purge the estimate of inflationary influences, NERA take out observed economy wide inflation. However, this may not be an appropriate measure of actual inflation observed by the LECs. Our calculations of the overall efficiency change which don't rely on SFA use instead specific price indices for differing inputs. We believe this approach is likely to more accurately account for inflation actually observed

<sup>40</sup> NERA (December 2008), page 11 expresses preference for the 1999-2006 sample period.

<sup>41</sup> Appendix E.1.1 replicates NERA's specification and results for the full sample period 1996-2006, and Appendix E.1.2 shows the result for NERA's preferred specification and sample period 1999-2006.

**Table 11: NERA's preferred regression results**

Variable	1996-1998 sample		1999-2006 sample	
	Coefficient	Z-ratio	Coefficient	Z-ratio
ln(Stranded Asset Ratio)	0.481	0.000	0.504	0.000
ln(Leased lines per Switched Line)	0.055	0.000	0.023	0.133
ln(Leased lines per switched line) × D96-98	-0.044	0.004		
ln(Switched minutes per switched line)	-0.004	0.706	0.008	0.537
ln(Switched minutes per switched line) × D96-98	0.012	0.479		
ln(Total sheath per switched line)	0.243	0.000	0.201	0.000
ln(Total sheath per switched line) × D96-98	0.061	0.000		
ln(Duct per switched line)	0.005	0.600	0.000	0.968
ln(Duct per switched line) × D96-98	-0.022	0.069		
ln(Population density)	0.036	0.035	0.029	0.062
ln(Business-residential ratio)	0.029	0.313	0.075	0.029
ln(Fibre proportion)	0.102	0.000	0.116	0.000
NYNEX dummy variable	0.126	0.012	0.148	0.002
Mid-West dummy variable	-0.205	0.000	-0.184	0.000
Verizon Anomaly dummy variable	-0.021	0.238	-0.044	0.026
Time	-0.007	0.044	-0.001	0.888
Time × D96-98	-0.025	0.000		
Constant	0.044	0.830	-0.225	0.246
Log-Likelihood	906.2		625.5	

Source: NERA (December 2008)

### A.1.1 Data used in NERA's analysis

NERA's estimates are based on a different dataset to that compiled by Deloitte. Differences in datasets can have a significant effect on the preferred model specification, estimation approach and the results used. In our original report, Deloitte (2008), we used the identical data to estimate the frontier shift and relative efficiency of BT Network. NERA reviewed our dataset and in their comments document set out several discrepancies we discuss in turn below:

- NERA have applied a WACC of 11.4%, which is the ROCE permitted for BT Wholesale, where as in our report we have used a weighted WACC of BT Wholesale and BT Openreach. Since our efficiency analysis concerns products supplied by both Wholesale and Openreach, we believe a weighted WACC is more appropriate. However, we note that the impact of utilising a WACC of 11.4% is likely to improve BT's relative efficiency from our initial estimation. The impact of this change on the time trend is less clear, however it



would only have an impact to the extent that capital and labour mix changes over time and therefore we assume the impact to be minimal;

- We have excluded payphones from our analysis since they are a retail service. We are unable to comment on NERA's calculation of depreciation and GRC and why they may differ from our own calculations; however, we agree with NERA that these differences are not large. NERA further states that they have some concerns over our data for 1996. In response to this, we have rechecked the processing of our data and can reconcile it to the source data. As such, we are unable to explain why differences may have arisen between our dataset and NERA's, other than stating that we believe our dataset to be correct;
- In some cases NERA point to their output data being superior because using this data yields a higher log likelihood. However, we disagree with this statement. Firstly, the data should be preferable only if it is accurate. Only then should we try to find a model that fits. We have rechecked our data on total sheath and can reconcile this to the FCC data. We therefore support the use of our dataset but note that a move to the NERA set has little material impact on either the comparative efficiency or time trend;
- As this is a time series model and the other explanatory variables are changing over time, then we believe that the population density should also be included as time variant. Again we note that a higher log-likelihood value should not be used as a sole determinant of whether one variable or other should be chosen, as many unintuitive variables could have a positive effect on goodness of fit. In this case, we believe a time variant population density figure is the appropriate measure to include in the analysis;
- We have excluded Verizon Washington from the sample due to it being an outlier in respect of its operating characteristics, including lower population characteristics and higher business to residential line ratio as a result of it being a city-based operator. We do not believe that an analysis of its management structure would provide an adequate reason to readmit it to the sample; and
- SNCT has been excluded by us due its data being inconsistent. NERA has chosen to adjust its data, however we are unsure how such data could be adjusted without imposing arbitrary assumptions on the dataset. Given the relatively large sample size and its ability to cope with reduced degrees of freedom, we believe it is better to remove SNCT than to approximate its data. It is unlikely that this has a large impact on the results, as it is one company out of 67.

In the remainder of this section we discuss the effect of the data differences on results reported by NERA, as well as various issues with NERA's modelling approach and preferred specifications drawn from it.

## A.2 Investigation of NERA's specifications

We investigated in further detail:

- NERA's estimates based on NERA's replication of Deloitte dataset and Deloitte equations;

- The constant returns to scale (CRS) assumption required for NERA's preferred models;
- Effect of removing insignificant variables from NERA's specifications;
- The nature of the "stranded assets" variable and its inclusion in total cost models; and
- The existence and implications of a second structural break in NERA's specifications.

### A.2.1 NERA's reported results on "Deloitte values"

NERA reports estimations of their preferred model specifications in Table 4.3, Table 4.4 and Table 4.5 in NERA (December 2008), using their dataset adjusted for "Deloitte values". The "Deloitte values" are obtained through various adjustments to the NERA dataset. However, applying the specifications provided by NERA to the actual Deloitte dataset does not produce the results shown by NERA. We therefore cannot accept the results shown in Table 4.3, Table 4.4 or Table 4.5 as being based on Deloitte data or specifications.

Section A.4 below discusses updated results on Deloitte dataset and specifications, amended to take into account suggestions by NERA we accept.

### A.2.2 CRS restriction (required for unit cost models)

NERA's preferred specifications are estimated on the basis of unit costs, calculated as total costs divided by the number of switched access lines. NERA justify this on the basis of the results presented in Table 4.1 and Table 4.2 of the NERA (December 2008), particularly it seems on the basis that the log-likelihood of their estimation of the unit cost model is near that of the total cost model.

However, invariance of the log-likelihood is not a sufficient condition to justify the CRS restriction on the total cost model. Imposing CRS on the model restricts the coefficient on the log of switched access lines to one. This restriction can be tested with a simple t-test based on the results reported by NERA as well as our exact replication of them reported in Appendix E.1. In all cases the restriction is rejected: the coefficient on the log of switched access lines is near 0.5, with relatively small standard error. To reach a conclusion that the coefficient is actually unity, we would have to add the standard error 10 times on top of the estimate. As the conventional 95% confidence bands are based on variation of 2 times the standard error, we can be very confident that the CRS restriction is in fact *not* supported by the NERA results.

The indication is that the models in (log) total cost terms should be considered. The time trend from these models reported by NERA varies between zero (insignificant) on the full sample and 1% cost *increase* through time on NERA's preferred sample of 1999-2006<sup>42</sup>. However, NERA has not included the stranded assets variable in these specifications. Based on the unit cost models,

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<sup>42</sup> NERA (December 2008), Table 4.1 and Table 4.2

NERA reports that including the stranded variable increases time trend further by little over 1% per year<sup>43</sup>.

### A.2.3 Insignificant variables in NERA's specifications

NERA's preferred models are over specified, including various statistically insignificant variables based on the standard Z-ratios reported by NERA in Table 4.1 and Table 4.2 of the December 2008 report. The Z-ratios indicate that the variables do not have useful explanatory power in the model, and should therefore be excluded from it. NERA does not discuss justification of retaining the variables or any tests conducted to investigate the relevance of their inclusion.

We employed standard likelihood ratio tests to investigate the effect the insignificant variables were having on the other results. The tests accepted the exclusion of the variables in question, consistently with their Z-ratios. Their exclusion did not in this case lead to material changes in the coefficients on the other variables.

### A.2.4 The “stranded assets” variable

NERA advocate the use of a variable to capture the cost of assets that have become stranded due to fall in demand for switched lines. This variable is defined for each company as<sup>44</sup>:

#### Equation 3: Stranded asset calculation

$$\text{Stranded Asset Ratio} = \text{Max (Switched Lines)} \div \text{Switched Lines}$$

Typically the number of switched lines for the LECs peaked in either 2000 or 2001, being, of course, lower in the years leading up to the peak as well as in those following the peak. The variable produces a value of 1 (or 0 after the log transformation) for the year with most active switched lines, and higher than zero for all other years, typically in a shallow U-shape. Therefore, it does not distinguish between increasing demand (or network utilisation) from falling demand – or from asset stranding as meant for by NERA. Instead, the end result is another variable that captures movements in network utilisation relative to the peak.

The variable does have clear explanatory power for both unit and total costs (see immediately below). It is not clear, however, that it should be interpreted as a variable capturing the effect of asset stranding.

Possible shortcomings of the variable aside, NERA only reports results with the stranded assets variable included in the unit cost models. We tested it in the models of total cost – the standard tests advocate the inclusion of the “stranded asset ratio” here also. The caution about this variable therefore stems not from its explanatory power, but from the uncertain interpretation of what cost driver of telecommunications operations it actually represents.

<sup>43</sup> NERA (December 2008), page 14

<sup>44</sup> Page 6, NERA (December 2008): “NERA's Analysis of the Efficiency of BT's Network Operations”

Further, NERA suggests that the stranded assets variable would control for effects captured by the second structural break identified by Deloitte. However, as discussed further below, the second structural break is found consistently significant in models with the stranded variable included.

The stranded variable is therefore not a substitute for a second structural break.

### A.2.5 Existence and implications of a second structural break

One of the standard tests of model stability is testing for structural breaks. Deloitte analysis found two structural breaks in the time trend estimate from the SFA models, one in 1999 and another in 2004.

NERA dismisses the inclusion of a second structural break on grounds of “good econometric practice”, preferring, it seems, the “richer specifications” reported in NERA (December 2008). In particular, NERA corrects for “Verizon Anomaly” and includes the stranded asset variable as well as various other variables not included in the original Deloitte specifications. However, stability tests of NERA’s preferred specifications on data provided to us consistently find the second structural break to be significant, particularly on the estimate of the time trend.

We tested the inclusion of structural break in 2004, both in the specifications on the full dataset from 1996 to 2006 (as a second structural break) and on the NERA’s preferred restricted sample from 1999 to 2006. The detailed results based on NERA’s dataset and specifications are shown in Appendix E.2.

The results show that none of the “Verizon Anomaly”, stranded assets variable, or the other variables included for a richer specification by NERA sufficiently control for the effects leading to the instability of the time coefficient. Good econometric practice dictates that structural breaks are investigated, and taken into account where indicated significant by the data — either leading to improvement of the model, or to caution in the interpretation of the results. Ignoring a significant structural break could lead to the estimated coefficient being biased and inconsistent, or interpreted incorrectly.

Introducing the later structural break to NERA’s main models makes the coefficient on time trend insignificant during the main estimation sample (although it was already insignificant in NERA’s preferred specification using data from 1999 to 2006). However, after the structural break, the coefficient indicates an overall trend increase in nominal unit costs of around 4% per annum (see Appendix E.2.1 and E.2.2).

In other words, the time trend – meant to capture gradual TFP frontier improvement – is highly unstable also in NERA’s preferred specifications. This could imply that the models remain mis-specified for TFP analysis, in that they are not controlling for the effect of some variable that should be included in the model, which is therefore reflected in the time coefficient. An alternative conclusion, if the coefficients are believed, is that in recent years there has been around 4% trend increase in unit costs according to these models. We note that the instability of the time coefficient does not necessarily invalidate the relative efficiency estimates produced by the analysis.

The above results are, however, based on NERA's unit cost models, whereas we have shown above that the CRS assumption required for it is not supported by the model results. Therefore, we have investigated further development of the alternative NERA specifications, below.

### A.2.6 Modelling approach used for NERA's preferred models

SFA econometric specifications have to make an assumption about the basic nature of the company-specific inefficiency terms: they are either restricted to be constant through time ("time invariant specification"), or they are allowed to vary through time ("time variant specification"). The validity of the restriction can be tested following estimation of the time varying models.

NERA has used the time invariant specification of the company-specific inefficiency – the efficiency score of the companies in sample is assumed to be constant throughout the sample. This imposes NERA's a priori expectation that there is no general catch-up of the companies towards the frontier in the sample of the LECs used.

Deloitte (2008) did not impose this restriction a priori, and we continue to believe this is the preferable approach.

However, the restriction does not seem to make material difference to NERA's results. As this section comments and builds on NERA's specifications, we report results employing the restriction preferred by NERA.

## A.3 Further development of NERA's SFA specifications

Finally, we investigated the result of using standard modelling practise from the starting point of the model specifications put forward by NERA in Table 4.1 and Table 4.2. In doing so, we took into account the following lessons from the above discussion:

- As the CRS restriction is not supported, the model should be estimated in terms of total cost rather than unit costs;
- The starting point models did not include the insignificant variables in the original NERA specifications;
- As the purpose of this exercise is to find the preferable result using NERA's specifications, we did not exclude the "stranded asset ratio" from the analysis a priori, regardless of the uncertainty of its interpretation; and
- We tested for the relevance of the second structural break.

Finally, the models were pared down using the likelihood ratio test. Table 12 below shows the results of the estimation. The full estimation results are shown in Appendix E.3.

**Table 12: Implied nominal annual cost change and comparative efficiency in refined NERA specifications**

Model	Implied annual cost change (nominal)		BT efficiency relative to top decile in 2006/07
	<i>main estimate</i>	<i>2004-2006</i>	
Full sample 1996 – 2006, time invariant inefficiency model <sup>1</sup>	-0.7%	3.6%	6.3% above decile
Restricted sample 1999 – 2006, time invariant inefficiency model <sup>2</sup>	0%	3.9%	3.3% above decile

Source: Deloitte analysis of NERA data and specifications

1) Appendix E.3.1

2) Appendix E.3.2

Overall, these results are largely consistent with those produced by NERA in December 2008 from the unit cost models, with the exception of the structural break on the time trend estimate.

The results again show that the time trend estimate, and therefore the implied TFP change, is unstable in the later part of the estimation period. In the full sample the effect of the implied TFP change, in nominal terms, varies from 2.3% in the early years (1996-1998) to 0.7% in the mid sample (1999-2003), again to an implied TFP of -3.6% per annum (implying an *increase* in costs) in the later years of the sample (2004-2006). The variance is also shown in the results estimated on the restricted sample from 1999 to 2006 preferred by NERA.

As above, two alternative conclusions might be drawn from the above results. First, the results could imply that the models remain misspecified for TFP analysis, in that they are not controlling for the effect of some variable that should be included in the model, which is therefore reflected in the time coefficient in the later part of the period. Given the second structural break, the time trend estimate used should be the main estimate from a model with two structural breaks, as it provides the estimated time trend purged from influence of the instability in the later sample. Alternatively, if the model and coefficients are believed, the conclusion would be that in recent years there has been between 3.6% and 3.9% trend increase in costs (lessening of productivity) as captured by the TFP frontier).

## A.4 Amended Deloitte results after NERA comments

In the discussion of NERA's critique in this section, we have accepted the potential relevance of the stranded asset variable. Also, there seem to be a valid basis for including the dummy variables used by NERA for the NYNEX and Mid-West companies. We have therefore re-estimated our original equations with those variables added in.

We estimated the Deloitte model with the additional variables both taking into account the second structural break and ignoring it (so as to produce results on a consistent basis with NERA). We did not use the a priori restriction of time invariant inefficiency; instead, we tested for its relevance. For both models, the time variant specification was indicated preferable by the regression output.

The full econometric results are shown in Appendix F. Table 13 summarises the results for the time trend and BT efficiency relative to the top decile.

**Table 13: Implied nominal annual cost change and BT relative efficiency from Deloitte data**

Model	Implied annual cost change (nominal)		BT efficiency relative to top decile in 2006/07
	main estimate	2004-2006	
Full sample 1996 – 2006, two structural breaks <sup>1</sup>	0.0%	4.0%	6.3% above decile
Full sample 1996 – 2006, one structural break <sup>2</sup>	0.0%	N/A <sup>3</sup>	7.6% above decile

Source: Deloitte analysis

1) Appendix F.1

2) Appendix F.2

3) There is no separate estimate for the later period from a model without the second structural break

The results are largely consistent with those found by NERA on their preferred specifications in December 2008. Also, the effect of the second structural break on the time trend is consistent with the results based on NERA's dataset. Given the second structural break, the time trend estimate used should be the main estimate from a model with two structural breaks, as it provides the estimated time trend purged of the influence of the instability in the later sample.

## A.5 Summary of SFA findings

We have several concerns regarding NERA's approach, preferred models and the variables included in the SFA:

- NERA's preferred models are based on calculation of unit costs, employing an assumption of CRS with respect to the number of switched access lines. NERA argues that this assumption (restriction of the model) is supported by the data and tests undertaken. However, analysis of results reported in Table 4.1 and Table 4.2 of NERA (December 2008) show the assumption is not supported. The preferred approach should therefore be to estimate the models in terms of total instead of unit costs per switched access line.
- NERA advocates the use of a variable to capture the cost of assets that have become stranded due to fall in demand for switched lines. However, the variable as constructed is perhaps better interpreted as a proxy for network utilisation (increasing as well as decreasing) than asset stranding. The variable does improve the model fit of the models, so we suggest it is included in the estimations, although the ambiguity over its interpretation should be kept in mind.
- NERA has critiqued Deloitte's inclusion of a second structural break in the model in 2004, preferring instead to introduce a Verizon sub-group specific dummy variable (VDummy), the stranded asset variable, and a "richer specification" in the models. Our analysis of NERA models show that none of the above are a sufficient substitute to allowing a structural break in 2004, particularly on the time trend.



- NERA's preferred "richer specifications" lead to their models being over specified. Although we found that the inclusion of the variables in this case does not materially affect the estimated coefficient on the time variable, there is no reason to retain them.
- Adjustments to the NERA's estimation, using their models as starting point, would lead us to conclude that the time trend during the main sample period (1999-2003) is between zero and -0.7%, and strong positive between 3.6% and 3.9% during the later sample period (2004-2006). However, the later sample period estimate is affected by some factors not controlled for by the model.

The above comments notwithstanding, our analysis finds that improvements to the NERA total cost specifications lead to results that are consistent with those reported by NERA from their main models. The main difference between the original and refined models is the treatment of the structural break in 2004. NERA has not included this in their specifications, on a point of principle. Deloitte has included the break, as it shows instability in the time trend estimate that is used to draw the TFP estimate from the SFA analysis.

Table 14 below summarises the implied annual real cost change from SFA specifications preferred by NERA. The results from the revised NERA specifications discussed above are consistent with the results shown here.

**Table 14: Implied annual real cost change from NERA (December 2008) SFA models**

Model	Estimated time trend	Annual real cost change
NERA main model (1999-2006)	0%	-2.2%
NERA main model (1996-2006)	-0.7%	-2.9%

*Source: NERA (December 2008), Tables 4.1, 4.2, 4.7 and page 14 for the inflation adjustment*

The overall conclusion from the SFA analysis regarding TFP estimates should be that the estimate is not stable. Further, the instability seems to affect mainly the estimated time trend. Although it is likely that the TFP improvement has slowed down significantly since the 1990s, as also reported by NERA, it is perhaps not credible to think that TFP is worsening at a rate above 3% per year as implied by the structural break taken at face value. It is more likely that some additional variable, not included in the model, is influencing the estimated time trend between 2004 and 2006, and therefore appears as a structural break. Given the second structural break, the time trend estimate used should be the main estimate from a model with two structural breaks, as it provides the estimated time trend purged from the influence of the instability in the later sample.

To have full confidence in the TFP results from the SFA analysis, the causes of the later structural break should be analysed further, and captured by additional variables introduced in the model. Otherwise, less weight should be placed on the SFA time trend relative to other methods, as it is known to be influenced by variables not controlled for in the estimation.



## Appendix B Econometric Total Factor Productivity Model

In Deloitte (2008), summarised in section 2, we estimated the real annual cost change using three differing techniques. Comments on the TFP approach were outlined in NERA (May 2008). As part of these comments NERA outlined several alternative specifications, based on our original dataset. This section focuses on amending our original models to reflect certain proposed adjustments from NERA and provides a critique of NERA's comments.

Our amended models find a revised estimated of TFP growth between 0% and 1.9%, this compares to between 0.5% and 1.1% presented previously. This translates to an annual real cost change of between 0% and -1.9%.

### B.1 The model

The econometric total factor productivity approach is derived from the classical economic growth literature<sup>45</sup>. It involves estimating a model of output, aggregated using a Törnqvist index, that controls for:

- Factor inputs;
- Economies of scale;
- Capital; and
- Labour.

Given the model controls for these factors; the coefficient on a time trend included in the model can be interpreted as the shift in total factor productivity. Formally, the model is of the form:

#### Equation 4:

$$\ln \bar{y}_{it}^{AdjustedTörnqvist} = a + w_i + \alpha \ln K_{it} + \beta \ln L_{it} + \lambda \ln M_{it} + gt + v_{it}$$

where:

$a$  is a constant;

$K$  the stock of capital measured in real terms;

$L$  labour input measured in real terms;

$M$  materials measured in real terms;

$w_i$  time invariant firm heterogeneity; and

$v_{it}$  an error term.

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<sup>45</sup> Islam N, 1995: 'Growth Empirics: A Panel Data Approach', The Quarterly Journal of Economics, Vol. 110, No. 4 (Nov 1995), pp. 1127-1170; also Mankiw NG, Romer D, Weil DN (1992): 'A Contribution to the Empirics of Economic Growth', The Quarterly Journal of Economics, Vol. 107, No. 2 (May 1992), pp. 407-437

In order to estimate this model, data was drawn from the ARMIS database for the US LECs. A full account of all adjustments made to this data is included in Deloitte (2008) appended to this report.

Throughout this section all estimates and variables are stated in real terms. This differs from the SFA analysis in the previous section where variables in the model were in nominal terms.

## B.2 Specification of unobserved heterogeneity

NERA's comments on our original approach discuss that the 'fixed effects' (FE) estimation procedure we have employed is not necessarily appropriate, and that alternative models including random effects (RE) and GLS should additionally be considered. In order to address these concerns we have begun by firstly ensuring we have modelled the time invariant unobserved heterogeneity correctly.

Firm-specific heterogeneity has been found to be statistically significant across a broad range of specifications. This suggests the appropriate estimation procedure is either RE or FE estimation. The appropriateness of each technique is contingent on the underlying relationship between the heterogeneity and other explanatory variables in the model. If for example heterogeneity is correlated to the variables, parameter estimates from FE will be consistent<sup>46</sup>, whilst inconsistent using RE. Conversely if no correlation is prevalent, both estimators will be consistent, but RE will be efficient<sup>47</sup>. Formally, our choice of estimation therefore relies on whether the following condition holds:

### Equation 5:

$$\text{corr}(w_i : K_{it}, L_{it}, M_{it}) = 0$$

We can test for this condition by calculating the correlation observed between the estimated firm heterogeneity and explanatory variables. Alternatively, we can deploy the Hausman test<sup>48</sup>.

The Hausman test has been used extensively throughout the literature, for example see Forbes (2000)<sup>49</sup>. The test is based on exploiting the distance between parameter estimates under FE and RE. If Equation 5 holds there should be little parameter estimate divergence between RE and FE as both will be consistent. However, if the condition fails parameter estimates will significantly differ. The formal statement of the test statistic is:

### Equation 6:

$$\text{Hausman} = N(\Phi_{FE} - \Phi_{RE})' \text{Var}(\Phi_{FE} - \Phi_{RE})^{-1} (\Phi_{FE} - \Phi_{RE}) \sim \chi_K^2$$

<sup>46</sup> A parameter estimate is consistent if as the sample size tends to infinity the estimator converges to the true underlying parameter value.

<sup>47</sup> The efficient estimator is the estimator with the lowest variance.

<sup>48</sup> Hausman JA, 1978: 'Specification Tests in Econometrics'. *Econometrica*, Vol. 46, No. 6 (November 1978), pp. 1251-1271

<sup>49</sup> Forbes KJ, 2000: 'A reassessment of the relationship between inequality and growth'. *American Economic Review*, Vol. 90, No. 4, pp. 869-887

where:

$\Phi$  is a vector of parameters  $\alpha$ ,  $\beta$  and  $\lambda$ ;

$\chi_K^2$  is the chi-squared distribution with  $K$  degrees of freedom;

$K$  is the number of explanatory variables;

$VAR$  is the variance; and

$N$  is the total number of observations.

In Table 15 we have tested between random and fixed effects procedures, calculating both the Hausman test and correlation.

**Table 15: Tests between random and fixed effects models**

Test specification	Hausman Statistic	Correlation
Test 1 – No structural break or account for serial correlation	8.08 (0.0886)	0.57
Test 2 – Structural break 2001, no account for serial correlation	9.59 (0.0876)	0.63
Test 3 – Structural break and account for serial correlation	21.70 (0.0006)	0.73

*Source: Deloitte Analysis. The P-value is reported in brackets. This is the probability of incorrectly rejecting the null that the differences in parameters across the model are not systematic. Correlation is calculated using estimates of the heterogeneity from the fixed effects procedure.*

The Hausman test and correlation results show that broadly there appears to be a reasonable degree of correlation between the unobserved heterogeneity and explanatory variables. Correlation is particularly apparent in the model allowing for serial correlation. This result suggests the fixed effects procedure should be applied in preference to random effects and other models requiring heterogeneity to be random. The regression output behind these tests is reported in Appendix E.

The finding of correlation is not unsurprising as a priori we would expect static firm specific factors to effect factor inputs decisions. For example, differing geographic characteristics will partially determine the capital investment required to achieve a given number of lines or minutes.

Based upon the analysis presented here, we reject the results derived from models in NERA (May 2008) reliant on the condition in Equation 5 holding. This leaves primarily their estimation based fixed effects and including a structural break. The frontier shift from this estimation is calculated to be 1.9%. It should be noted that despite our rejection of their alternative models on average across all NERA models, the frontier shift is still estimated to be 1.9%<sup>50</sup>.

<sup>50</sup> This is based on NERA's models 2 to 8 in table 2, and the coefficients on the latter time trend. Model 1 has been excluded due to their preference for a structural break, whilst model 9 is omitted as it is clearly an outlier.

## B.3 Structural breaks

NERA introduce a structural break in 2002 by establishing that the output index reached a peak in 2001 whilst other variables do not follow this pattern. They have specified the break on the time trend by allowing the coefficient on this variable to vary across the two periods. In order to assess whether the break should be included we have tested the hypothesis that the coefficient is identical across the periods in a simple fixed effects model. Using standard Wald test we find that the null hypothesis that the coefficients are identical can be rejected at the 1% level of significance.

Despite the structural break being present, it is not immediately obvious that:

- Firstly the break should only be entertained across the time trend; and
- Secondly the relevant position of the break.

In Table 16 we look at varying the model across both of these dimensions and report the resulting estimates of TFP growth<sup>51</sup>. Consistent to the findings in Section B.2 we have estimated these models using the fixed effects procedure. Full regression output for these models is included in Appendix E.

**Table 16: Results varying the identified structural break**

TFP growth after the structural break	Structural break: 2001/02	Structural break: 2002/03
Structural break applied to time trend only	1.86% (-137.6)	1.6% (-143.3)
Structural break applied to whole model	0.0% (-151.6)	0.0% (-147.2)

*Source: The Schwarz Bayesian information criterion is reported in brackets in each row. The more negative the value, the better model fit. Results for the Akaike criterion are analogous. The finding of 0% TFP growth is results from a lack of statistical significance.*

In order to assess the different models' fit we have reported the Schwarz Bayesian information criterion. This criterion weights both the estimated model fit and the degrees of freedom consumed by the specification as follows<sup>52</sup>.

### Equation 7:

$$Schwarz = -2(Likelihood) + (k + 1)Ln(N)$$

<sup>51</sup> The results we have reported looked at moving the break forward by a year. We additionally looked at moving the break back a year but found this to produce an inferior fit.

<sup>52</sup> We have concentrated on the Schwarz criterion given it leads to more parsimonious specifications than the Akaike criterion. For a discussion on this see Koehler A and Murphree E, 1998: 'A Comparison of the Akaike and Schwartz Criteria for Selecting Model Order'. Applied Statistics Vol. 37:2, 187-195

where *Likelihood* is the model's likelihood given the parameters in the model.

From the criterion it is apparent that there is not a clear and distinguishable preference for any particular model. On the basis of these result we conclude that a structural break is apparent, and once account is made for this, TFP growth per year is estimated to lie in the range 0.0% to 1.9%.

## B.4 Response to other NERA comments

This section outlines some further responses to other comments in NERA (May 2008).

### B.4.1 Accounting for serial correlation

NERA stated that our estimation procedure does not account for within-panel serial correlation or cross-panel contemporaneous correlation. NERA primarily focus on the former, showing in the standard fixed effects model the error term tends to be correlated by around 0.7. This is suggestive of serial correlation, although it is calculated on the basis of ten observations for each panel.

As a sensitivity we have estimated a model accounting for serial correlation but retaining the fixed effects specification of the unobserved heterogeneity. Starting with our original model in Equation 4, the error term therefore becomes:

#### Equation 8

$$v_{it} = \rho v_{it} + e_{it}$$

where:

$e_{it}$  is independent and identically distributed, with zero mean and constant variance; and  $\rho$  is a parameter determining the speed of decay of previous errors.

We ran this model using a variety of specifications but found it not to be robust. As parameter estimates remain consistently estimated given serial correlation, we have continued to place more weight on the results outlined in Section B.3<sup>53</sup>.

### B.4.2 Opportunity cost

NERA correctly identified the omission of the opportunity cost of capital in the measurement of capital inputs. However, it is not clear whether the opportunity cost of holding capital be included when we are considering payments to capital. It could be argued that the opportunity cost should be included; however, this would be difficult to calculate and we have not seen this adjustment performed in other studies.

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<sup>53</sup> Consistency is maintained as long there are no omitted variables correlated to the variables in the model. Given the model includes variables suggested from standard theory of the firm, we do not expect this to be the case.

In any case, we envisage that the impact of this change would however be minimal. Testament to this, in NERA (December 2008) they report (in table 4.2 and 4.5) SFA results with varied return on capital assumptions. Across the differing assumptions only marginal different coefficient are observed.

### B.4.3 Typographical error

NERA's correctly identify that there is a typographical error in equation 13 in the original draft reported we delivered to BT. This has not been carried into our numerical analysis and is corrected in the version appended to this report.

## B.5 Summary and implications for BT efficiency adjustment

Our "Econometric TFP" model estimates that TFP growth observed by the US LECs is between 0.0% to 1.9% per year. This range is driven by varying the specification of an identified structural break. Our model is estimated using a fixed effects procedure accounting with heteroscedasticity across the error term.

The estimated range for TFP is marginally wider than that previously reported in Deloitte (2008) of 0.5% to 1.1%<sup>54</sup>. This increase is attributable to the addition of a structural break into the specification. Our findings remain marginally lower than NERA's 2% point estimate<sup>55</sup>. This slightly higher estimate is attributable to the differing estimation techniques they have deployed.

The final BT annual efficiency adjustments required from BT are estimated set out in Table 17. These adjustments reflected the updated TFP growth estimates established in this section, and estimates of the relative efficiency from the stochastic frontier analysis in section 3. In regards to the latter, we have assumed that the decile is allowed to catch-up with BT over the period of the charge control. Further discussion on this point is included in section 3.

**Table 17: BT's required efficiency adjustment**

Model: BT relative to top decile in 2006/2007	Estimated time trend	Annual real cost change
Deloitte FE TFP <sup>1</sup>	N/A	0% to -1.9%
NERA (December 2008) SFA Table 4.7 <sup>2</sup>	0%	-2.2%
NERA (May 2008) TFP Table 5.2 <sup>3</sup>	N/A	-2.0%

1) Using Deloitte specification with two structural breaks in Table 13.

2) Based on NERA (December 2008) SFA Table 4.7. Using NERA's main model and their stated preferred sample from 1999 to 2006. Preference discussed on page 11.

3) NERA's stated point estimate from NERA (May 2008) page 8.

The efficiency adjustment required from BT, using our estimates, ranges from 0.0% to -2.2%. This does not take into account any future catch-up, as discussed in Appendix D.

<sup>54</sup> This range was based on our alternative measures of TFP growth from the SFA analysis and our 'Standard TFP analysis' founded on a indices approach.

<sup>55</sup> See NERA (May 2008) page 8.

## Appendix C Review of Ofcom's Efficiency Model

Appendix 9 of the Ofcom consultation document introduced an alternative approach to calculating the shift in the productivity frontier. In summary, this approach involves measuring the average real unit cost change from 2003/2004 to 2006/2007 for individual network components, whilst holding volumes constant and controlling for BT's historical catch-up to the frontier. Finally, a weighted average is calculated across all included components, where the weights used are the share of operating costs of each component in total.

Using this method, Ofcom produced a range of 0% to 5% for the annual reduction in the real unit operating costs in the period from 2003/2004 to 2006/2007. Throughout this report, we refer to this approach as the 'Ofcom model'.

The approach taken by Ofcom appears to be unusual and does not accord with the econometric SFA approach that is more usually employed by Ofcom for calculating the frontier shift. We have concerns over the rigour of this technique for calculating a robust value for the frontier shift assumption in the price control, particularly as the value of "X" in a price control is typically highly sensitive to this value<sup>56</sup>. However, we understand that Ofcom intends to continue to rely upon this approach alongside more traditional econometric measures. Therefore, in this chapter we focus on providing a series of adjustments that may increase the robustness of the Ofcom model outputs.

### C.1 Comments on the Ofcom model

Ofcom has provided us with the spreadsheet model containing all the data and calculations used to arrive at the range of 0% to 5% efficiency improvement.

We have analysed the Ofcom model and provided Ofcom with a verbal critique of this model, this critique is summarised below. Our critique focuses on both the robustness of the calculation and the wide range of the frontier shift estimates that it produces:

- Since the efficiency factor is applied as a single variable in the price cap model, Ofcom will need to reduce the range to a single value and our adjustments are intended to provide a basis upon which Ofcom can perform this task; and
- BT has provided us with data that has allowed us to improve the robustness of the Ofcom model in terms of ensuring cost elements are correctly captured.

#### C.1.1 Inputs and calculations

Using cost and volume data from 2003/2004 to 2006/2007, the Ofcom model divides the PPC service provision into 20 different network components, and calculates a weighted average unit cost trend across the components in several stages:

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<sup>56</sup> X refers to price controls set as a "RPI-X" where X represents the real annual change in price that a regulated company is permitted to make for a particular regulated product

- First, the nominal cost of each component is converted into real terms using a simple RPI inflation adjustment;
- The real cost data is used to calculate both the annual percentage change in real costs for each component, and the proportion of the total cost attributable to each component, separately for each year in the sample. These proportions are used in the weighted average taken in the final stage of the calculation, and have a large effect on the results;
- Similarly, the Ofcom model calculates an annual growth rate for the volumes relating to each component;
- Next, the “real constant volume unit cost change” is calculated, as described in Appendix 9 (A9.23) of Ofcom (2008), separately for each network component. The annual “constant volume” unit cost changes are used to calculate a constant average growth rate (CAGR) separately for each component. This single figure for each component, giving equal weight to each of the annual growth rates, is used in all of the following calculations;
- The CAGR of unit costs of each component is adjusted for factor price change and BT catch-up of the top efficiency decile as estimated by NERA. This produces 9 possible measures of the frontier shift from 2003/04 to 2006/07, reflecting possible combinations of the start and end points of BT’s relative efficiency; and
- The figure is finally adjusted for annual future expected catch-up, calculated as the remaining inefficiency in 2006/2007 divided equally across the six years from 2006/2007 to 2012/2013. However, where it is found that BT has overtaken the top decile, Ofcom’s analysis requires BT to maintain that level of efficiency instead of allowing the decile to catch up to BT.

The application of the above methodology in the Ofcom model produces 9 alternative measures of the frontier shift for each of the individual network components from 2007/2008 to 2012/2013. The final stage of the Ofcom model is to combine each of these to a weighted average for the total assumption, using share of operating expenditure in each of the years within the period.

The final output of the Ofcom model is a set of 5 matrices of each of the 9 possible frontier shift estimates (45 alternative measures in total), where each matrix corresponds to a different base year weight used in the final step.

Our analysis of the Ofcom model shows that the results are highly sensitive to key modelling parameters used and the data included in the model. Table 18 below illustrates the variation in Ofcom’s results depending on the choice of base year weights and (assumed) BT position relative to the top efficiency decile.



**Table 18: Ofcom estimates of real unit cost change on different base year weights**

<b>BT relative to top decile 2003/2004 and 2006/2007</b>	<b>2003/2004 weights</b>	<b>2006/2007 weights</b>	<b>Median of 2003/2004 - 2006/2007 weights</b>	<b>Average of 2004/2005 - 2006/2007 weights</b>
0.5% and 0.8%	-6.2%	-2.9%	-4.6%	-3.7%
2.2% and -3.8%	-4.1%	-0.7%	-2.4%	-1.6%
3.8% and -4.5%	-3.4%	0.1%	-1.6%	-0.8%
Average across all relative efficiency combinations	-4.6 %	-1.2%	-2.9%	-2.0%

Source: Deloitte analysis of Ofcom's frontier shift calculation spreadsheet

The wide range is driven primarily by the choice of the base year for the operating expenditure weights and the start and end points of efficiency of BT relative to the top decile. These estimates need to be more robust and narrowed to a smaller range in order to assist Ofcom in reaching a decision on a single frontier shift measure for inclusion in the price cap model.

## C.2 Proposed adjustments to the model

The Ofcom model is based a number of key assumptions. Whilst we have made adjustments to many of these, there are a number of assumptions that we have either not tested or adjusted. These are listed below, along with our rationale for not performing an adjustment:

- The Ofcom model is based on operating costs (excluding depreciation) of the whole network. This approach places more weight on the copper components of the network than the cost base attributable to leased lines service provision. We have not adjusted this assumption as we have no basis of assuming which proportion of copper costs should properly be included in the analysis;
- The approach uses estimates of operating costs (excluding depreciation). This does not capture substitution between capital and labour and this may bias the estimate if it is used as a TFP measure. We have not adjusted for this as it would require a structural change to the model and is therefore considered outside the scope of our analysis; and
- The model applies a single weighted average correction factor for input price change across all components. Adjusting this assumption would require substantially more detailed analysis than was in the scope of this report.

We propose three adjustments to the Ofcom model which we believe will increase the robustness of the estimate. These are:

- Including distance-related elements in the analysis;
- Using a Törnqvist index, instead of the alternative base year weights; and

- Updating and refining the estimates of BT efficiency relative to the top decile, based on NERA estimates for 2003/2004 and 2006/2007..

### C.2.1 Impact of including distance-related components

The Ofcom model used for the 0% to 5% range quoted in the Ofcom consultation document did not include costs and volumes of distance-related components, due to inconsistency in the volume measures for the component. This implies only capacity and copper related costs were considered, covering only 65% of the cost base relevant for the LLCC. The results could therefore be biased if cost trends of components included in the analysis differ from the omitted components. We understand that BT has now provided the distance-related component information to Ofcom and therefore these components should be included in the Ofcom model.

The volume data on the distance-related components in the original Ofcom model were affected by a change in the basis of measurement from a route distance to a radial distance measure. This affected the 2005/2006 and the 2006/2007 volume measures. BT provided us with conversion factors so that the affected data could be converted to a consistent basis with the early sample, and the distance-related components included in the analysis. Table 19 below shows the results after including them.

**Table 19: Ofcom model including distance-related components**

BT relative to top decile 2003/2004 and 2006/2007	2003/2004 weights	2006/2007 weights	Median of 2003/2004 - 2006/2007 weights	Average of 2004/2005 - 2006/2007 weights
0.5% and 0.8%	-4.4%	4.4%	0.0%	1.7%
2.2% and -3.8%	-2.2%	6.8%	2.3%	4.0%
3.8% and -4.5%	-1.5%	7.6%	3.1%	4.8%

*Source: Deloitte analysis of Ofcom model*

The inclusion of distance-related elements leads to a wide range of estimated values from -4.4% to 7.6%.

This highlights the limitations of the static weighting approach employed by Ofcom. The static approach does not adequately reflect the changing importance of individual components through time. There are two main sources of variations in the importance (weight) of individual components: natural growth of the services and costs, and cost reallocation between individual components. For example, with the static weighting approach, these can lead to a situation where a high cost growth in a small component in the early sample is given too much weight, when the weights are based on cost proportions in the last year of the sample.

Instead of the static approach, the overall unit cost trend estimate should be based on a continuously weighted index, such as the Törnqvist index.

## C.2.2 Impact of using the Törnqvist index

Törnqvist index is a standard measure used in productivity analysis. The particular attraction of its use in this case is that it is designed for continuously weighted multi-component analysis, using both the current and the base year cost shares in weighting each component in the index.<sup>57</sup> This continuous weighting calculated by the index correctly accounts for variations in the importance of different components through time.

### Equation 9: Calculation of the Törnqvist index

$$C_t^T = \prod_{m=1}^M \frac{x_t^m}{x_0^m}^{0.5(w_0^m + w_t^m)}$$

Where:

$$w_t^m = \frac{p_t^m x_t^m}{\sum_{m=1}^M p_t^m x_t^m} \text{ is cost } m\text{'s nominal cost share}$$

$p$  is the price of inputs

$x$  is the quantity of inputs

We used the cost and volume data in the Ofcom model to calculate a Törnqvist input index (cost) and a Törnqvist output index (volumes). Törnqvist unit cost index is achieved simply by dividing the input index with the output index. We then measured the CAGR of the Törnqvist unit cost index to form a single starting point for the factor price and catch-up adjustments.

Table 20 below confirms that the Törnqvist index does not suffer from the same variation as the static weighting approach, and produces meaningful and reliable estimates.

**Table 20: Ofcom model applying the Törnqvist index**

BT relative to top decile 2003/04 and 2006/07	2003/04 weights	2006/07 weights	Median of 2003/2004 - 2006/2007 weights	Average of 2004/2005 - 2006/2007 weights	Törnqvist Index
0.5% and 0.8%	-4.4%	4.4%	0.0%	1.7%	-3.2%
2.2% and -3.8%	-2.2%	6.8%	2.3%	4.0%	-1.1%
3.8% and -4.5%	-1.5%	7.6%	3.1%	4.8%	-0.4%

Source: Deloitte analysis of Ofcom model

The Törnqvist results in the above table are based on a pure Törnqvist index. In the language of the Ofcom approach of “constant volume unit cost change”, this carries an implicit assumption that the CVE is one. The input and output indices can also be used in a calculation which allows for

<sup>57</sup> Revenue shares would be preferable to cost shares as the weights used in the output index. However, the data on revenue weights of the components considered here does not exist, so cost weights have been used instead for both the input and output indices.

Ofcom's CVE assumption of 0.24 be included in the analysis. This, however, moves the analysis away from using a Törnqvist index to measure efficiency. Also, it produces final estimates of unit cost increase, calling the assumed value of the CVE in question.

Our preference is therefore to use the pure Törnqvist index in the context of Ofcom's approach.

### C.2.3 Updating BT relative efficiency estimates

The results in Table 20 above include a correction for BT's catch-up to the top efficiency decile between 2003/04 and 2006/07. The adjustment for catch-up controls for improvement in BT's relative efficiency during the period that influences unit cost trends measured by the model. The adjustment acknowledges efficiency improvement by BT over and above the efficiency improvement driven by technological progress (frontier shift).

The calculation of the catch-up adjustment requires a measure of BT's efficiency relative to the benchmark (top decile) at the start and the end of the period. For example, if BT moved from 1% below the decile to 1% above the decile over two years, and the cumulative unit cost trend observed from data was 3% over the two years, we would know that the efficiency frontier change was 1% (other things being equal).

The range for the starting point of BT's relative efficiency used by Ofcom, between 0.5% and 3.8% less efficient than the decile in 2003/04, is sourced from NERA estimates in 2005<sup>58</sup>. The range from 0.8% higher to 4.5% lower costs than the decile is sourced from NERA estimates in March 2008<sup>59</sup>. We have sought to refine these estimates based on NERA's preferred specifications in up-to-date studies.

NERA 2005<sup>60</sup> reports the final range of estimates of BT's relative inefficiency to vary between 0.5% and 3.8%, used by Ofcom in their analysis, without explicitly selecting a preferred point estimate. In the interest of achieving a narrow range based on best estimates, we have identified the estimate of the inefficiency from the preferred specification. The estimate indicates BT is 1.05% less efficient than the top decile<sup>61</sup>. We have used this as the assumed starting point for the catch-up adjustment<sup>62</sup>.

The end point of the catch-up adjustment used in the Ofcom analysis comes from a report focusing on the relative efficiency of BT Openreach. NERA has since provided updated estimates based on analysis focusing on efficiency of BT's network operations.<sup>63</sup> This analysis finds that the efficiency

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<sup>58</sup> NERA (2005), table 5.1

<sup>59</sup> NERA (March 2008), table 6.1

<sup>60</sup> NERA (2005)

<sup>61</sup> NERA (2005), page 35

<sup>62</sup> Alternative would be to use the relative efficiency assumed in the previous LLCC determination as the starting point, adjusted to reflect the catch-up by the start of the 2003/2004 – 2006/2007 period under consideration here. Although we have not investigated this possibility, it should not be rule out.

<sup>63</sup> NERA (December 2008)

of BT's network operations is between 3.8% and 6.8% higher than the top decile. The estimate from NERA's main model is that BT is 6.0% above the decile.<sup>64</sup>

In cost terms, we have used a single starting point of 1.05% and a range from -3.8% to -6.8% for the catch up adjustment. Table 21 below shows the resulting estimate of frontier shift in terms of unit costs.

**Table 21: Törnqvist index estimate of the frontier shift in terms of unit cost change**

BT relative to top decile 2003/04 and 2006/07	Törnqvist Index frontier shift
1.05% and -3.8%	-1.5%
1.05% and -6.0%	-0.7%
1.05% and -6.8%	-0.5%

*Source: Deloitte analysis of Ofcom model*

This gives a measure of efficiency frontier shift between -0.5% and -1.5%. Using NERA's preferred specification, as identified above, implies a frontier shift estimate of -0.7%.

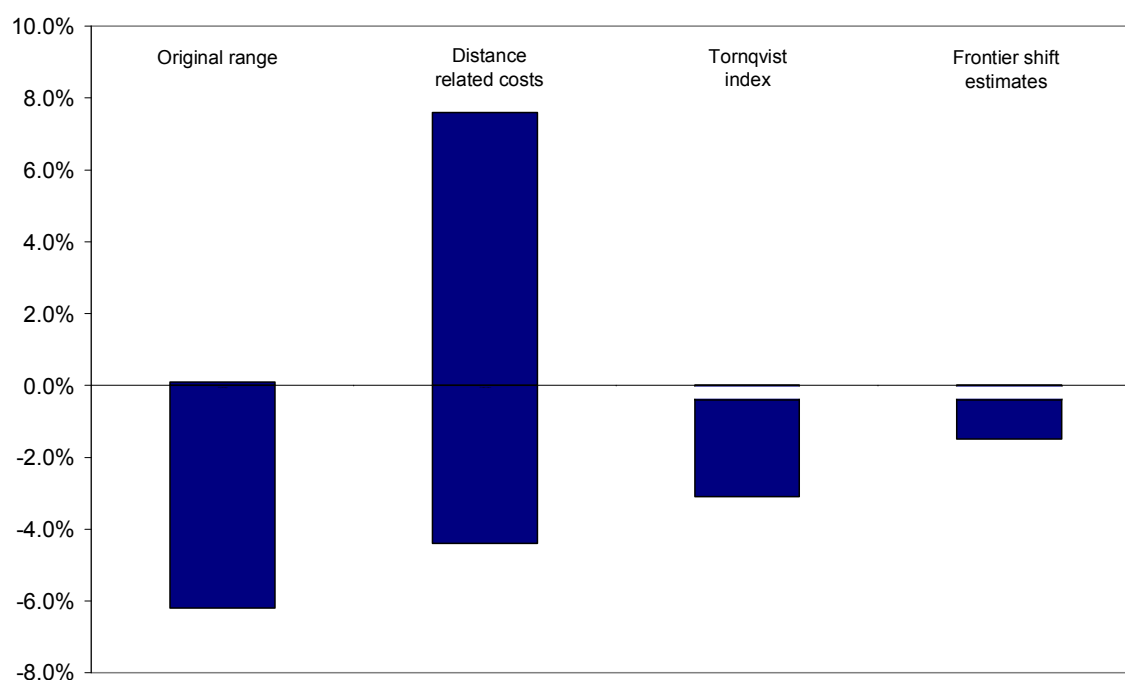
It should be noted that using the full range of estimates of BT's relative efficiency for 2003/04 would have little effect on the top of the range (raising it to -1.7%), but would change the bottom of the range to indicate a cost *increase* of +0.5% rather than a cost change of -0.5% as estimated above.

<sup>64</sup> NERA (December 2008), table 4.6 and table 4.7

### C.3 Summary of findings from Ofcom's model

Figure 2 below summarises the cumulative effect of the three adjustments made to the Ofcom model.

**Figure 2: Cumulative effect of the adjustments to the Ofcom model**



Source; Deloitte analysis

The original range for unit cost reduction estimated by Ofcom varies from 0.1% to -6.2%. Including the distance-related components produces even wider range due to the static weighting approach discussed above. Using the Törnqvist index to address the issues with the static weighting approach produces a reasonable range of unit cost CAGR for the time period. The update to the start and end points of BT relative efficiency leads to a further narrowing of the range, towards the lower end, resulting in a range of real unit cost change estimates between -0.5% and -1.5%.

## Appendix D Future Catch-up

Typically, price controls in the telecommunications sector require the regulated company, in this case BT to increase efficiency, by reducing real annual unit costs, to allow for three separate effects:

- Future catch-up: The change in real unit costs that is required so that BT's comparative efficiency is equal to that of the top decile efficiency, assuming that the efficiency of the top decile is constant;
- Annual real cost change: Referred to as the productivity gain, this is the annual increase in productivity, assuming constant volumes, that companies within the telecommunication sector are assumed to incur during the price control period; and
- Economies of scale: The change in BT's unit costs that result from a change in volumes.

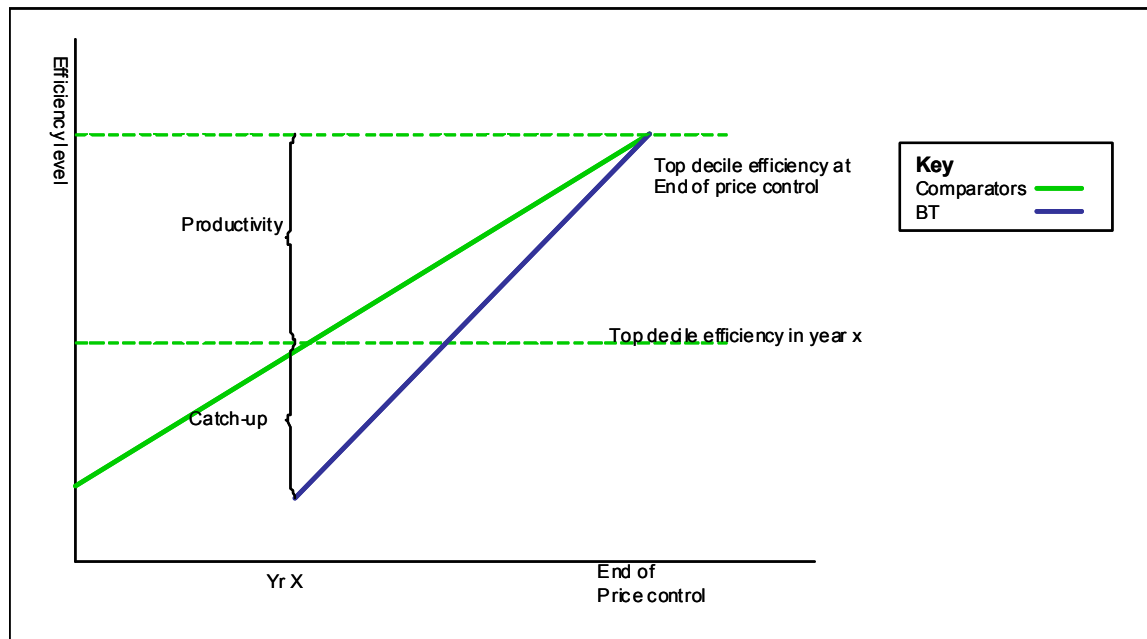
Economies of scale are captured by Ofcom through the application of cost volume and asset volume elasticities within the price control model. Therefore our efficiency study only considers the first two measures.

### D.1 Calculating a catch-up factor

Ofcom has traditionally computed a single measure for BT's efficiency assumption by combining the catch-up requirement with the annual real cost change.

In previous efficiency studies, including NERA (2005), BT's comparative efficiency has been below the top decile at the start of the price control period. In this case, Ofcom has required to BT to catch-up to the top decile over the period of the price control.

This catch-up efficiency requirement has been applied in addition to the annual real cost change is due to technological and general telecoms productivity improvements that are causing the efficient frontier to shift outwards. By combining both these measures, Ofcom has taken a stance that BT should be at the efficiency level of the top decile by the end of the price control period. The combination of these two effects to achieve a single efficiency measure is shown in the diagram below.

**Figure 3: Calculation of the efficiency assumption**

In the above figure, BT is required to catch-up to the efficiency level of the comparators as they were at the start of the price control. In addition, BT is required to undertake an annual productivity improvement that is equal to that assumed for the comparators. By the end of the price control, BT's efficiency level is equal to that of the comparators.

Numerically, if BT is found to be 5% below the decile's efficiency at the start of a 5-year price control and the productivity of the telecoms sector is increasing at 2% per year, then BT's annual efficiency assumption should be:

- Annual efficiency factor = (inefficiency at start ÷ duration of price control) + productivity
- Annual efficiency factor = (5% ÷ 5 yrs) + 2% = 3%

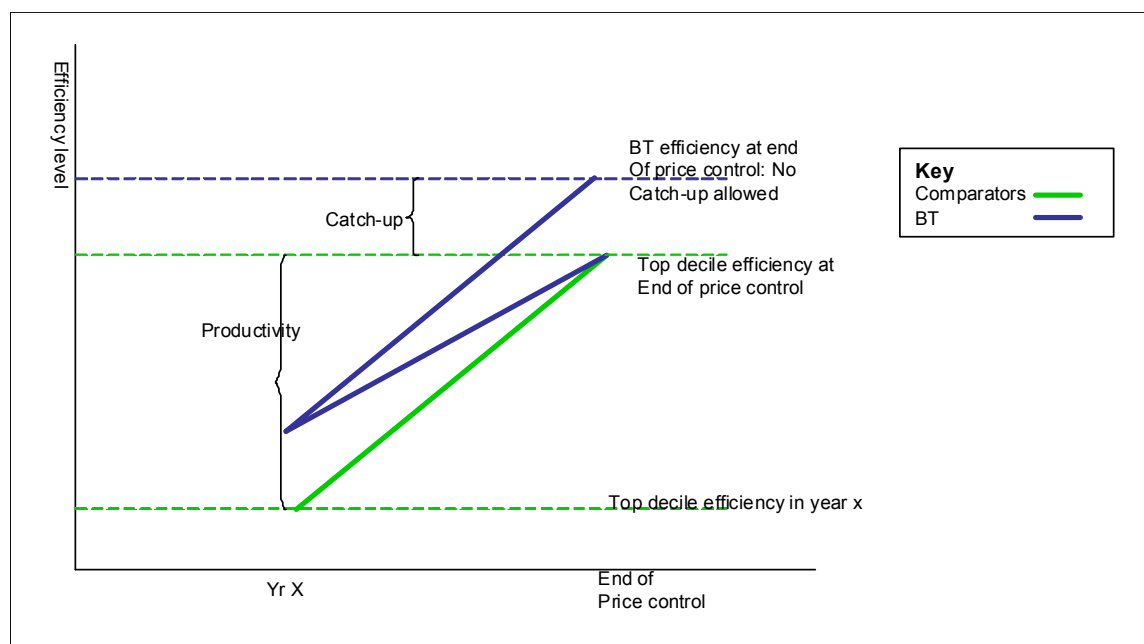
Catch-up can also be applied in cases where BT is above the decile at the start of the price control period. In this case, catch-up is used to allow the comparators to catch-up to BT over the price control period. For example if BT was found to be 5% above the decile at the start of the price control period, then the above calculation would apply:

- Annual productivity factor = (-5% ÷ 5 years) + 2% = 1% per year

This is shown diagrammatically below.



**Figure 4: Application of catch-up when BT is above the efficiency decile at the start of the price control**



Price caps are a particular form of price regulation that is intended to control prices whilst also rewarding, or penalising, a regulated company for its performance. A price cap therefore seeks to control prices, allowing them to rise when production costs unavoidably rise so that a company can continue to make a normal economic profit, whilst also promoting improvements in productive efficient.

The inclusion of a catch-up parameter, where BT is found to be below the decile at the start of the price control period, is accepted by Ofcom as a means for encouraging efficiency improvements and has been consistently applied within price controls.

Following the theoretical arguments of a price control as set out above, we believe that a catch-up element should also be applied where BT is found to be above the decile at the start of the period since this also provides positive incentives to BT to continue to improve efficiency. This is because:

- Without a catch-up element, BT would be required to be more efficient than the decile at the end of the price control period. Requiring BT to be more efficient than the decile would be inconsistent with Ofcom's previous approach and would imply a more stringent definition of efficiency being applied to BT;
- The reward and penalty structure is an integral part of any price control. Incentive regulation should be applied in a symmetric manner. This recognises that BT should be rewarded for good performance as well as penalised for bad performance;
- It increases the incentives for BT to continue to make efficiency reductions. Where catch-up is not permitted then the rewards to BT only incur within the current price and BT is less incentivised to over-perform in future price control periods. Allowing the company to

choose its goals and rewarding a regulated company with greater rewards to more ambitious goals is an important part of price control regulation; and

- Without the application of catch-up, it could be argued that BT should be permitted a higher weighted average cost of capital within the price control model. This is due to increase of effect of systematic risks on BT (due to tighter targets), as well as substantial increase in regulatory uncertainty.

Further arguments surround the measurement of efficiency itself. Each estimate of the efficiency of BT has looked only at the efficiency of BT at a point in time. Since the efficiency is based on comparing BT to the performance of other operators, it is likely that there would be some variations in efficiency in each year. It is possible that the current high performance of BT has occurred since other operators are in an early stage of cost-reducing programmes which BT has completed. It would, in this case, be unfair to expect BT to continue at this high relative efficiency level, as other operators would be expected to become relatively more efficient over time.

## D.2 Application of catch-up to current price controls

Our revised SFA analysis summarised in Section 2.1 showed that BT's comparative efficiency in 2006 is better than that of the top decile by 6.3%. This aligns with NERA's analysis which places BT at 6% above the decile. These comparative efficiency estimates are computed based on BT's position in 2006 and are to be used in price controls which end in 2012.

As stated above, in previous price controls, Ofcom has required BT to be at the decile at the end of the price control and has applied a constant glide path approach to achieve this. A key question must therefore be where BT's efficiency will lie at the start of the price control. Given the analysis outlined above and in detail in Appendix A, we believe it is probable that BT will lie above the decile at the start of the regulatory period.

Therefore, BT may wish to contend that it should be allowed some form of catch-up in the calculation of the efficiency assumption for the price control. This catch-up amount should be subtracted from the annual real cost change to give the final BT efficiency assumption.

## Appendix E Econometric Results of SFA analysis

This appendix provides the full econometric results referred to in the SFA analysis section above. All the models were estimated in the Stata econometrics package.

### E.1 Replication of NERA's preferred SFA models

#### E.1.1 Full sample from 1996 to 2006

```
Iteration 0: log likelihood = -505.24234 (not concave)
Iteration 1: log likelihood = 62.68838 (not concave)
Iteration 2: log likelihood = 142.54518 (not concave)
Iteration 3: log likelihood = 302.85115 (not concave)
Iteration 4: log likelihood = 662.67824 (not concave)
Iteration 5: log likelihood = 849.75381
Iteration 6: log likelihood = 865.95379 (not concave)
Iteration 7: log likelihood = 904.90563 (not concave)
Iteration 8: log likelihood = 905.84135 (not concave)
Iteration 9: log likelihood = 905.84778 (not concave)
Iteration 10: log likelihood = 905.85275 (not concave)
Iteration 11: log likelihood = 905.93974 (not concave)
Iteration 12: log likelihood = 906.03181
Iteration 13: log likelihood = 906.11158
Iteration 14: log likelihood = 906.14913
Iteration 15: log likelihood = 906.14932
Iteration 16: log likelihood = 906.14932
```

Time-invariant inefficiency model  
Group variable: cref

Number of obs = 726  
Number of groups = 66

Obs per group: min = 11  
avg = 11  
max = 11

Log likelihood = 906.14932

wald chi2(18) = 2613.36  
Prob > chi2 = 0.0000

ln_ncostpsl	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
ln_strand	.4810694	.0498135	9.66	0.000	.3834367 .5787021
ln_llpsl	.0549611	.0122521	4.49	0.000	.0309475 .0789747
ln_llps~9698	-.0436845	.0149634	-2.92	0.004	-.0730123 -.0143567
ln_swmnpsl	-.0042766	.0113505	-0.38	0.706	-.0265232 .0179699
ln_swmnps~8	.0122222	.017249	0.71	0.479	-.0215852 .0460296
ln_tspsl	.2426738	.0327746	7.40	0.000	.1784368 .3069108
ln_tsps~9698	.0608047	.0122343	4.97	0.000	.0368259 .0847834
ln_ductpsl	.0046733	.0089072	0.52	0.600	-.0127845 .022131
ln_ductpsl~8	-.0218325	.0120044	-1.82	0.069	-.0453607 .0016957
ln_popden	.0359079	.0169906	2.11	0.035	.002607 .0692088
ln_busres	.0294056	.0291743	1.01	0.313	-.027775 .0865862
ln_fibreprop	.1024179	.0241151	4.25	0.000	.0551532 .1496825
nyne	.1261071	.050009	2.52	0.012	.0280912 .224123
midwest	-.205131	.042011	-4.88	0.000	-.287471 -.1227911
vdummy	-.0211001	.0178715	-1.18	0.238	-.0561276 .0139275
time	-.0065201	.0032441	-2.01	0.044	-.0128785 -.0001618
time_d9698	-.0251523	.0061734	-4.07	0.000	-.0372519 -.0130526
d9698	.0437183	.2039709	0.21	0.830	-.3560573 .4434938
_cons	.0256715	.1668988	0.15	0.878	-.3014441 .3527871
/mu	.2535532	.0569463	4.45	0.000	.1419404 .3651659
/lnsigma2	-4.20384	.1906064	-22.06	0.000	-4.577421 -3.830258
/ilgtgamma	1.186073	.2600041	4.56	0.000	.6764741 1.695672
sigma2	.0149381	.0028473			.0102814 .021704
gamma	.766038	.0465989			.6629513 .8449686
sigma_u2	.0114432	.0028562			.0058452 .0170412
sigma_v2	.003495	.0001944			.003114 .0038759

## E.1.2 Restricted sample 1999 - 2006

```

Iteration 0: log likelihood = -381.21285 (not concave)
Iteration 1: log likelihood = 67.979875 (not concave)
Iteration 2: log likelihood = 222.41193 (not concave)
Iteration 3: log likelihood = 471.83524
Iteration 4: log likelihood = 546.85124
Iteration 5: log likelihood = 591.11566
Iteration 6: log likelihood = 619.83442
Iteration 7: log likelihood = 623.70258
Iteration 8: log likelihood = 624.29125
Iteration 9: log likelihood = 625.30536
Iteration 10: log likelihood = 625.51905
Iteration 11: log likelihood = 625.53371
Iteration 12: log likelihood = 625.53385
Iteration 13: log likelihood = 625.53385

```

Time-invariant inefficiency model  
Group variable: cref

Number of obs = 528  
Number of groups = 66

Obs per group: min = 8  
avg = 8  
max = 8

Log likelihood = 625.53385

wald chi2(12) = 2023.65  
Prob > chi2 = 0.0000

ln_ncostpsl	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
ln_strand	.5035134	.0585964	8.59	0.000	.3886665 .6183603
ln_llpsl	.0230668	.0153618	1.50	0.133	-.0070418 .0531754
ln_swmnpsl	.0081511	.0131914	0.62	0.537	-.0177036 .0340057
ln_tspsl	.2009153	.0333277	6.03	0.000	.1355942 .2662363
ln_ductpsl	-.0003938	.0098323	-0.04	0.968	-.0196647 .0188772
ln_popden	.028566	.0152822	1.87	0.062	-.0013866 .0585186
ln_busres	.0745398	.0340734	2.19	0.029	.0077571 .1413225
ln_fibreprop	.1164365	.0302407	3.85	0.000	.0571658 .1757071
nyne	.1475671	.0488088	3.02	0.002	.0519036 .2432305
midwest	-.1835292	.0414299	-4.43	0.000	-.2647303 -.1023282
vdummy	-.0441536	.0198257	-2.23	0.026	-.0830112 -.005296
time	-.0005275	.0037325	-0.14	0.888	-.007843 .0067881
_cons	-.2246036	.193724	-1.16	0.246	-.6042958 .1550885
/mu	.2860958	.0712929	4.01	0.000	.1463643 .4258272
/lnsigma2	-4.248888	.1620465	-26.22	0.000	-4.566494 -3.931283
/ilgtgamma	1.053007	.2350357	4.48	0.000	.5923451 1.513668
sigma2	.0142801	.002314			.0103943 .0196185
gamma	.7413518	.0450679			.643903 .8196042
sigma_u2	.0105866	.0023231			.0060333 .0151398
sigma_v2	.0036935	.0002465			.0032104 .0041767

## E.2 Structural break in 2004 in NERA's preferred specifications

### E.2.1 Full sample from 1996 to 2006

```

Iteration 0: log likelihood = -503.75166 (not concave)
Iteration 1: log likelihood = 90.522955 (not concave)
Iteration 2: log likelihood = 220.36765 (not concave)
Iteration 3: log likelihood = 321.70081 (not concave)
Iteration 4: log likelihood = 662.70802 (not concave)
Iteration 5: log likelihood = 858.69984
Iteration 6: log likelihood = 893.83115
Iteration 7: log likelihood = 918.55234
Iteration 8: log likelihood = 944.44303
Iteration 9: log likelihood = 945.91263
Iteration 10: log likelihood = 946.16942
Iteration 11: log likelihood = 946.25448
Iteration 12: log likelihood = 946.26822
Iteration 13: log likelihood = 946.26876
Iteration 14: log likelihood = 946.26876

```

Time-invariant inefficiency model  
Group variable: cref

Number of obs = 726  
Number of groups = 66

Obs per group: min = 11  
avg = 11  
max = 11

Log likelihood = 946.26876

wald chi2(24) = 3046.16  
Prob > chi2 = 0.0000

ln_ncostps1	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
ln_strand	.4648867	.0497353	9.35	0.000	.3674072 .5623661
ln_llps1	.0458223	.0129217	3.55	0.000	.0204962 .0711484
ln_llps~9698	-.0363952	.0145606	-2.50	0.012	-.0649333 -.007857
ln_llps~0406	.0195253	.0128653	1.52	0.129	-.0056902 .0447408
ln_swminps1	-.000385	.0126615	-0.03	0.976	-.0252011 .024431
ln_swminps~8	.0123851	.0168801	0.73	0.463	-.0206994 .0454695
ln_swminps~6	.0005811	.0160005	0.04	0.971	-.0307793 .0319415
ln_tsps1	.2465749	.0341053	7.23	0.000	.1797298 .31342
ln_tsps~9698	.0625521	.0120156	5.21	0.000	.039002 .0861021
ln_tsps~0406	.0153615	.0125708	1.22	0.222	-.0092768 .0399997
ln_ductps1	.0217328	.0167948	1.29	0.196	-.0111844 .05465
ln_ductps1~8	-.0274247	.011903	-2.30	0.021	-.0507543 -.0040952
ln_ductps1~6	-.0124744	.011789	-1.06	0.290	-.0355804 .0106315
ln_popden	.0377972	.0192006	1.97	0.049	.0001646 .0754298
ln_busres	-.0104329	.0282279	-0.37	0.712	-.0657586 .0448929
ln_fibreprop	.1037387	.0239975	4.32	0.000	.0567045 .1507728
nyne	.1138662	.0518848	2.19	0.028	.0121738 .2155585
midwest	-.2085298	.0435641	-4.79	0.000	-.2939138 -.1231458
vdummy	.0008148	.0189319	0.04	0.966	-.036291 .0379207
time	-.0049052	.0038913	-1.26	0.207	-.0125321 .0027217
time_d9698	-.0246537	.0062761	-3.93	0.000	-.0369546 -.0123528
time_d0406	.0397855	.0061133	6.51	0.000	.0278037 .0517674
d9698	.0197839	.1999207	0.10	0.921	-.3720534 .4116212
d0406	-.3957265	.179233	-2.21	0.027	-.7470167 -.0444363
_cons	.04064	.1969154	0.21	0.836	-.345307 .426587
/mu	.2524172	.0612702	4.12	0.000	.1323298 .3725047
/lnsigma2	-4.173287	.213656	-19.53	0.000	-4.592045 -3.754529
/ilgtgamma	1.387033	.2779403	4.99	0.000	.8422804 1.931786
sigma2	.0154016	.0032906			.0101321 .0234115
gamma	.8001182	.0444507			.6989453 .873447
sigma_u2	.0123231	.0033002			.0058548 .0187913
sigma_v2	.0030785	.0001713			.0027428 .0034142

## E.2.2 Restricted sample 1999 - 2006

```

Iteration 0: log likelihood = -379.19374 (not concave)
Iteration 1: log likelihood = 111.34649 (not concave)
Iteration 2: log likelihood = 359.51967 (not concave)
Iteration 3: log likelihood = 518.84864
Iteration 4: log likelihood = 540.05651
Iteration 5: log likelihood = 661.02722
Iteration 6: log likelihood = 662.79765
Iteration 7: log likelihood = 664.2554
Iteration 8: log likelihood = 664.56015
Iteration 9: log likelihood = 664.60191
Iteration 10: log likelihood = 664.60218
Iteration 11: log likelihood = 664.60218

```

Time-invariant inefficiency model  
Group variable: cref

Number of obs = 528  
Number of groups = 66  
obs per group: min = 8  
avg = 8  
max = 8

Log likelihood = 664.60218

wald chi2(18) = 2488.31  
Prob > chi2 = 0.0000

ln_ncostpsl	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
ln_strand	.4960516	.0582998	8.51	0.000	.3817861 .6103171
ln_llpsl	.010225	.0153236	0.67	0.505	-.0198087 .0402587
ln_llps~0406	.0197452	.0133175	1.48	0.138	-.0063567 .045847
ln_swmnpsl	.0139088	.0143604	0.97	0.333	-.0142371 .0420547
ln_swmnps~6	.0128119	.0165498	0.77	0.439	-.0196251 .0452489
ln_tspsl	.1946731	.0342971	5.68	0.000	.1274521 .2618941
ln_tspsl~0406	.0105841	.0128483	0.82	0.410	-.0145981 .0357664
ln_ductpsl	.0058021	.0186741	0.31	0.756	-.0307984 .0424027
ln_ductpsl~6	-.001323	.0125096	-0.11	0.916	-.0258413 .0231953
ln_popden	.0262691	.0157446	1.67	0.095	-.0045896 .0571279
ln_busres	.0223351	.0326072	0.68	0.493	-.0415738 .086244
ln_fibreprop	.123112	.0299476	4.11	0.000	.0644157 .1818083
nyne	.1415793	.0499531	2.83	0.005	.043673 .2394856
midwest	-.1810938	.0423268	-4.28	0.000	-.2640527 -.0981348
vdummy	-.023066	.0208741	-1.11	0.269	-.0639785 .0178466
time	.0019253	.0042272	0.46	0.649	-.0063598 .0102104
time_d0406	.04142	.0061882	6.69	0.000	.0292914 .0535487
d0406	-.4871438	.1834465	-2.66	0.008	-.8466924 -.1275952
_cons	-.3055831	.2245113	-1.36	0.173	-.7456172 .1344509
/mu	.2908417	.0659797	4.41	0.000	.1615239 .4201595
/lnsigma2	-4.252853	.1692795	-25.12	0.000	-4.584634 -3.921071
/ilgtgamma	1.277063	.2327084	5.49	0.000	.8209626 1.733163
sigma2	.0142236	.0024078			.0102075 .0198199
gamma	.7819494	.0396778			.6944406 .8498165
sigma_u2	.0111221	.0024167			.0063855 .0158588
sigma_v2	.0031015	.0002069			.0026959 .003507

## E.3 Further development of NERA's specifications

### E.3.1 Model 1996 – 2006 (time invariant inefficiency specification)

```

Iteration 0: log likelihood = -504.02353 (not concave)
Iteration 1: log likelihood = 253.89597 (not concave)
Iteration 2: log likelihood = 258.57539 (not concave)
Iteration 3: log likelihood = 467.29369 (not concave)
Iteration 4: log likelihood = 824.10347 (not concave)
Iteration 5: log likelihood = 915.86876
Iteration 6: log likelihood = 930.40042
Iteration 7: log likelihood = 938.57238
Iteration 8: log likelihood = 942.99572
Iteration 9: log likelihood = 943.30997
Iteration 10: log likelihood = 943.33215
Iteration 11: log likelihood = 943.33313
Iteration 12: log likelihood = 943.33313

```

```

Time-invariant inefficiency model
Group variable: cref

Number of obs      =      726
Number of groups   =       66

Obs per group: min =      11
                avg  =      11
                max  =      11

wald chi2(15)      =    10211.72
Prob > chi2        =     0.0000

Log likelihood     =    943.33313

```

ln_ncost	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
ln_strand	.4717573	.042322	11.15	0.000	.3888077 .5547069
ln_sl	.6823419	.0347897	19.61	0.000	.6141553 .7505285
ln_ll	.0558007	.0094101	5.93	0.000	.0373573 .0742441
ln_ll_d9698	-.0450141	.007622	-5.91	0.000	-.059953 -.0300752
ln_ts	.2547156	.037767	6.74	0.000	.1806938 .3287375
ln_ts_d9698	.0324248	.0113539	2.86	0.004	.0101715 .054678
ln_popden	.0424712	.0199879	2.12	0.034	.0032955 .0816468
ln_fibreprop	.1203838	.0234647	5.13	0.000	.0743939 .1663738
nyne	.110795	.0513918	2.16	0.031	.0100691 .211521
midwest	-.2076555	.0442565	-4.69	0.000	-.2943966 -.1209145
time	-.0071953	.0032561	-2.21	0.027	-.0135771 -.0008135
time_d9698	-.0236158	.0057671	-4.09	0.000	-.0349192 -.0123124
time_d0406	.0432842	.0053942	8.02	0.000	.0327117 .0538567
d9698	.351111	.0514858	6.82	0.000	.2502008 .4520213
d0406	-.4000848	.0457846	-8.74	0.000	-.489821 -.3103486
_cons	.1075612	.1836292	0.59	0.558	-.2523454 .4674679
/mu	.2423245	.0602742	4.02	0.000	.1241893 .3604597
/lnsigma2	-4.144356	.2323519	-17.84	0.000	-4.599757 -3.688954
/ilgtgamma	1.413169	.2995566	4.72	0.000	.826049 2.000289
sigma2	.0158536	.0036836			.0100543 .0249981
gamma	.8042653	.047157			.6955189 .8808275
sigma_u2	.0127505	.0036941			.0055103 .0199908
sigma_v2	.0031031	.0001726			.0027648 .0034414

## Relative efficiency scores

eff_rank	company	eff_sfa	eff_best	eff_decile
1	Qwest-Idaho South	0.019	0.000	-0.078
2	<b>BT Network</b>	<b>0.035</b>	<b>0.016</b>	<b>-0.063</b>
3	Verizon NE-Rhode Island	0.050	0.031	-0.050
4	Wisconsin Bell	0.056	0.036	-0.044
5	Qwest-Iowa	0.072	0.052	-0.030
6	Verizon-Delaware	0.086	0.065	-0.018
7	Qwest-North Dakota	0.102	0.081	-0.003
.				
.				
.				
64	Illinois Bell	0.499	0.471	0.356
65	Verizon NW-West Coast California	0.516	0.487	0.371
66	Verizon-New York Telephone	0.639	0.609	0.483
67	Contel-Arizona	0.664	0.632	0.505

## E.3.2 Model 1999 – 2006 (time invariant inefficiency specification)

```

Iteration 0: log likelihood = -378.86513 (not concave)
Iteration 1: log likelihood = 122.95408 (not concave)
Iteration 2: log likelihood = 393.47318 (not concave)
Iteration 3: log likelihood = 461.5965 (not concave)
Iteration 4: log likelihood = 534.85342 (not concave)
Iteration 5: log likelihood = 634.11694
Iteration 6: log likelihood = 652.66206
Iteration 7: log likelihood = 660.09383
Iteration 8: log likelihood = 662.5756
Iteration 9: log likelihood = 662.72065
Iteration 10: log likelihood = 662.72677
Iteration 11: log likelihood = 662.72689
Iteration 12: log likelihood = 662.72689

```

Time-invariant inefficiency model  
Group variable: cref

Number of obs = 528  
Number of groups = 66  
Obs per group: min = 8  
                  avg = 8  
                  max = 8

Log likelihood = 662.72689

wald chi2(12) = 11595.50  
Prob > chi2 = 0.0000

ln_ncost	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
ln_strand	.530273	.049985	10.61	0.000	.4323043 .6282418
ln_sl	.8200147	.0323098	25.38	0.000	.7566886 .8833407
ln_sl_d0406	-.0406637	.0174726	-2.33	0.020	-.0749093 -.0064182
ln_ll	.017509	.0138158	1.27	0.205	-.0095695 .0445874
ln_ll_d0406	.0319722	.0139723	2.29	0.022	.0045869 .0593575
ln_ts	.1561848	.0344275	4.54	0.000	.0887081 .2236615
ln_fibreprop	.1240753	.0300282	4.13	0.000	.0652211 .1829296
nyne	.1418538	.0472467	3.00	0.003	.0492519 .2344556
midwest	-.1415822	.0378027	-3.75	0.000	-.2156742 -.0674902
time	.0013556	.0040234	0.34	0.736	-.00653 .0092413
time_d0406	.0390869	.0060733	6.44	0.000	.0271834 .0509903
d0406	-.2514053	.0936129	-2.69	0.007	-.4348832 -.0679273
_cons	-.1890616	.2119691	-0.89	0.372	-.6045134 .2263902
/mu	.3267706	.078995	4.14	0.000	.1719432 .4815979
/lnsigma2	-4.243673	.1578494	-26.88	0.000	-4.553052 -3.934294
/ilgtgamma	1.2831	.2171707	5.91	0.000	.8574535 1.708747
sigma2	.0143548	.0022659			.010535 .0195595
gamma	.782977	.0369025			.7021283 .8466737
sigma_u2	.0112395	.0022711			.0067882 .0156907
sigma_v2	.0031153	.0002073			.0027089 .0035217



**Relative efficiency scores**

eff_rank	company	eff_sfa	eff_best	eff_decile
1	Qwest-Idaho South	0.043	0.000	-0.134
2	Qwest-North Dakota	0.129	0.082	-0.063
3	Qwest-Iowa	0.159	0.111	-0.038
4	Wisconsin Bell	0.160	0.112	-0.038
5	<b>BT Network</b>	<b>0.164</b>	<b>0.116</b>	<b>-0.033</b>
6	Verizon NE-Rhode Island	0.181	0.132	-0.020
7	Contel-Nevada	0.198	0.149	-0.005
8	Qwest-Montana	0.205	0.155	0.000
.				
.				
65	Verizon-New York Telephone	0.714	0.643	0.423
66	Verizon SO-North Carolina	0.716	0.645	0.425
67	Contel-Arizona	0.848	0.772	0.534

## Appendix F Deloitte SFA Results After NERA Comments

All the models were estimated in the Stata econometrics package.

### F.1 Amended model with two structural breaks

```
Iteration 0: log likelihood = -185.59636 (not concave)
Iteration 1: log likelihood = 771.59134 (not concave)
Iteration 2: log likelihood = 791.53982 (not concave)
Iteration 3: log likelihood = 825.03362 (not concave)
Iteration 4: log likelihood = 845.01747
Iteration 5: log likelihood = 851.97746
Iteration 6: log likelihood = 852.1466
Iteration 7: log likelihood = 852.14713
Iteration 8: log likelihood = 852.14713
```

```
Time-varying decay inefficiency model      Number of obs      =      727
Group variable: company_code              Number of groups   =      67
```

```
Time variable: time                      Obs per group: min =      1
                                           avg   =     10.9
                                           max   =     11
```

```
Log likelihood = 852.14713              wald chi2(20)      = 14621.12
                                           Prob > chi2        = 0.0000
```

log_total~t	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
log_strand~s	.5604923	.0385779	14.53	0.000	.484881	.6361036
log_sheath~e	.1448632	.0285172	5.08	0.000	.0889704	.200756
D_log_shea~e	.0952998	.016207	5.88	0.000	.0635347	.1270649
D2_log_shea~e	-.0180695	.0164593	-1.10	0.272	-.0503291	.0141902
log_tota~tes	-.0015811	.0141232	-0.11	0.911	-.029262	.0260999
D_log_to~tes	-.0249355	.0192308	-1.30	0.195	-.0626272	.0127562
D2_log_t~tes	-.0046251	.0168565	-0.27	0.784	-.0376631	.028413
log~d_lines	.1116909	.0162072	6.89	0.000	.0799254	.1434564
D_lo~d_lines	-.0056036	.0137709	-0.41	0.684	-.032594	.0213868
D2_lo~d_lines	.0516961	.0169201	3.06	0.002	.0185333	.0848588
l~cess_lines	.896831	.0205828	43.57	0.000	.8564894	.9371727
D_total_sw~s	.0212094	.0225472	0.94	0.347	-.0229824	.0654012
D2_total_s~s	-.0562437	.0229373	-2.45	0.014	-.1012	-.0112875
time	.0055996	.0040107	1.40	0.163	-.0022613	.0134605
D_time	.0256472	.0072345	3.55	0.000	.0114679	.0398266
D2_time	.0402548	.0070704	5.69	0.000	.0263971	.0541124
dummy	.6764215	.1965603	3.44	0.001	.2911704	1.061672
dummy2	-.3117821	.2223431	-1.40	0.161	-.7475666	.1240024
NYNE	.1694509	.0380516	4.45	0.000	.0948712	.2440307
Midwest	-.1240954	.0312473	-3.97	0.000	-.185339	-.0628518
_cons	-.4964967	.219198	-2.27	0.024	-.9261169	-.0668765
/mu	.1973825	.0323104	6.11	0.000	.1340552	.2607097
/eta	.0300956	.0080692	3.73	0.000	.0142802	.0459109
/lnsigma2	-4.332223	.1695996	-25.54	0.000	-4.664632	-3.999814
/ilgtgamma	.789715	.253637	3.11	0.002	.2925957	1.286834
sigma2	.0131383	.0022283			.0094227	.018319
gamma	.6877701	.0544666			.5726315	.7836109
sigma_u2	.0090361	.002223			.0046791	.0133931
sigma_v2	.0041022	.0002267			.0036578	.0045465

## Relative efficiency scores

eff_rank	company	eff_sfa	eff_best	eff_decile
1	WTWI	0.021	0.000	-0.068
2	MSID	0.025	0.004	-0.065
3	<b>BTOP</b>	<b>0.026</b>	<b>0.005</b>	<b>-0.063</b>
4	NERI	0.033	0.012	-0.057
.				
.				
61	NYNY	0.383	0.355	0.263
62	SWMO	0.399	0.371	0.277
63	GTSC	0.412	0.383	0.289
64	COAZ	0.443	0.413	0.317
65	GNCA	0.470	0.440	0.342
66	NWNE	0.507	0.476	0.375
67	GTNC	0.602	0.569	0.462

## F.2 Amended model with one structural break

```

Iteration 0: log likelihood = -293.81711 (not concave)
Iteration 1: log likelihood = 745.94373 (not concave)
Iteration 2: log likelihood = 786.76393
Iteration 3: log likelihood = 806.01671
Iteration 4: log likelihood = 809.11253
Iteration 5: log likelihood = 809.44062
Iteration 6: log likelihood = 809.46057
Iteration 7: log likelihood = 809.46102
Iteration 8: log likelihood = 809.46102

```

```

Time-varying decay inefficiency model
Group variable: company_code
Number of obs      =      727
Number of groups   =      67

Time variable: time
Obs per group: min =      1
               avg  =     10.9
               max  =     11

Log likelihood = 809.46102
wald chi2(14)    = 14790.86
Prob > chi2      = 0.0000

```

log_total~t	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
log_strand~s	.6070503	.0389982	15.57	0.000	.5306153 .6834853
log_sheath~e	.0716626	.0258414	2.77	0.006	.0210144 .1223109
D_log_shea~e	.1035314	.0164538	6.29	0.000	.0712825 .1357803
log_tota~tes	-.0185894	.0128556	-1.45	0.148	-.043786 .0066071
D_log_to~tes	-.0199003	.020099	-0.99	0.322	-.0592937 .019493
log~d_lines	.1193252	.0163242	7.31	0.000	.0873305 .15132
D_lo~d_lines	-.0229613	.0139097	-1.65	0.099	-.0502239 .0043012
l~cess_lines	.8858197	.0188867	46.90	0.000	.8488024 .922837
D_total_sw~s	.0414224	.0232257	1.78	0.075	-.0040991 .0869439
time	.0047594	.0040924	1.16	0.245	-.0032617 .0127804
D_time	.0302836	.0073048	4.15	0.000	.0159664 .0446008
dummy	.5268837	.1997735	2.64	0.008	.1353348 .9184326
NYNE	.1583475	.041037	3.86	0.000	.0779163 .2387786
Midwest	-.1162887	.0304163	-3.82	0.000	-.1759034 -.0566739
_cons	-.3492016	.2248667	-1.55	0.120	-.7899323 .0915291
/mu	.2306134	.0480019	4.80	0.000	.1365314 .3246954
/eta	.0348308	.0091393	3.81	0.000	.016918 .0527436
/lnsigma2	-4.347172	.1477004	-29.43	0.000	-4.63666 -4.057685
/ilgtgamma	.5790375	.2371888	2.44	0.015	.114156 1.043919
sigma2	.0129434	.0019117			.00969 .017289
gamma	.6408459	.0545919			.528508 .7396055
sigma_u2	.0082947	.001901			.0045687 .0120207
sigma_v2	.0046487	.0002575			.0041439 .0051534

**Relative efficiency scores**

eff_rank	company	eff_sfa	eff_best	eff_decile
1	WTWI	0.030	0.000	-0.100
2	NERI	0.038	0.008	-0.093
<b>3</b>	<b>BTOP</b>	<b>0.058</b>	<b>0.027</b>	<b>-0.076</b>
4	MSID	0.072	0.041	-0.063
5	CONV	0.096	0.065	-0.042
.				
.				
59	NYNY	0.406	0.365	0.228
60	MSWY	0.415	0.374	0.237
61	SWTX	0.432	0.390	0.251
62	GTSC	0.433	0.392	0.252
63	COAZ	0.441	0.399	0.259
64	SWMO	0.443	0.401	0.261
65	GNCA	0.462	0.419	0.277
66	NWNE	0.546	0.501	0.351
67	GTNC	0.646	0.598	0.438

## Appendix G Regression Output for Econometric TFP Model

This appendix provides the regression outputs for the models used in Section Appendix A. All the models were estimated in the Stata econometrics package.

### G.1 Models used in Hausman test

#### G.1.1 Test 1 Model - Fixed Effects

```
Fixed-effects (within) regression              Number of obs   =       286
Group variable: company_code                  Number of groups =        26

R-sq:  within = 0.1115                        Obs per group:  min =        11
        between = 0.9025                      avg   =       11.0
        overall = 0.8768                      max   =        11

corr(u_i, Xb) = 0.5704                        F(4,256)        =        8.03
                                                Prob > F         =       0.0000
```

ln_Törnqvist	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
ln_materials	.0231216	.1064322	0.22	0.828	-.1864726 .2327157
ln_staff	.2007008	.0916982	2.19	0.030	.0201219 .3812796
ln_capital	.505918	.1010557	5.01	0.000	.3069117 .7049243
time	.0114339	.0050502	2.26	0.024	.0014887 .0213791
_cons	-12.6042	2.445171	-5.15	0.000	-17.41941 -7.788992
sigma_u	.41950149				
sigma_e	.19601528				
rho	.82079588	(fraction of variance due to u_i)			
F test that all u_i=0:		F(25, 256) =	31.04	Prob > F = 0.0000	

#### G.1.2 Test 1 Model - Random Effects

```
Random-effects GLS regression              Number of obs   =       286
Group variable: company_code              Number of groups =        26

R-sq:  within = 0.1094                        Obs per group:  min =        11
        between = 0.9042                      avg   =       11.0
        overall = 0.8786                      max   =        11

Random effects u_i ~ Gaussian              wald chi2(4)    =       274.80
corr(u_i, X) = 0 (assumed)                 Prob > chi2     =       0.0000
```

ln_Törnqvist	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
ln_materials	.1197455	.0835537	1.43	0.152	-.0440166	.2835077
ln_staff	.2502065	.0692252	3.61	0.000	.1145277	.3858854
ln_capital	.5568306	.091042	6.12	0.000	.3783915	.7352696
time	.015457	.0040251	3.84	0.000	.0075679	.0233461
_cons	-15.37639	.8212514	-18.72	0.000	-16.98601	-13.76676
sigma_u	.32817344					
sigma_e	.19601528					
rho	.73705145	(fraction of variance due to u_i)				

### G.1.3 Test 2 Model - Fixed Effects

Fixed-effects (within) regression  
 Group variable: company\_code

Number of obs = 286  
 Number of groups = 26

R-sq: within = 0.1697  
 between = 0.9030  
 overall = 0.8787

Obs per group: min = 11  
 avg = 11.0  
 max = 11

corr(u\_i, Xb) = 0.6276

F(5,255) = 10.42  
 Prob > F = 0.0000

ln_Törnqvist	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
ln_materials	.0878996	.1042183	0.84	0.400	-.1173386 .2931377
ln_staff	.2173097	.0889025	2.44	0.015	.042233 .3923863
ln_capital	.389253	.101692	3.83	0.000	.1889899 .5895162
time_d9601	.0446743	.0092575	4.83	0.000	.0264434 .0629051
time_d0206	.0186161	.0051778	3.60	0.000	.0084193 .0288129
_cons	-12.07662	2.371592	-5.09	0.000	-16.74702 -7.406219
sigma_u	.44246372				
sigma_e	.18985367				
rho	.84451454	(fraction of variance due to u_i)			
F test that all u_i=0:		F(25, 255) =	32.98	Prob > F = 0.0000	

### G.1.4 Test 2 Model - Random Effects

Random-effects GLS regression  
 Group variable: company\_code

Number of obs = 286  
 Number of groups = 26

R-sq: within = 0.1662  
 between = 0.9044  
 overall = 0.8806

obs per group: min = 11  
 avg = 11.0  
 max = 11

Random effects u\_i ~ Gaussian  
 corr(u\_i, X) = 0 (assumed)

wald chi2(5) = 293.50  
 Prob > chi2 = 0.0000

ln_Törnqvist	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
ln_materials	.1918585	.0831583	2.31	0.021	.0288713	.3548458
ln_staff	.2764623	.0676416	4.09	0.000	.1438873	.4090374
ln_capital	.4526045	.0918944	4.93	0.000	.2724947	.6327143
time_d9601	.0487368	.0089293	5.46	0.000	.0312356	.066238
time_d0206	.0230718	.0043245	5.34	0.000	.014596	.0315477
_cons	-15.26348	.8185195	-18.65	0.000	-16.86775	-13.65921
sigma_u	.32850259					
sigma_e	.18985367					
rho	.74961916	(fraction of variance due to u_i)				

### G.1.5 Test 3 Model - Fixed Effects

FE (within) regression with AR(1) disturbances  
 Group variable: company\_code

Number of obs = 260  
 Number of groups = 26

R-sq: within = 0.1319  
 between = 0.8499  
 overall = 0.5997

Obs per group: min = 10  
 avg = 10.0  
 max = 10

corr(u\_i, Xb) = 0.7326

F(5,229) = 6.96  
 Prob > F = 0.0000

ln_Törnqvist	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
ln_materials	-.0552642	.0752097	-0.73	0.463	-.2034556 .0929273
ln_staff	.1958152	.0858936	2.28	0.024	.0265724 .3650579
ln_capital	.0419581	.1130388	0.37	0.711	-.180771 .2646871
time_d9601	-.0391023	.0160557	-2.44	0.016	-.0707381 -.0074665
time_d0206	-.0440607	.0133031	-3.31	0.001	-.0702729 -.0178486
_cons	-4.070047	.6831724	-5.96	0.000	-5.416154 -2.72394
rho_ar	.70517647				
sigma_u	.90309889				
sigma_e	.13156607				
rho_fov	.97921757				

(fraction of variance because of u\_i)

F test that all u\_i=0: F(25,229) = 8.67 Prob > F = 0.0000

### G.1.6 Test 3 Model - Random Effects

RE GLS regression with AR(1) disturbances  
 Group variable: company\_code

Number of obs = 286  
 Number of groups = 26

R-sq: within = 0.1513  
 between = 0.9027  
 overall = 0.8785

Obs per group: min = 11  
 avg = 11.0  
 max = 11

corr(u\_i, Xb) = 0 (assumed)

wald chi2(6) = 273.49  
 Prob > chi2 = 0.0000

ln_Törnqvist	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
ln_materials	.0986612	.0680422	1.45	0.147	-.034699 .2320214
ln_staff	.2833936	.0689035	4.11	0.000	.1483453 .418442
ln_capital	.5169003	.0891928	5.80	0.000	.3420857 .6917149
time_d9601	.0320953	.0081754	3.93	0.000	.0160718 .0481189
time_d0206	.0189763	.0057809	3.28	0.001	.0076461 .0303066
_cons	-14.97994	.8122454	-18.44	0.000	-16.57191 -13.38797
rho_ar	.70517647				
sigma_u	.30742749				
sigma_e	.14390478				
rho_fov	.82026957				
theta	.62889993				

(estimated autocorrelation coefficient)

(fraction of variance due to u\_i)

## G.2 Structural break models

### G.2.1 Regression output for structural break 2001/02 across time trend

Fixed-effects (within) regression  
 Group variable: company\_code

Number of obs = 286  
 Number of groups = 26

R-sq: within = 0.1697  
 between = 0.9030  
 overall = 0.8787

Obs per group: min = 11  
 avg = 11.0  
 max = 11

corr(u\_i, Xb) = 0.6276

F(5,255) = 10.42  
 Prob > F = 0.0000

ln_Törnqvist	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
ln_materials	.0878996	.1042183	0.84	0.400	-.1173386 .2931377	
ln_staff	.2173097	.0889025	2.44	0.015	.042233 .3923863	
ln_capital	.389253	.101692	3.83	0.000	.1889899 .5895162	
time_d9601	.0446743	.0092575	4.83	0.000	.0264434 .0629051	
time_d0206	.0186161	.0051778	3.60	0.000	.0084193 .0288129	
_cons	-12.07662	2.371592	-5.09	0.000	-16.74702 -7.406219	
sigma_u	.44246372					
sigma_e	.18985367					
rho	.84451454	(fraction of variance due to u_i)				
F test that all u_i=0:		F(25, 255) =	32.98	Prob > F = 0.0000		
Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	286	59.18469	85.7792	6	-159.5584	-137.6224

### G.2.2 Regression output for structural break 2002/03 across time trend

Fixed-effects (within) regression  
 Group variable: company\_code

Number of obs = 286  
 Number of groups = 26

R-sq: within = 0.1860  
 between = 0.9006  
 overall = 0.8768

Obs per group: min = 11  
 avg = 11.0  
 max = 11

corr(u\_i, Xb) = 0.6375

F(5,255) = 10.07  
 Prob > F = 0.0000

(Std. Err. adjusted for clustering on company\_code)

ln_Törnqvist	Coef.	Robust Std. Err.	t	P> t	[95% Conf. Interval]	
ln_materials	.1417712	.1117006	1.27	0.206	-.078202 .3617445	
ln_staff	.2806341	.1045705	2.68	0.008	.0747024 .4865659	
ln_capital	.2584291	.1572227	1.64	0.101	-.0511911 .5680494	
time_d9602	.0415934	.0086773	4.79	0.000	.0245051 .0586818	
time_d0306	.0155618	.0064209	2.42	0.016	.0029171 .0282066	
_cons	-11.69769	3.292945	-3.55	0.000	-18.18252 -5.212861	
sigma_u	.45285759					
sigma_e	.18798033					
rho	.85301928	(fraction of variance due to u_i)				
Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	286	59.18469	88.61526	6	-165.2305	-143.2946



### G.2.3 Regression output for structural break 2001/02 across all variables

Fixed-effects (within) regression		Number of obs	=	286		
Group variable: company_code		Number of groups	=	26		
R-sq: within	= 0.2549	Obs per group: min	=	11		
between	= 0.9089	avg	=	11.0		
overall	= 0.8828	max	=	11		
corr(u_i, Xb) = 0.8032		F(8,252)	=	9.26		
		Prob > F	=	0.0000		
(Std. Err. adjusted for clustering on company_code)						
ln_Törnqvist	Coef.	Robust Std. Err.	t	P> t	[95% Conf. Interval]	
ln_mate~9601	.3189913	.1194838	2.67	0.008	.0836772	.5543053
ln_mate~0206	-.1774059	.138238	-1.28	0.201	-.4496548	.0948431
ln_staf~9601	.2232787	.1336119	1.67	0.096	-.0398596	.4864169
ln_staf~0206	.1138494	.1052322	1.08	0.280	-.0933973	.3210961
ln_capi~9601	-.0007323	.2027437	-0.00	0.997	-.4000202	.3985557
ln_capi~0206	.5743274	.1897417	3.03	0.003	.2006459	.948009
time_d9601	.0606902	.0098751	6.15	0.000	.041242	.0801383
time_d0206	-.0017811	.0182443	-0.10	0.922	-.0377119	.0341497
_cons	-9.611936	3.82655	-2.51	0.013	-17.14803	-2.075843
sigma_u	.57422555					
sigma_e	.18091587					
rho	.90970024	(fraction of variance due to u_i)				
Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	286	59.18469	101.2629	9	-184.5258	-151.6218

## G.2.4 Regression output for structural break 2002/03 across all variables

Fixed-effects (within) regression		Number of obs	=	286		
Group variable: company_code		Number of groups	=	26		
R-sq: within	= 0.2432	obs per group: min	=	11		
between	= 0.9060	avg	=	11.0		
overall	= 0.8807	max	=	11		
corr(u_i, Xb) = 0.7841		F(8,252)	=	8.29		
		Prob > F	=	0.0000		
(Std. Err. adjusted for clustering on company_code)						
ln_Törnqvist	Coef.	Robust Std. Err.	t	P> t	[95% Conf. Interval]	
ln_mate~9602	.3221854	.1185553	2.72	0.007	.0886999 .5556709	
ln_mate~0306	-.206156	.1542798	-1.34	0.183	-.5099981 .0976861	
ln_staf~9602	.1814848	.1279155	1.42	0.157	-.0704349 .4334046	
ln_staf~0306	.1489761	.1131731	1.32	0.189	-.0739095 .3718616	
ln_capi~9602	.0575203	.2030358	0.28	0.777	-.3423431 .4573836	
ln_capi~0306	.586126	.1996792	2.94	0.004	.1928733 .9793787	
time_d9602	.0474842	.0083862	5.66	0.000	.0309683 .0640001	
time_d0306	.0087589	.0224228	0.39	0.696	-.0354011 .0529188	
_cons	-9.949313	3.865304	-2.57	0.011	-17.56173 -2.336897	
sigma_u	.55630644					
sigma_e	.18232768					
rho	.9030014	(fraction of variance due to u_i)				
Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	286	59.18469	99.03968	9	-180.0794	-147.1754

## Appendix H References and glossary

### H.1 References

A list of sources, along with their citation convention for this report, are set out in Table 22.

**Table 22: References**

Citation in Report	Reference
Deloitte (2008)	Deloitte, 2008: ' <i>The Efficiency of BT's Network Operations</i> '
Forbes (2000)	Forbes KJ, 2000: ' <i>A reassessment of the relationship between inequality and growth</i> '. American Economic Review, Vol. 90. No. 4, pp. 869-887
Hausman (1978)	Hausman JA, 1978: ' <i>Specification Tests in Econometrics</i> '. Econometrica, Vol. 46, No. 6 (November 1978), pp. 1251-1271
Islam (1995)	Islam N, 1995: ' <i>Growth Empirics: A Panel Data Approach</i> ', The Quarterly Journal of Economics, Vol. 110, No. 4 (Nov 1995), pp. 1127-1170
Koehler & Murphree (1998)	Koehler A and Murphree E, 1998: ' <i>A Comparison of the Akaike and Schwartz Criteria for Selecting Model Order</i> '. Applied Statistics Vol. 37:2, 187-195
Mankiw et al (1992)	Mankiw NG, Romer D, Weil DN (1992): ' <i>A Contribution to the Empirics of Economic Growth</i> ', The Quarterly Journal of Economics, Vol. 107, No. 2 (May 1992), pp. 407-437
NERA (2005)	NERA, 2005: ' <i>The Comparative Efficiency of BT in 2003</i> '
NERA (March 2008)	NERA, 2008: ' <i>The Comparative Efficiency of BT Openreach</i> ', available from <a href="http://www.ofcom.org.uk/consult/condocs/llcc/efficiency.pdf">http://www.ofcom.org.uk/consult/condocs/llcc/efficiency.pdf</a>
NERA (May 2008)	NERA, 2008: ' <i>Comments on the Deloitte paper on the efficiency of BT's network operations</i> ', available from <a href="http://www.ofcom.org.uk/consult/condocs/llcc/operations.pdf">http://www.ofcom.org.uk/consult/condocs/llcc/operations.pdf</a>
NERA (December 2008)	NERA, 2008: ' <i>NERA's Analysis of the Efficiency of BT's Network Operations</i> '
Ofcom (2008)	Ofcom, 2008: ' <i>Leased Lines Charge Control</i> ', available from <a href="http://www.ofcom.org.uk/consult/condocs/llcc/leasedlines.pdf">http://www.ofcom.org.uk/consult/condocs/llcc/leasedlines.pdf</a>
Romer (2000)	Romer D, 2000: Advanced Macroeconomics McGraw-Hill/Irwin

## H.2 Glossary

Abbreviation	Reference
BT	British Telecom
FCC	Federal Communications Commission
LECs	Local Exchange Carriers
SFA	Stochastic Frontier Analysis
TFP	Total factor productivity

## Appendix I      Our Previous Report

This appendix contains our previous report, referred to above as Deloitte (2008).



# **The Efficiency of BT's Network Operations**

**A Report for BT**

**9 May 2008**

**Private and Confidential**

**Audit.Tax.Consulting.Corporate Finance.**

This report has been prepared on the basis of the limitations set out in the engagement letter and the matters noted in the Important Notice From Deloitte on page 1.

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This report has been prepared for British Telecommunications plc ("BT") in accordance with the services agreement of the 7<sup>th</sup> February 2008 (the "Services Agreement") and on the basis of the scope and limitations as set out below.

This report has been prepared solely for the purposes of supporting BT in understanding its relative efficiency using a particular econometric modelling approach and to support its discussions with Ofcom. Efficiency benchmarks are used in this report solely for informing BT in their prospective role in supporting Ofcom's determination of BT's efficient cost basis for regulatory purposes. It is for BT to determine the suitability of this report for distribution outside of BT, and Deloitte accepts no responsibility for consequences of any action which Ofcom may choose to take as a consequence of BT releasing this report to them. These benchmarks should not be used for any other purpose and Deloitte accepts no responsibility for their use in any other context. Any results from our efficiency modelling are reliant on information available at the time of our writing this report and should not be relied upon in subsequent periods.

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As set out in the Services Agreement, the scope of our work has been limited by the time, information and explanations made available to us. We have relied upon the documents and data provided by BT and on third party data sources which have been clearly referenced in the appropriate sections of this report. We have no responsibility for the accuracy or completeness of this information and have neither sought to corroborate it, nor reviewed its overall reasonableness.

## Executive Summary

This study finds BT to be not comparatively inefficient compared to the top decile of US operators ranked by efficiency, and calculates that the efficiency (or productivity) frontier of telecommunications network operations is shifting at between 0.5% and 1.1% per year.

Ofcom is currently seeking to apply a set of price controls to products provided by BT Openreach and BT Wholesale. Typically, price controls in the telecommunications sector require the regulated company, in this case BT, to increase their productivity to allow for three separate but often related effects:

- Catch-up: Where BT is expected to increase its relative efficiency in the current time period to match that of some benchmark representing an efficient comparator.
- Productivity gain: The annual increase in productivity, assuming constant volumes, that BT may be expected to experience during the price control period.
- Economies of scale: The change in BT's unit costs that results from a change in volumes.

This study looks at the first two of these measures. In considering this question we focus on the efficiency of BT's entire network operations including both BT Openreach and BT's wholesale network business, as previous analysis has shown that it is not practical to disaggregate the efficiency effects of these two operations.

Accurate data is key to the techniques utilised for the empirical estimation of efficiency. As is normal in estimating the catch-up component, BT data has been compared to that of the US Local Exchange Carriers (LECs). LEC data has been taken from the ARMIS database maintained by the Federal Communications Commission (FCC), which is provided in response to highly detailed descriptions and has been taken as accurate. A number of adjustments are made to the BT data to provide for better comparability and these are reconciled to published accounts in the report.

In estimating the productivity gain, less disaggregated data is required and we include European incumbent operators alongside the LECs and BT in our estimates. This data is collated from annual reports but has been stripped of any mobile operations.

The approach to modelling BT's comparative efficiency has become well established over recent years: stochastic frontier analysis (SFA) is run over a panel dataset to estimate a Cobb-Douglas production function specifying the relationship between costs and outputs. We followed this approach estimating a specification including two structural breaks and including total sheath per line as a proxy for population distribution. Our results suggest that BT lies slightly above the decile of the list of US LECs ranked by efficiency. Following from Ofcom's previous work, this implies that BT should be considered efficient, therefore not requiring a catch-up component of efficiency.

In previous studies, Ofcom has used the time trend resulting from panel data SFA to set the productivity gain that may be expected in each year. We believe that in the current climate, with rapidly changing volumes (both increasing and decreasing) and shifts in the market and general

economy, this may give an inaccurate description of productivity movements. Our alternative has been to estimate two separate measures to calculate the annual productivity gain:

- A calculation of total factor productivity (TFP) using the rate of change of input and output indices. Our work is based on the use of a Törnqvist index which provides a growth rate for both outputs and inputs, which we calculate for each of the LECs and European incumbents. The subtraction of the annual input growth rate from the annual output growth rate then provides a measure of the TFP.
- Estimation of the TFP, where a standard fixed effects growth model is estimated with an econometric specification which allows for a general technology trend. This trend provides an estimate of TFP after allowing for fixed heterogeneity across firms, idiosyncratic technology effects, and other movements in output from changes in production.

We have found that our two methods of calculating TFP give very similar and consistent results. Our results show that productivity growth lies in the range of 0.5% (calculated using the first method above) to 1.1% (estimated using the second method). However, we believe that the upper end of this range may be biased by capacity utilisation effects, as it was not possible to fully remove these in our regression specification.

Similar results were found when we added European incumbents to our original data set based on the US LECs, supporting the robustness of our results.

# 1 Introduction

This report builds upon previous work undertaken by Ofcom, updating previous studies into BT's efficiency<sup>1</sup> and moving on to expand the methods used to examine how the efficiency frontier moves over time.

## 1.1 Background

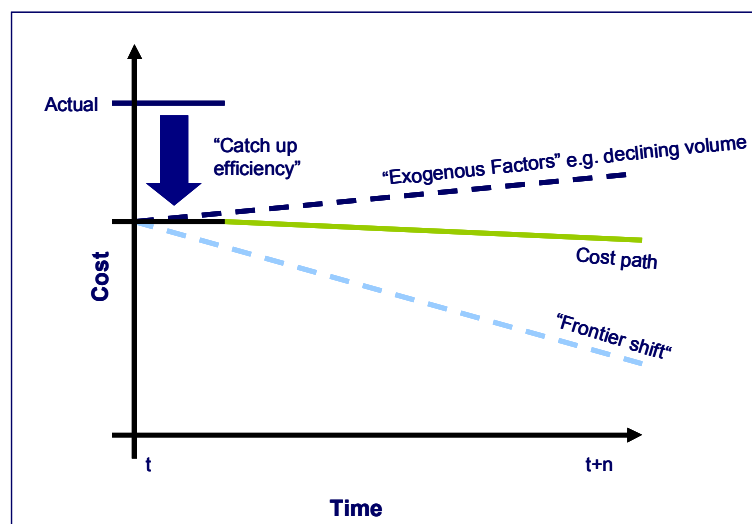
Ofcom is currently seeking to apply a series of network charge controls (NCCs) to services provided by BT. These will be applied to products supplied by BT's network operations such as partial private circuits (PPCs) and termination. As part of this NCC process Ofcom proposes to estimate the extent to which BT Wholesale and Openreach, both jointly and independently, perform efficiently.

There are usually three types of efficiency factors incorporated into NCC models:

- The initial inefficiency, or "catch-up" component, which is included as a percentage reduction in the base year costs to reflect the current level of BT's relative inefficiency;
- An annual productivity target to reflect technological improvements that lead to continuous shifts in the efficiency frontier at constant volume levels; and
- An adjustment to the annual productivity target to reflect exogenous impacts on BT's costs, for example declining volumes.

These three elements are illustrated in Figure 1, leading to a final cost path.

**Figure 1: Incorporation of efficiency factors into NCC models**



<sup>1</sup> Including NERA Economic Consulting (2005): 'The Comparative Efficiency of BT in 2003' from <http://www.ofcom.org.uk/consult/condocs/charge/main/nera.pdf>

In previous studies undertaken for Ofcom and Oftel<sup>2</sup>, the US LECs were chosen for comparison due to the disaggregated information that they are required to provide and which is not published by, for example, European incumbent operators. Sophisticated econometric models were then employed to seek a like-for-like comparison between BT and the US LECs.

We understand from meetings with Ofcom that they have commissioned a similar study to conduct an external analysis of the comparative efficiency of BT and that this study would adopt a top-down approach to this issue based on a comparison of BT with the US Local Exchange Carriers (LECs). This study has not been published, but the approach described is consistent with that previously adopted by Ofcom and Oftel in considering the relative efficiency of BT as a whole for previous price controls.

This top-down approach has difficulty in drawing meaningful conclusions over the frontier shift component described above, since not every change in cost over time can be allocated directly to shifts in productivity. For this reason we have investigated further methodologies that can be used to estimate the movement in the efficient frontier, including utilising the available LEC data for other statistical purposes to calculate productivity changes.

## 1.2 Terms of reference

BT has commissioned Deloitte to estimate BT's relative efficiency and the annual productivity gain, where BT is defined as "BT network" comprising of both Openreach and BT's wholesale network business. We have not been asked to calculate the impact of change in volumes and understand that BT will be providing Ofcom with a separate series of cost volume elasticities (CVEs) and asset volume elasticities (AVEs) to allow for the calculation of volume effects outside of our modelling.

Taking a standard approach to that has been used in previous NCCs, but expanding it to make greater use of the available data, we have:

- Calculated the relative efficiency of BT. Utilising disaggregated data from 1996-2006 available from the US LECs, we tested various modelling specifications to allow for a consideration of the full range of possible efficiency results. Our approach contains analysis using stochastic frontier analysis (SFA) which is consistent with the approach used previously.
- Calculated the rate of change of input and output indexes for both the US LECs and a selection of European incumbent telecommunication operators. Using the rate of change of these indexes and by separating out volume effects, we calculate the average total factor productivity (TFP) growth rate over the period.
- Estimated the improvements in technology, consistent with a TFP definition, by using a fixed effects growth model which can be estimated using the US LEC data. We tested

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<sup>2</sup> Including NERA (2005), but also NERA Economic Consulting (2004): 'BT Efficiency: Private Circuits Study', National Economic Research Associates (2001): 'The Comparative Efficiency of BT: An Update' and National Economic Research Associates (2000): 'The Comparative Efficiency of BT'

various modelling specifications to allow for consideration of a broad range of technological improvements.

## **1.3 Structure of this report**

This report is structured as follows:

- Section 2 provides a summary of the theoretical background to the approach adopted to econometrically estimate the relative efficiency of BT;
- Section 3 discusses the comparability of BT, the US LECs and the European Incumbents and the approaches we sought to adopt to make the comparison meaningful;
- Section 4 presents the results of our relative efficiency modelling;
- Section 5 presents the results of our productivity analysis;
- Section 6 outlines our main conclusions; and
- The final section discusses the implication of the results of this report and their application to the regulatory environment.

Appendices provide additional information on BT data, including reconciliation to audited financial or regulatory accounts alongside further econometric and statistical outputs.

## 2 Methodology

This section sets out the theoretical framework around which we have based our study:

- Firstly, we examine the comparative efficiency analysis, used to compare BT's costs to those of a theoretically efficient frontier.
- Secondly, we look at productivity analysis, used to estimate the movement in the efficient frontier over time. We propose two methods for measuring this.

### 2.1 Comparative efficiency

The concept of efficiency of a company can be thought of as a company's ability to minimise its costs, for a given level and set of outputs, taking into account the environment in which the company operates.

The idea of comparative efficiency analysis is to estimate the inefficiency of a company without the need for a detailed bottom-up analysis of costs and production. To do this for a particular period, the costs of a company are compared against the costs of a similar company, with differences in the company size, outputs, operations and operating environment adjusted for. To compare efficiency over time, it is possible to compare between companies although an alternative method is to observe time trends within individual companies. These techniques are therefore top-down approaches, requiring far less data and are often considered to be quicker and easier to implement than a bottom-up approach.

#### 2.1.1 Efficiency drivers

There exist several methods that can be used to measure and compare efficiencies of different companies. The simplest method would be to choose a basic unit cost indicator and compare this across the companies in the sample to produce an efficiency ranking. For example, for a telecom company, indicators could be cost per subscriber line, or the number of employees per line. However, this method has serious drawbacks, as it does not allow us to take into account technological and strategic choices made by the operators and the environment in which they operate. Nor does it allow us to consider operators who incur costs through providing more than one service, unless we are able to specifically split out the costs.

To overcome this problem that exists with unit costs or key performance indicators (KPIs), there are several statistical and mathematical programming techniques that can be applied in order to measure comparative efficiency levels across all products. Using total costs in this way allows us to take account of the fact that companies are able to trade off costs between differing inputs, such as labour and capital.

Statistical techniques involve the use of regression analysis, which uses companies' past data to estimate a relationship between costs, output variables (such as number of switched lines and minutes) and environmental variables (such as population density, and wage level). The most commonly statistical techniques used are described later in this section.

## 2.1.2 Relationship of cost to outputs

The primary driver of cost will be the size of the company, or, more accurately, the outputs it produces. For telecommunications companies, the outputs we consider include:

- Switched (PSTN) lines;
- Leased lines;
- Call minutes;
- Data services; and
- Interconnection services.

For each of these, as the output amount increases we would expect the cost to also increase. If all other outputs were held constant, we would be able to directly measure the effect of an increase in one output on cost.

$$\begin{aligned}
 \text{cost} = & a \\
 & + \beta^1 \times \text{switched lines} \\
 & + \beta^2 \times \text{leased lines} \\
 & + \beta^3 \times \text{call minutes} \\
 & + \beta^4 \times \text{data services} \\
 & + \beta^5 \times \text{interconnection services}
 \end{aligned}
 \tag{Equation 1}$$

In this function, each of the coefficients  $\beta^k$  represents the change in cost caused by a change in the relevant variable when all other variables are held constant.

In practice, it is not possible for us to measure the relationship between costs and outputs in this way. In order to estimate such a cost function, we instead use econometric statistical methods over a large number of observations.

As we are seeking to use this analysis to estimate a level of comparative efficiency, we cannot simply look at past of the company we are examining since this would tell us only whether the efficiency of that company had improved or worsened over time, and nothing about its actual levels of efficiency. By using other companies' data, our analysis will provide an indication about the company's relative level of efficiency compared to other telecommunication operators. The choice of comparators is described in section 2.1.10.

## 2.1.3 Relationship of cost to environmental factors

This analysis can be expanded to include other drivers of cost. The cost of a telecommunications operator will depend not only on the outputs it produces, but also on the environment it operates in. An operator in a densely populated area will require a smaller network (and thus lower costs) to provide the same number of lines.



We therefore include further variables in our cost function, to allow for these differences. Such variables include:

- Population density (or proxies thereof);
- Relative wage costs; and
- Weather and temperature differentials.

These would be included in our econometric analysis in the same way as the outputs outlined above, giving an overall cost function as shown below.

$$\begin{aligned}
 \text{cost} = & a \\
 & + \beta^k \times \text{switched lines} \\
 & + \dots \\
 & + \alpha^l \times \text{population density} \\
 & + \dots
 \end{aligned}
 \tag{Equation 2}$$

Each of the variables must then be tested within the econometric framework to see whether it is a statistically significant driver of costs.

## 2.1.4 Movements in efficiency over time

As well as differences in costs between companies (cross-sectional differences) there are also differences in efficiency over time. Companies are generally expected to become more efficient over time and this is true of telecommunications companies where improvements in technology have led to declining unit costs. Our analysis will therefore allow for the estimate of the movement in efficiency over time.

While this method gives some indication of how costs are moving over time, it is not possible to state that this is completely to technology effects. Any other exogenous factor, such as changes in regulation, general economic wellbeing, and income effects will also be included within this time trend. It is important for the sake of our econometric analysis that such factors are removed. However, it does mean that the extent to which this time trend can be used to look at technology shifts in the efficiency frontier is limited.

We have undertaken a more detailed analysis of the movement of the efficiency frontier, as set out in section 2.2.

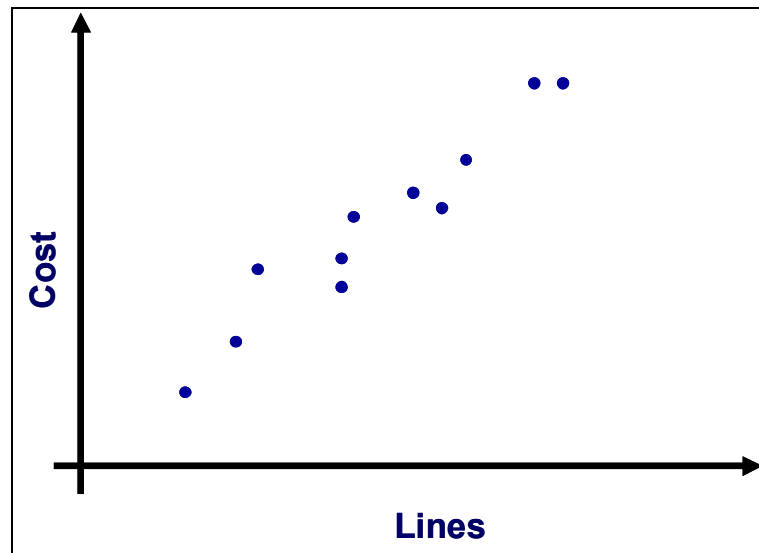
## 2.1.5 Econometric techniques to estimate comparative efficiency

The comparative efficiency ("catch up component") may be calculated using cross-sectional comparisons between companies. If this cross-sectional data is available for a number of time periods then a "panel" can be created that allows for the use of a larger range of econometric techniques and can be expected to provide more robust results. The econometric techniques that may be used are explained below.

Given a sufficiently large number of observations<sup>3</sup>, econometric analysis allows us to model a relationship between costs and other variables by considering the differences between operators.

To understand how econometric analysis works, it is initially useful to consider a simple two-variable model, where costs are dependent only on the number of switched lines supplied. Measuring the costs and lines for a number of operators, we can plot observations as follows.

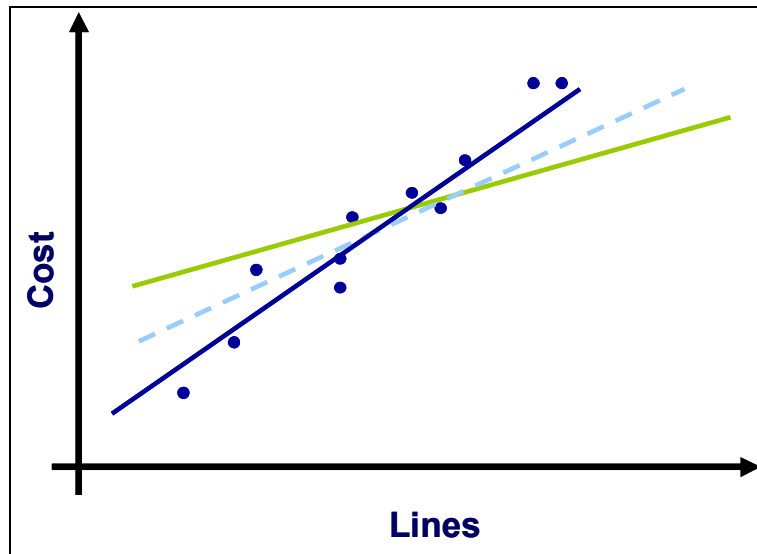
**Figure 2: Example simple relationship between costs and lines**



The above figure demonstrates that, in general, as the number of lines supplied increases, so does the cost of providing those lines. In order to estimate a relationship between costs and lines, we can draw a 'line of best fit' through our observations, and look at the slope and intersection point of that line. There exist a number of potential methods for determining the line of best fit (represented by the various lines in Figure 3)<sup>4</sup>.

<sup>3</sup> The definition of 'sufficient' is dependent upon the number of variables in the regression and the standard deviation of the observations. Theoretically this is set by the law of large numbers where the central limit theorem applies and the distribution may be determined to be normally distributed. However, in practice such variable numbers are rarely available to econometric investigations and an accepted guide of around 30 observations is typically considered sufficient to provide statistical confidence.

<sup>4</sup> For example, methods include least squares, quantile regression, non-parametric regression and Bayesian methods.

**Figure 3: Illustrative lines of best fit between costs and lines**

The most common method of performing this analysis is through least squares analysis. This method measures the distance of each point from a linear line and then sums the square of these distances. A line of best fit is then reached by adjusting the line until the sum of squares is minimised.

The line of best fit can be interpreted as estimating the average cost of a company given its output level.

Although the two-dimensional case illustrated above is useful to understand the principles of Ordinary Least Square (OLS) analysis, there are generally more than two explanatory variables included in a regression. A cost function for an operator is likely to include other cost drivers in addition to the number of lines, such as number of call minutes. In the case of multiple explanatory variables, an econometrics package will compute the coefficients that produce the 'best-fit' across all dimensions. The multivariate regression that we model takes the following general form:

$$C_i = a + \sum_{k=1}^K \beta^k Y_i^k + \sum_{l=1}^L \alpha^l N_i^l + \varepsilon_i \quad (\text{Equation 3})$$

where:

subscript  $i$  represents the firm under consideration;

$C_i$  is the total cost;

$Y^k$  are the output variables;

$N^l$  are network (environmental) variables;

$a$  represents the fixed costs;

$\beta^k$  and  $\alpha^l$  are the coefficients of the output and network variables respectively; and

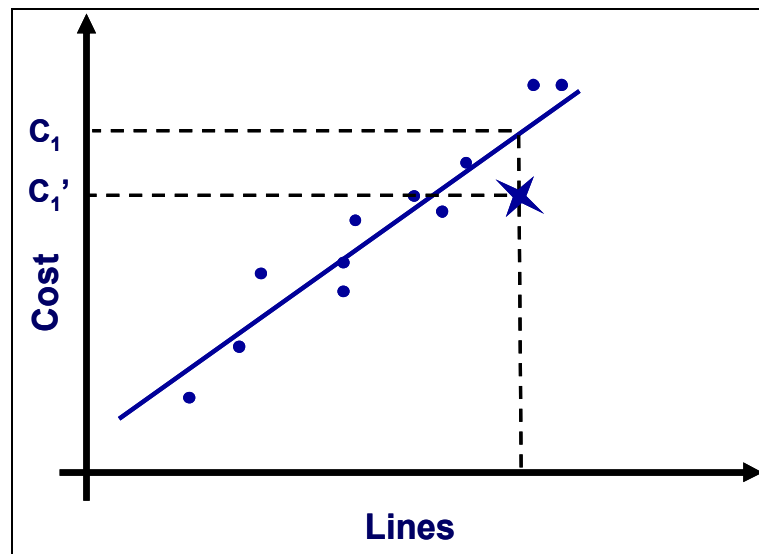
$\varepsilon_i$  is the residual.

We return to the simple example above to illustrate how this analysis can be used to estimate a firm's comparative efficiency.

The figure below shows a relatively efficient firm, marked by the star. Given a level of output  $I_1$ , our regression analysis predicts that that operator should have costs of  $c_1$ . However, we can see from the position of the operator on the graph that costs are at  $c_1'$ , which is lower than expected. This firm therefore has a higher than average cost efficiency.

The opposite is also true. If the firm lay above the line of best fit, then its costs would be higher than predicted, meaning that it would be more inefficient than the average firm.

**Figure 4: Efficiency and the relationship between costs and lines**



Depending on the aim of our exercise, it may be optimum to compare operators not against the average company (as represented by this line of best fit), but against the most efficient company or a particular subset of efficient companies. This may be achieved either by shifting the line downwards, or by excluding some operators from the regression. However, the second of these options may lead to issues of incomparability and a less robust model.

We discuss below a number of econometric techniques which may be used to estimate relative efficiency and provide details of the functional form and comparators to which we have applied these techniques.

### 2.1.6 OLS analysis

The major advantages of OLS analysis is that it is easy to estimate and to interpret. However, it also has several drawbacks some of which are particularly relevant in the context of this study:

- The validity of OLS analysis depends on some strict statistical assumptions which might not always be satisfied. In particular the assumption of independence between explanatory variables and residuals might not be appropriate in the case of efficiency studies where the residual is interpreted as the measure of inefficiency of a company.
- In OLS regression, the whole of the residual from the regression is interpreted as inefficiency. In reality this is unlikely to be the case due to factors such as data errors,

omitted variables and non-linearity of relations between variables possibly introducing some randomness in the error term. In other words, the residuals should be thought of as made up by two components, one of which being the inefficiency of the company and the other a random white noise error. OLS regressions are unable to separate these two components.

- The output of an OLS regression can only be interpreted as inefficiency compared to a company of “average” efficiency. Generally, however, the relative efficiency compared to more efficient companies is of more interest. Moreover, the use of average efficiency as a benchmark can reduce incentives for any improvements in performance.
- OLS is only efficient when using only one year of data (a cross-sectional dataset)<sup>5</sup>. However, limiting the amount of data used for the analysis reduces the robustness and accuracy of the model estimation<sup>6</sup>. Since over ten years of data are publicly available for the US LECs, using the entire available dataset seems appropriate.

Given these considerations, we have concluded that it is inappropriate to use OLS as our preferred regression analysis technique in this study. This conclusion is consistent with previous studies published by Ofcom<sup>7</sup> and ComReg in Ireland<sup>8</sup>, and is also consistent with our experience in other countries.

## 2.1.7 Panel data

Panel datasets incorporate observations for a number of companies (the cross-sectional dimension) over a number of time periods (the time series dimension). Panel data models are able to utilise a considerably larger amount of data than simple cross-sectional models, leading to more robust model estimates. Moreover, using data over a number of years for the same companies reduces the impact of irregularities in the data as these will tend to average out over a period of time.

The simple OLS techniques described above are not generally appropriate to deal with these models. This is due to two main reasons:

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<sup>5</sup> Technically ‘Pooled OLS’ can be used to estimate models with multiple time periods. This approach however is undesirable as it essentially pools all the time series elements into one period.

<sup>6</sup> Generalised least squares (GLS) is a multi-period version of OLS, which takes into account the fact that multiple observations from a single operator are likely to not be independent – that is, it corrects for the problem of autocorrelation. Further information on this is contained in section 2.1.7.

<sup>7</sup> NERA (2005), NERA (2004) et al

<sup>8</sup> Europe Economics (2004): ‘Operating Costs for the Access Network in Ireland: an Econometric Approach’ from [http://www.comreg.ie/\\_fileupload/publications/ComReg0421a.pdf](http://www.comreg.ie/_fileupload/publications/ComReg0421a.pdf)

- Panel datasets exhibit heterogeneity<sup>9</sup> both between observations ('between-variation') and within the same observation over time ('within-variation'); and
- Observations over time for the same cross-sectional unit will generally be correlated, which will cause a false relationship to be shown in estimation.

OLS generally only gives unbiased estimates<sup>10</sup> when the assumption that the regression residual is statistically independent from the explanatory variables holds true; however this assumption is likely to be violated when using a panel dataset.

There are however other regression techniques, such as Generalised Least Squares (GLS) and Maximum Likelihood Estimation (MLE) which are able to deal with the two issues above. In our case, the multivariate panel model takes the following general form:

$$C_{it} = a + \sum_{k=1}^K \beta^k Y_{it}^k + \sum_{l=1}^L \alpha^l N_{it}^l + \delta t + \varepsilon_{it} \quad (\text{Equation 4})$$

where:

$i$  indicates the cross-sectional unit,

$t$  indicates the time period of the observation; and

$t$  is also included as a separate explanatory variable, which will capture the effect of technology changes or other factors that might cause costs to change over time even if outputs remain unchanged<sup>11</sup>.

Despite using a larger amount of data, GLS or MLE estimation of Equation 4 still presents some of the drawbacks highlighted for simple OLS models. In particular, in the context of this study, the main disadvantage of these models is that the entire error term is implicitly attributed to the relative inefficiency of a company, while in reality part of the error term will be due to random variation caused by either errors in the data or omission of explanatory variables from the regression.

The following section presents an econometric technique that is able to overcome this issue and produce more accurate measurements of relative inefficiency.

## 2.1.8 Stochastic Frontier Analysis

Stochastic Frontier Analysis (SFA) is a regression technique that is widely used in efficiency studies conducted by regulators and is extensively covered in academic literature<sup>12</sup>. Regulators

<sup>9</sup> That is, the observations within the dataset are materially different from each other, generally to a greater extent than using a single-year cross-sectional dataset

<sup>10</sup> An estimate is unbiased if the expected value of the residual or error term (the gap between the predicted value and the actual value) is zero. If a model consistently predicts values that are higher than the true values, it is said to be biased.

<sup>11</sup> As this time trend is not firm specific the other factors captured will only be cost movements common across all firms.

<sup>12</sup> A good academic discussion of SFA for cost efficiency estimation can be found in 'Cost Efficiency in Network Industries: Application of Stochastic Frontier Analysis' (M Kuenzle, 2005) available at

that have used this technique include Ofcom and Oftel for BT, Comreg and ODTR for Eircom, OPTA for KPN<sup>13</sup>, the Communications Commission for Telecom New Zealand and the ACCC for Telstra.

The estimation process used in SFA involves a MLE approach, but differs to the procedure used to estimate the specification in Equation 4. The SFA estimation procedure overcomes two of the main drawbacks of the methods described above:

- Distinguishing between two components of the error term: an inefficiency component and a random noise component. This distinction is based on assumptions about the statistical distribution properties of the two error components. The accuracy of the inefficiency calculation is therefore improved.
- Estimating a line of best fit which describes a theoretically efficient frontier; that is, the theoretical maximum efficiency reachable by the companies in the sample. The residuals are therefore not calculated against a firm of average efficiency, but instead represent the distance of a firm from its theoretical most efficient status.

### 2.1.8.1 Single-year and panel data SFA

SFA models can be estimated using only one year of data (cross-sectional SFA) or multi-year dataset (panel data SFA). Panel data SFA, however, presents some major advantages over single year models:

- Panel data models utilise more information and generally produce more robust results; and
- The assumptions required on the statistical properties of the error components are less strict if a multi-year model is used.

The general form of the regression equation, when panel data is used, can be represented as follows:

$$\ln C_{it} = a + \sum_{k=1}^K \beta^k \ln Y_{it}^k + \sum_{l=1}^L \alpha^l \ln N_{it}^l + \delta t + u_{it} + v_{it} \quad (\text{Equation 5})$$

where:

- ln is the natural logarithm;
- $i$  represents the individual company observation;
- $t$  represents the year of the observation;
- $u_{it}$  is the inefficiency component; and

---

<http://e-collection.ethbib.ethz.ch/ecol-pool/diss/fulltext/eth16117.pdf>. Some recent interesting uses of SFA in regulation are 'Estimating the Distribution of Plant-Level Manufacturing Energy Efficiency with Stochastic Frontier Regression' (G Boyd, 2007) and 'Institutions and Bank Performance: A Stochastic Frontier Analysis' (R Lensink & A Meetsers, 2007). Another interesting case, where comparators are particularly unequal and where costs are uncertain, is 'A Stochastic Frontier Analysis of English and Welsh Universities' (P Stevens in Education Economics, vol 13, 2005)

<sup>13</sup> NERA Economic Consulting (2006): 'The Comparative Efficiency of KPN'

$v_{it}$  is the random error component.

It is important to note that the inefficiency component will always be positive, since it is measured against a theoretical maximum efficiency.

In SFA models, whether single or multi-year, the inefficiency component of the error term is assumed to follow a strictly nonnegative distribution, while the random error component is assumed to follow a symmetric distribution. This distinction is the base for the decomposition of the error term in the two components. In practice, the technique tries to fit a nonnegative distribution on the errors and identifies as inefficiency the portions of these errors that can be explained by the distribution.

As mentioned above, in the single-year specification it is necessary to make precise assumptions on the type of distribution of the inefficiency component (such as half-normal or truncated-normal distributions). However, there is generally insufficient information to establish precisely what distribution best fits the data and the validity of the assumption cannot typically be tested. In panel data models, on the other hand, such precise assumptions are not required as the technique is able to better identify the inefficiency component by observing each company repeatedly over time.

### 2.1.8.2 Inefficiency varying with time

There are two variants of the panel data SFA models, which differ in the parameterisation of the inefficiency component  $u_{it}$ :

- In the time-invariant model, the inefficiency term  $u_{it}$  is assumed to be constant over time, so that  $u_{it} = u_i$ ; and
- In the time-variant model, the inefficiency term is allowed to change over time. The parameterisation of the inefficiency term in this case takes the following form:

$$u_{it} = u_i \times e^{-\eta(t-T)} \quad (\text{Equation 6})$$

where  $\eta$  is the decay parameter and  $T$  is the last period in the panel.

We are able to test within our econometric analysis whether the decay parameter is significantly different from zero, and therefore whether efficiencies change over time.

### 2.1.8.3 Robustness of models

One further advantage of panel data SFA is that it tends to produce relatively robust models. Experience of using SFA models in estimating the efficiency of fixed-line telecoms networks shows that, in general, a consistent estimate of inefficiency is found even when the specification is varied (when the variables used are changed, or when the assumption over time variant efficiencies is



changed). Certainly, this has previously been the case when Ofcom has estimated the efficiency of BT, as can be seen from the sensitivities set out in previous reports submitted to Ofcom<sup>14</sup>.

### 2.1.9 Form of model and interpretation of results

Prior to deciding on the most appropriate statistical technique to be used from those described above, it is important to consider the functional form of the model to be estimated. In particular, we do not wish to constrain ourselves to considering only a linear relationship between costs and outputs. For the purpose of this analysis we have used a Cobb-Douglas (log-log) specification for the cost function. While this functional form does assume a certain type of relationship between cost and the explanatory variables, it has several advantages:

- It allows for non-constant returns to scale;
- It is linear in the explanatory variables, allowing for simpler econometric techniques;
- The coefficients on the explanatory variable can be interpreted as elasticities, so that they indicate the proportional change in costs derived from a 1% change in the explanatory variable, holding everything else constant;
- It reduces the impact of heteroskedasticity<sup>15</sup>; and
- It is consistent with previous studies into the comparative efficiency of BT.

The general form of the cost function estimated in this study is therefore:

$$\begin{aligned}
 \ln(\text{cost}) = & a \\
 & + \beta^k \times \ln(\text{switched lines}) \\
 & + \dots \\
 & + \alpha^l \times \ln(\text{population density}) \\
 & + \dots \\
 & + u
 \end{aligned}
 \tag{Equation 7}$$

In this equation,  $u$  is the stochastic error term, the difference between the line of best fit and the actual observation. This term will be different for each operator and is the variable used to measure comparative efficiency.

### 2.1.10 Comparators

We have chosen the US Local Exchange Carriers (LECs) as comparators for the purposes of estimating BT's efficiency. This choice is motivated by three reasons:

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<sup>14</sup> Including NERA (2005)

<sup>15</sup> Heteroskedasticity occurs when observations from random variables (such as the residual in our models) have different variances, hence non-identical distributions. While this does not affect whether the estimators are biased, it may instead affect the apparent significance of variables, making some variables appear significant when they are not and vice versa.

- US LECs are generally considered to be reasonably efficient operators, due to the competitive environment in which they have been operating for a large number of years;
- The Federal Communications Commission (FCC) requires the US LECs to annually publish a large set of data related to their costs, outputs and quality of service. Such detailed data is not available for any other comparable operators; and
- Previous studies of BT's efficiency carried out by Ofcom have used US LECs as comparators, thus using this dataset in our analysis ensures consistency.

Although the US LECs are generally regarded as comparable companies, it is important to note that BT, and more generally the BT Group, have considerably larger scale of operations than the average LEC. Therefore, consistent with previous studies carried out by Ofcom, we have excluded BT from the sample on which the econometric estimation is based, since its large scale could have biased the estimated relationship between costs and outputs towards itself.

One key consideration for this study is the extent to which BT can be compared with the LECs. Certainly, the scope of operations of the LECs is generally far greater than that of BT, with each of the LECs providing retail and data services in addition to the communications network. Given the lack of publicly-available data, and indeed the lack of operators structured in the same way as BT, it would not be possible to run such a comparison only against identical operators. Therefore, we have adjusted our dataset in order to make the US LECs and BT comparable. A further discussion of this is contained in section 3.1.

## 2.2 Techniques to estimate the annual productivity gain

The techniques set out above allow for the calculation of the comparative efficiency of a company at a particular point in time. However, it is expected that the efficient frontier is moving outwards over time as technological advances make it possible to produce more outputs from the same level of inputs. This is particularly true in the telecommunications sector where constant innovation has led to decreasing asset prices, particularly for switches and transmission equipment. Finally, the overall reduction in the cost based may be lessened to some extent by increasing labour prices.

In previous regulatory exercises, Ofcom has used the time trend as predicted by SFA (as set out in section 2.1.4) to define how productivity varies over time. However, we believe that this method is less robust than the alternative methods we describe below. We recognise that the time trend predicted by SFA is not only capturing changes in productivity, but also changes in any other factors affecting costs (since cost is the dependent variable in the regression analysis). Therefore, while regulatory changes or changes in accounting standards should not be considered to be a true technology shift, they will be included when looking at the overall time trend in SFA.

Furthermore, the costs used in the SFA analysis are in nominal terms, which is not a true reflection of the value of output. We would therefore be introducing inflation fluctuations into the general trend.

Rather than considering it as a shift in costs, the movement in the efficiency frontier may be interpreted as the change in productivity. There are a number of statistical and econometric techniques that may be used to estimate the productivity gain, calculating productivity shifts both

within and across companies over time. However, in estimating this gain in practice two effects may bias estimation and need to be excluded:

- Economies of scale: Without economies of scale exclusions we may attribute large output increases as a result of scale realization, to productivity advances.
- Capacity utilisation: If factor inputs are not fully or efficiently used changes in outputs may not be reflected by efficient changes to inputs. This lack of efficiency may create both positive and negative bias in TFP estimation; a positive bias could occur where a firm is under utilising capacity before increasing output by reaching efficient utilisation, a negative bias could occur from falling outputs in the face of sticky inputs<sup>16</sup>.

Our methodologies described below are designed to eliminate this potential biases.

We have used two approaches to estimate productivity gain outlined below. These approaches have differing robustness contingent on data availability and how they can deal with the two effects identified above.

## 2.2.1 Indexation

Total Factor Productivity (TFP) refers to the additional output that can be produced from a given set of inputs, where all other variables are kept constant.

A TFP index is generated by subtracting a firm's output growth from its input growth. This then indicates by how much output is growing through increases in productivity as opposed to simple changes in the scale of inputs. Given the divergent range of a firm's outputs (for example access lines, local minutes, long-distance minutes) and inputs (for example labour, materials and physical capital) it is necessary to combine all output and input into a single output and input index, which can then be used in the TFP calculation.

This approach is similar to that used by the FCC to measure annual productivity changes for the LECs and to inform the rate of X within their network price controls<sup>17</sup>.

### 2.2.1.1 Output index

An output indices is constructed based on actual physical output of each type of output (such as call minutes), or by using deflated revenue (revenue adjusted for inflation). A single index is then estimated from the different categories of output based on the weights of each category's contribution to total revenue.

<sup>16</sup> Capacity utilisation has been discussed at length in the Real Business cycle literature. For example see Jeremy Greenwood, Zvi Hercowitz, Gregory W Huffman: 'Investment, Capacity Utilization, and the Real Business Cycle', The American Economic Review, Vol. 78, No. 3 (Jun., 1988), pp. 402-417

<sup>17</sup> It should be noted that price controls are typically set on a different basis in the USA to the UK and Europe. The price cap is of the form RPI-X where X is set to be equal to the historic total factor productivity plus a stretch factor. The stretch factor may incorporate exogenous factors and, for example, anticipated volume changes.

The output quantities are based on actual historic quantities from the regulatory and statutory accounts of the benchmark companies. Since TFP seeks to estimate the change in productivity over time within companies it is not necessary for the output categories to be consistent across companies; however, it remains pragmatic to define the total output measure on a consistent basis. The total output measure defined for this study includes the following:

- Access lines: Total number of PSTN and ISDN channels;
- Local and internet minutes: Total local and internet minutes including fixed to mobile minutes and calls to ISPs;
- Long-distance minutes; and
- Leased circuits: The number of leased circuits in 64kbps equivalents.

All volumes not associated with fixed line services, for example those mobile services supplied by an integrated fixed and mobile operator, are excluded from the measure of output.

### 2.2.1.2 Choice of indices

There are a number of appropriate models for creating a single index.

Given the changing balance of outputs in our dataset, we believe that the best theoretical index is the Törnqvist index. To construct this index we firstly take each of the operators' outputs separately and weight them by their corresponding output in 2006, our assumed base year. For a representative output  $k$  we thus calculate for firm  $i$ :

$$\bar{y}_{it}^k = \frac{y_{it}^k}{y_{i0}^k} \quad (\text{Equation 8})$$

The output indices are then weighted, based on the average revenue share of each service in relation to total revenue in year  $t$  ( $w_t^k$ ) and in 2006 ( $w_0^k$ ) respectively to form a single weighted output index:

$$\bar{y}_{it}^{\text{Törnqvist}} = \prod_{k=1}^K (\bar{y}_{it}^k)^{0.5(w_{i0}^k + w_{it}^k)} \quad (\text{Equation 9})$$

This index is preferable over alternative indices such as the Paasche and Laspeyres indices given *a priori* it weights outputs based on an average of a base year and current year revenues. This lends stability to the model (and ensures that this stability is based around the most recent year in our dataset), but recognises that the importance of various outputs will vary over time by attaching equal importance to the outputs in both periods.

The Fisher relative quantity index is defined as the geometric mean of the Paasche and Laspeyres indices<sup>18</sup> and used in general economic statistics. Similarly to the Törnqvist index, the Fisher

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<sup>18</sup> These are defined in Appendix B.2.

relative quantity index is often referred to as a symmetric index, providing the model with stability by attaching equal importance to the two situations being compared.

$$\bar{y}_{it}^{Fisher} = (\bar{y}_{it}^{Laspeyres} \bar{y}_{it}^{Paasche})^{\frac{1}{2}} \quad (\text{Equation 10})$$

In this report we use the Törnqvist index given that this index is commonly used to measure volumes changes for purposes of productivity measurement<sup>19</sup>. While there is theoretically little difference between the Fisher and Törnqvist indices, the latter has been more widely used in the estimation of productivity. Certainly, both of these indices will give a more robust measure of productivity than the Paasche and Laspeyres indices in industries with fast-moving variations in inputs or outputs. Further discussion on indices can be found in Appendix B.2.

We also looked at chained indices, where the index for one period depends on indices for future periods directly. However, given the large changes in volumes these quickly diverge from sensible levels, and are therefore not appropriate for use in this case.

### 2.2.1.3 Input index

Factor inputs are classified into three broad categories: labour, materials, and capital service. Indices reflecting changes in the quantity of labour, material and capital service are constructed and weighted based on their income to form a single input index. The types of input indexes which are calculated are consistent with those used to calculate the output index and the total input growth rate for each company was thus calculated using the Törnqvist index.

The labour input index measures the amount of labour that is used to produce the output. Since there is no way to account for the contribution of different categories of employees, for example part-time versus full-time workers or skilled versus non-skilled, the total number of employees is used as the factor input measure for labour. The cost of labour is deflated using the general economy wide inflation rate.

$$\bar{L}_{it} = \left( \frac{c_{it}^L}{c_{i0}^L} \right) \quad (\text{Equation 11})$$

The material input index measures the quantity of materials (often referred to as “non-pay operating expenditure”) that is used to produce the output. Material expense is derived as the residual of total operating costs in this analysis, that is, the total operating expenses less the sum of labour compensation and depreciation expenses. The material index is deflated using either the general economy-wide inflation rate or the asset price index, calculated below. This is in recognition of the linkage between the level of assets in a company and expenditure on material items.

$$\bar{M}_{it} = \left( \frac{c_{it}^M}{c_{i0}^M} \right) \quad (\text{Equation 12})$$

<sup>19</sup> See UN statistics division <http://unstats.un.org/unsd/sna1993/tocLev8.asp?L1=16&L2=3>

The capital input index measures the quantity of capital that is used by the company to produce the output. Capital service represents the contribution capital makes to the production of output. The Perpetual Inventory Method is used to estimate the capital stock employed, with the opening capital balance to be used in the perpetual inventory method is referred to as the benchmark capital stock. All categories of fixed assets were considered in this analysis with data on GBV and NBV obtained directly from the companies' accounts. The stock of capital employed during a year is estimated by adding the deflated capital additions less disposals to the net asset balance at the start of the year (net of the current year's depreciation) as can be seen in Equation 13 below.

$$\begin{aligned} \text{Capital Stock}_{it} = & \text{NBV}_{\text{Opening}_{it}} \\ & + \left( \Delta \text{NBV}_{\text{Opening}_{it} - \text{Closing}_{it}} \right) \times \frac{\text{Asset price index}_{i0}}{\text{Asset price index}_{it}} \\ & - \frac{1}{n} \sum_{n=1}^n \text{Depreciation}_{it} \end{aligned} \quad (\text{Equation 13})$$

A critical data requirement for the computation of the perpetual inventory method is investments in constant prices. In order to translate the reported capital expenditure into constant prices, we used an asset price index. However, the rate at which prices change differ by types of asset, and so companies with a larger cable share of assets would experience a different asset price index to companies with less cable. We therefore weighted the individual asset price indices (for each type of asset) by the amount of each asset type in the asset base for each LEC, to calculate an overall individual asset price trend. An average of the company specific asset price indices was applied to non US companies.

Having calculated the capital stock using the perpetual inventory method for each year, we then go on to calculate the capital stock quantity index.

$$\bar{K}_{it} = \left( \frac{c_{it}^K}{c_{i0}^K} \right) \quad (\text{Equation 14})$$

Finally, having constructed the input quantity indices for each factor of production, a composite input index is then computed by weighting each factor of production using relative shares of payments.

To compute the relative shares of payments,  $w_i$ , payments to each factor are used:

- Payment to labour is total staff costs;
- Payment to material input are material and residual operating costs; and
- Payments to capital are depreciation.

A Törnqvist Input quantity index is then calculated as follows:

$$\bar{C}_{it}^{\text{Törnqvist}} = (\bar{L}_{it})^{0.5(w_{i0}^L + w_{it}^L)} (\bar{M}_{it})^{0.5(w_{i0}^M + w_{it}^M)} (\bar{K}_{it})^{0.5(w_{i0}^K + w_{it}^K)} \quad (\text{Equation 15})$$

## 2.2.2 TFP using a standard calculation

Once the output and input indices have been calculated for each company for each year, the annual growth rates of the respective indices are calculated. The TFP growth rate is obtained by deducting the growth of the input index from the growth rate of the output index. This provides the annual change in productivity, including volume effects.

The productivity index is likely to be relatively unstable over time within a company since it is volatile to a number of factors including:

- Economy of scale effects<sup>20</sup>;
- Capacity utilisation; and
- Financial reporting conventions including the labour capitalisation and write-off policies which may weaken the immediate link between movements in the output index and the input index.

To remedy the first and second deficiency we have deployed an econometric remedy discussed in more detail below.

In addition to this remedial measure we have averaged TFP estimates across time. However, as our indices are calculated over a maximum of 8 years since, 2 previous years worth of data are required to calculate the first year index, we further take an average of each company's average index. This provides a single annual average productivity across time and companies which will alleviate volatility created from the three factors identified above.

### 2.2.2.1 Total factor productivity adjustments for economies of scale and capacity utilisation

In order to reduce the effect on our TFP measure of capacity utilisation bias we model the relationship between changes in the output variables and the TFP estimate for each firm in each period:

$$\Delta TFP_{it} = A + \sum_{k=1}^K \beta^k \Delta \bar{y}_{it}^k + v_{it} \quad (\text{Equation 16})$$

where  $\Delta x_{it} = x_{it} - x_{it-1}$

The purpose of this regression is to examine whether our calculated TFP estimate is affected by the rate of growth of a company. For example, if outputs were falling over time, we might expect the resulting under-utilised network to show a lower productivity than would otherwise be the case. Using this approach all capital utilisation effect caused by changes in volume are netted out, leaving an adjusted TFP measure estimated as the intercept A.

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<sup>20</sup> By calculating TFP in this way returns to scale are implicitly assumed to be constant.

This approach can be extended by including further the output indices in time  $t$  to net out economies of scale effects.

$$\Delta TFP_{it} = A + \sum_{k=1}^K \beta^k \Delta \bar{y}_{it}^k + \sum_{k=1}^K \lambda^k \bar{y}_{it}^k + v_{it} \quad (\text{Equation 17})$$

### 2.2.3 TFP using econometric methods

Our second approach to estimate the total factor productivity uses a fixed effects growth model based on a Cobb Douglas production function.

The Cobb Douglas production function is specified in the standard way for firm  $i$  in period  $t$ . However, we measure out outputs using an adjusted Törnqvist index, as discussed in section 2.2.1.2. The Törnqvist index is adjusted to allow firms' relative size to be accounted for at the same time as price effects.

$$\bar{y}_{it}^{\text{Adjusted Törnqvist}} = \prod_{k=1}^K \left( \frac{y_{it}^k}{y_{jt}^k} \right)^{0.5(w_{i0}^k + w_{jt}^k)} \quad (\text{Equation 18})$$

where  $j$  represents one firm benchmarked off for all other firms.

Taking our adjusted Törnqvist index our Cobb Douglas production function becomes:

$$\bar{y}_{it}^{\text{Adjusted Törnqvist}} = A_{it} K_{it}^{\alpha} L_{it}^{\beta} M_{it}^{\lambda} \quad (\text{Equation 19})$$

where:

- $K$  is the stock of capital measured in real terms;
- $L$  is labour input measured in real terms;
- $M$  is materials measured in real terms; and
- $A$  is total factor productivity.

We then specify  $A$  to capture both firm specific shocks and movements in productivity  $v_{it}$ , a fixed firm element  $w_i$  and an industry wide shift in productivity  $g$ .

$$A_{it} = e^{v_{it} + w_i + gt} \quad (\text{Equation 20})$$

Taking logs and appending a constant yields the following linear model to estimate:

$$\ln \bar{y}_{it}^{\text{Adjusted Törnqvist}} = a + w_i + \alpha \ln K_{it} + \beta \ln L_{it} + \lambda \ln M_{it} + gt + v_{it} \quad (\text{Equation 21})$$

This specification of technology has been widely used at the country level in neoclassical econometric growth literature<sup>21</sup>. This literature has its roots in Solow's seminal theoretical papers<sup>22</sup>.

<sup>21</sup> This specification is specifically set out in N Islam (1995): 'Growth Empirics: A Panel Data Approach', The Quarterly Journal of Economics, Vol. 110, No. 4 (Nov 1995), pp. 1127-1170. However, it is also alluded to



This specification of technology has been widely used at the country level in neoclassical growth literature.

In order to estimate the coefficients in this equation, a suitable modelling procedure is a 'fixed effects approach' as this procedure allows correlation between the fixed effect and other input variables<sup>23</sup>. The parameter of interest in Equation 21 is  $g$ , the growth rate of TFP. It should be noted that, by specifying output in this way and inputs in real terms, price effects have been excluded.

This specification measures the TFP of the average company provided that two main conditions are met:

- If firms exhibit idiosyncratic productivity shifts to and away from the frontier that are correlated to the factor inputs, estimates of industry-wide TFP growth will be inconsistent.
- Although this specification allows for economies of scale by putting no constraints on the coefficients of the factor inputs, no account is made for capacity utilisation. If there is large-scale underutilisation, then the comparison of outputs and inputs may not reflect a true measure of productivity change.

The first condition is theoretically valid although we have no reason to believe on *a priori* grounds that such a correlation is likely to be insignificant. However, while estimating this specification we will be able to assess whether correlation is a problem.

The second condition is only pertinent if under utilisation is common across all the firms in the industry at the same point in time. This is because any firm's capacity utilisation effects will be subsumed into the error term.

To estimate TFP growth using this approach we have used data from the US LECs. Details on collection of this data are discussed in section 3.2.2.

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in the earlier work in N G Mankiw, D Romer, D N Weil (1992): 'A Contribution to the Empirics of Economic Growth', The Quarterly Journal of Economics, Vol. 107, No. 2 (May 1992), pp. 407-437

<sup>22</sup> R M Solow (1956): 'A Contribution to the Theory of Economic Growth', The Quarterly Journal of Economics, Vol. 70, No. 1 (Feb 1956), pp. 65-94

<sup>23</sup> This correlation is testable, and if no correlation is found then a random effects estimation is appropriate.

## 3 Data Collection

In any study of this type, it is important to use consistent data in order to get robust results. This study uses the US LEC dataset, since it is the most comprehensive dataset available to us outside of BT. Since this dataset has been used in previous studies, it allows us to ensure that our approach is consistent with that previously used by Ofcom. We have also used less disaggregated data for European operators.

In this section we:

- Discuss the US LEC and BT data used in the SFA analysis. This will include a discussion on methods deployed to facilitate comparability between BT and the LECs; and
- Outline data used in both our approaches for measuring TFP growth.

### 3.1 Data used in Stochastic Frontier Analysis

To calculate the initial efficiency estimate, we are focussing on the comparative efficiency of BT where we have defined BT as comprising of both BT Openreach and the network business. Our first consideration thus when looking at which data to collect is that of the comparability of operators themselves.

We are using the US LECs as comparator operators. However, the way in which these firms operate is not directly comparable to BT. Each of the US LECs provides a full telephony service, providing both retail and network services, conveying calls and operating leased lines. In this study, however, we wish to only consider BT's network operations.

In order to make our study directly relevant to BT's network business, therefore, we must make allowances for the wider scope of the LECs' operations. As much as possible we have tried to adjust the LEC data to flatten these differences. However, due to a lack of disaggregated data in some cases it has not been possible to simply adjust the LECs to match BT's network business, necessitating several adjustments to BT's data. The following provides an exhaustive account of firstly, LEC data collection and adjustments made, and secondly, BT data and adjustments made.

#### 3.1.1 US LEC data

Data for the US LECs was collected from the FCC's Automated Reporting Management Information System (ARMIS) database<sup>24</sup>. Data was available for all large and medium operating companies, at state level, for a period of eleven years (from 1996 to 2006 inclusive). All of our analysis described here is carried out over this period.

The data collected covers:

- Operating expenses;

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<sup>24</sup> Available at <http://www.fcc.gov/wcb/armis/>

- Depreciation;
- Asset values, on an historical cost accounting (HCA) basis, and accumulated depreciation;
- Outputs, including lines and minutes; and
- Network data, notably sheath and duct length.

Other data were collected for the US from various other sources, including:

- Population density;
- Average staff wages, both overall, and specifically for telecommunications workers; and
- Climate, including absolute temperatures and temperature variations.

While most variables can be extracted directly from these sources, we have had to calculate or estimate some variables to complete our dataset. These calculations are set out below.

#### **3.1.1.1 Excluded companies**

We excluded a number of companies from our analysis. Firstly we excluded companies which would not provide good comparators for BT due to the environment in which they operate. We excluded Verizon Washington DC, as it operates only within a city's boundaries, and we also excluded operators in Hawaii and Puerto Rico since the island-based operations require a significantly different network structure.

Secondly, we excluded AT&T Southern New England Telephone (previously known as SBC/SNET Connecticut) due to anomalies in that operator's output data. The operator appears to have large jumps in the number of special access lines provided, as well as unreasonable fluctuations in network data, over the period studied. Since we have been unable to verify which are the correct data, we have excluded the company.

#### **3.1.1.2 Cost data**

BT's network business is regulated in the basis of current cost accounting (CCA) costs. However, the ARMIS database produced by the FCC provides data on asset book values for the US LECs, reported on an historic cost accounting (HCA) basis. Under HCA, the reported Gross Book Value (GBV) for similar assets bought at different points in time can differ considerably, depending on the prices of the asset at the time it was bought. Under CCA, on the other hand, the reported value of an asset depends on the current replacement price of the asset, irrespective of the price at which it was bought.

Since we are looking to apply our analysis to a CCA environment, it is therefore necessary to adjust asset and depreciation figures for the US LECs to a CCA basis. This adjustment consists of the following steps:

Calculation of asset age: for each of the asset categories above and for each LEC separately, we have computed the average age of the asset according to the following formula:

$$\text{Average age of asset} = \text{Useful asset life} \times \left(1 - \frac{\text{NBV}}{\text{GBV}}\right) \quad (\text{Equation 22})$$

where NBV is the Net Book Value of the asset, defined as GBV minus accumulated depreciation.

Since the FCC issues guidelines on the range of asset life values that should be used by the LECs in their accounts, in our final calculation we used the midpoint of the range indicated by the FCC as our preferred asset life value.

Due to limited availability of data, the calculation of average asset age was conducted at the parent company level and the calculated average age of each asset for each parent has been subsequently applied to all the LECs belonging to that parent company.

Collection of data on asset price indices for each asset category: the US Statistical abstract provides data on price trends in the US for a number of asset categories. These data provide information on the proportional price change between any two points in time for a particular asset.

Calculation of Gross Replacement Cost (GRC): the final step in the CCA adjustment of US LEC asset value figures was to adjust the GBV figures for the price changes occurred for each asset between the year in which the asset has been bought (given by the average asset age figure calculated in the first step) and the current year. Effectively, this adjustment converts GBV to GRC. For example, the GRC figure for aerial cable for company  $i$  in 2006 was calculated as:

$$GRC_{\text{Aerial},i,2006} = GBV_{\text{Aerial},i,2006} \times \left( \frac{\text{Price}_{\text{Aerial},2006}}{\text{Price}_{\text{Aerial},2006 - \text{average age}}} \right) \quad (\text{Equation 23})$$

A figure for Net Replacement Cost (NRC) was obtained as:

$$NRC = GRC \times \left( \frac{\text{NBV}}{\text{GBV}} \right) \quad (\text{Equation 24})$$

We then calculated consistent CCA figures for depreciation:

- Using the LECs data on depreciation accruals and GBV, we have calculated the depreciation rate for each asset as the ratio between depreciation accruals and GBV; and
- Having calculated the GRC as described above, we have obtained CCA adjusted depreciation figures by multiplying the GRC figure by the depreciation rate.

This approach is consistent with previous efficiency studies that have used the US LECs' data as a benchmark, particularly those previously commissioned by Ofcom and Oftel<sup>25</sup>.

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<sup>25</sup> Including NERA (2005)

The depreciation figures reported by the US LECs are sometimes distorted by accounting practices. For example, we have noticed that in some instances the reported accumulated depreciation figures are higher than the GBV of the asset, leading to a negative NBV. In other cases, the reported depreciation figures were negative. Since it is not possible to know what accounting adjustments have led to these figures, we have adjusted the data by converting all negative figures to zero.

The total depreciation cost of a LEC was calculated as the sum of the CCA adjusted depreciation figures over the relevant asset categories. The relevant asset categories were those directly related to the network part of the LEC operations, as BT does not deal with the retail sector (this is explained in more detail in section 3.1.1.3 below).

Similarly, the total asset value for each LEC was calculated as the sum of the NRC figures over the same asset categories. In order to calculate the cost of capital, we have multiplied the sum of NRCs over all asset categories by the weighted average cost of capital (WACC). To facilitate comparability to BT we have taken a weighted average of BT's WACC, using the WACC of BT Wholesale (11.4%) and Openreach (10%). The weight attributed to each was calculated using the percentage split of BT's NRC between Openreach and Wholesale. Hence, the WACC was calculated as follows:

$$\begin{aligned} WACC_{lec} &= (\%NRC_{OR} \times WACC_{OR}) + (\%NRC_{WS} \times WACC_{WS}) \\ &= (69\% \times 10\%) + (31\% \times 11.4\%) \\ &= 10.44\% \end{aligned} \quad \text{(Equation 25)}$$

While this revaluation method is reasonably accurate for most asset types, it does not take into account the significant repair and maintenance costs for duct. The GBV of duct includes costs incurred in repairs, while the true GRC would be simply the cost of laying a cable for the first time. Adjusting the GBV in the above manner would therefore lead to an overestimation of the GRC for duct assets, and make it incomparable to the true GRC used for BT. We have therefore reduced the estimated GRC for the LECs by 10%, following the estimations previously agreed between Ofcom and BT<sup>26</sup>.

### 3.1.1.3 Network costs

We only considered the network part of the LECs, as we are calculating the comparative efficiency of BT excluding retail operations. However, some asset categories such as land and buildings, general purpose computers and furniture and fittings, are used both in the retail and network side of the business. The US LEC data does not provide this split. We have therefore made an assumption, based on assumptions used in previous studies and our experience of cost models for fixed-line operators worldwide, to allocate 40% of these cost to the network side of the business and 60% to the retail side. The values for these assets is, in any case, generally not large compared to the sum of all other network-side assets so that small variations to this assumption would therefore be expected to materially affect our analysis.

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<sup>26</sup> See NERA (2005)

### 3.1.1.4 Other cost adjustments

Three final adjustments were made to the cost figures:

- The US LECs report 'work-in-progress' as a single figure, not split by asset categories, under the name of "Telecommunications Plant Under Construction (TPUC)". Since no information is provided regarding which asset categories are included in this figure, we have apportioned it across all the asset categories, based on their proportion in total GBV. We add the resulting proportion of TPUC GBV directly to the NRC value of each asset, based on the consideration that GBV and GRC are equivalent for a new asset and that, being work-in-progress, the asset has not yet been depreciated, so that GRC and NRC are equal.
- We added the costs associated with leaseholds and leaseholds adjustments to the costs associated with buildings.
- We have excluded costs and outputs associated with public payphones. Payphones were excluded because of comparability concerns to BT's costs. As these items represent a minor fraction of cost and output this exclusion is insignificant.

### 3.1.1.5 Leased lines data

Data for the number of leased lines (64kbps equivalents) are available in the ARMIS database for the years 2002 onwards only. In order to use our full eleven-year dataset, we must make some assumptions over the number of leased lines in previous years. This calculation was performed in three different ways:

- Using the revenues obtained from leased line (data for which are available for all years), divided by the average price of a leased line;
- Using the reported number of leased lines for 2002 to 2006, and fitting a trend backwards to estimate the missing data; and
- Using a combination of these two methods, using real data for years back to 2002 and weighting a trend backwards using the estimates of leased lines calculated from revenues.

In previous studies carried out for BT and for Ofcom, the first of these methods has been used, primarily because of the lack of a long enough time series of actual data to allow for a sufficiently robust trend to be estimated. However, since the actual data now covers a five year period (from 2002 to 2006 inclusive), we believe that the second method now carries reasonable weight.

Furthermore, we have more confidence in the actual leased line data in later years. Inspection of the US leased line revenue data has revealed some abnormal variation in later years for some LECs, with reported revenues sometimes being implausibly low compared to previous years and to the reported number of leased lines reported in subsequent years. Since these abnormalities are possibly entirely due to accounting practices, calculating the number of leased lines based purely on these revenue figures could potentially result in highly inaccurate estimates.

Nevertheless, simply trending the data for 2002-2006 backwards places large assumptions on the growth of leased lines over time, particularly that growth (or decline) must be monotone. This may not be the case. Since we have some reasonable data on the relative growth of revenues in the early years of our dataset, we prefer the third methodology outlined above, as this uses all data available to us.

Using these three methods to calculate the number of leased lines should not, in theory, lead to very different conclusions being drawn, since:

- The difference between the number of leased lines calculated by the two methods is significant only for a very few LECs, and these LECs tend to be those with relatively few leased lines in any case;
- When using a combined lines variable in particular, the number of leased lines is much smaller than the number of switched plus special access lines;
- All of our regressions are performed in logarithms (see section 2.1.9), which will 'squeeze' the difference; that is, since the leased line volume estimates are all of the same order of magnitude, the logarithm values will be very similar; and
- The correlation between the leased lines variables calculated using the above methods is very high, indicating that the difference between the three methods is minimal.

**Table 1: Correlation between leased lines variables**

	Log of total lines (calculated using revenues)	Log of total lines (calculated using trended actuals)	Log of total lines (calculated using combination)
Log of total lines (revenues)	1		
Log of total lines (trended)	0.9738	1	
Log of total lines (combination)	0.9796	0.9956	1

Source: Deloitte analysis

The LECs provide data on two types of leased line products; "special access lines", and "private circuits". The first of these are analogous to partial private circuits (PPCs) operated by BT, which are typically used to connect a customer to a point-of-presence, therefore having only one 'end'. Private circuits have two 'ends'. In order to create a composite leased line variable, we have weighted special access lines by 1.5 and private local by 2. This weighting is based on the expected relative costs of each product: local private lines will have two customer ends and a main link whilst special access have a main link and one end. This is consistent with previous studies undertaken by Ofcom.

### 3.1.1.6 Total lines

Consistent with our working on leased lines set out above, we have calculated a total lines variable based on weighting each of the LEC reported lines to capture differences in the costs associated with each. Our total lines variable is calculated:

$$\text{total lines} = 1 \times \text{switched lines} + 1.5 \times \text{special access lines} + 2 \times \text{local private lines} \quad (\text{Equation 26})$$

This total lines variable is tested as a substitute to individual switched lines and leased lines variables.

### 3.1.1.7 Switch minutes data

The ARMIS database only provides the number of minutes for interLATA calls (incoming and outgoing together<sup>27</sup>). We have calculated the average length of a call as:

$$\text{average call length} = \frac{\text{interLATA minutes} \times 0.5}{\text{interLATA calls}} \quad (\text{Equation 27})$$

Since data on minutes for other types of calls is not available, we assumed that the average length of a call is the same for local intraLATA and interLATA calls. Although this assumption will tend to underestimate the total volume of call minutes, we do not expect it to influence the results of this study significantly since all LECs will be affected in the same way. The number of local and intraLATA minutes was then calculated by multiplying this average call length figure by the number of outgoing calls.

In order to ensure comparability between operators with different-sized operating areas, and to be consistent with previous studies, we have converted call minutes to switch minutes. In order to do this, we have used the following routing factors:

**Table 2: Routing factors**

	Local switch routing factor	Main switch routing factor
Local calls	1.54	0.02
IntraLATA calls	2.0	0.2
InterLATA calls	1.0	0.3

Source: Hatfield, Ofcom<sup>28</sup>

These routing factors have been used in previous efficiency studies conducted for Ofcom, and are derived from the Hatfield model, built for AT&T and MCI.

One remaining issue is that the number of main switch minutes and the number of local switch minutes are very highly correlated. As discussed in the next section, where explanatory variables are correlated then econometric models have some difficulty separating out the effect of each of them.

<sup>27</sup> An intraLATA call minute is counted twice, once as an outgoing call minute originating at customer A and once as an incoming call minute terminating on customer B. Therefore the number of call minutes must be divided by two to obtain the number of actual number of call minutes that occurred. The absolute number of calls is not affected in this way.

<sup>28</sup> See NERA (2005)



In previous studies, including those undertaken by Ofcom, a single 'switch minutes' variable has been constructed by giving main switch minutes a weighting of 0.5 and local switch minutes a weighting of 1, with the rationale that main switch minutes are estimated to cost half that of local switch minutes. We have adopted this methodology, and have also run a sensitivity examining the results if such a weighting were not applied (and therefore total switch minutes were simply the sum of local and main switch minutes).

### 3.1.1.8 Correlation in output data

One important issue to note regarding all of the output variables presented (switched lines, special access lines, leased lines and switch minutes) is that they are highly correlated with each other. This means that as one variable increases within the dataset, that similar, proportionate, changes exist in all other variables.

This issue of correlation can cause difficulties in estimation of our econometric models. In particular, econometrics may find it difficult to establish the effects of one variable from the effects of another, commonly referred to as collinearity or multicollinearity. This can make variables look spuriously insignificant or significant<sup>29</sup>, and can affect the coefficients in regressions; potentially leading to counterintuitive magnitudes and signs. Although this problem affects individual coefficients and associated standard errors making judgments over the responsiveness of costs to changes in outputs misleading, the overall model is still valid.

To correct as much as possible for this issue, there are a number of possible remedies:

- Create composite variables, which look at a single effect of output increasing in general. For example, we may consider a variable of "lines" instead of considering "switched lines" and "leased lines" variables separately. However, this does not allow each of the included variables to have a different impact on costs.
- Disregard one or more variables, and assume that their impact on costs is fully reflected in the impact of another variable. This is likely to reduce the goodness of fit of the model, and will make the interpretation of coefficients difficult.
- Create new variables by dividing one variable by another. For example, we consider "minutes per line" as opposed to "minutes". Again, this may make it more difficult to interpret the coefficients. Also, given our log-log specification, it may have a limited effect, since  $\log\left(\frac{A}{B}\right) = \log(A) - \log(B)$ . This therefore would simply rearrange the coefficients in our model.

Each of these possible remedies is tested when identifying the final specification.

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<sup>29</sup> In this report, all significance decisions are made at the 5% level.

### 3.1.2 BT data

Data has been collected from a variety of sources to produce a data set for BT. In this study we have looked to analyse BT comprising of both BT Openreach and BT's wholesale network business. Further, given that BT is excluded from the initial SFA regression estimation, and only inputted to calculate its comparative efficiency in the most recent period, we have collected data only for the year 2006.

In common with the LECs we have collected cost, output and network data whilst making several adjustments to facilitate comparability. In the following we provide an exhaustive breakdown of data sources and adjustments made.

#### 3.1.2.1 Cost data

The starting point for the cost data was the CCA Opex and depreciation data in Annex 5A of BT's 2007 Financial Statements<sup>30</sup>. The CCA operating costs reported, defined as the sum of OPEX and depreciation, are £8,774million. However, several items included required adjustment to facilitate comparability to the LEC data. Initial adjustments included:

- Exclusions of costs related to emergency "999" and public payphone calls: these have been removed because of systematic differences in how costs for these calls accrue;
- Removal of payments to other fixed line operators for international calls and mobile operator: costs related to these activities have been removed from BT data as the LECs do not report costs for these items; and
- Exclusion of data networks costs: again these costs have been removed as they are not included in the LEC costs<sup>31</sup>.

The above are standard adjustments which have been made in previous SFA studies. In total they reduce operating costs from £8,774 million to £5,874 million. However, we have identified two further alterations as a result of discussions with BT:

- Costs defined as 'Other Adjustments' in the CCA accounts have been excluded: these costs are exceptional and relate to changes in depreciation lives. Given that these are exceptional, we believe they fall outside of reported costs by the LEC, and in particular they should not be included when estimating the actual efficiency of BT. Taking these costs out reduced costs by £843 million.
- Costs related to business rates have been removed: these rates amount to £299 million and are excluded because of differences in their treatment between the US and UK. In the US these rates are treated as "below the line" tax and are hence not included in the LEC's operating cost.

<sup>30</sup> Available from <http://www.btplc.com/Thegroup/Regulatoryinformation/Financialstatements/2007/CurrentCostFinancialStatements.pdf>

<sup>31</sup> For a discussion see NERA (2005).

Taking account of these further adjustments comparable operating costs for BT are £4,732 million.

**Table 3: BT operating costs reconciliation**

Item	Cost (£m)
Total CCA operating costs	8,774
<i>Exclusions:</i>	
Business rates	(299)
Other adjustments	(843)
Other in BT Wholesale residual market	(2,893)
Other in other markets	(7)
<b>Total operating cost</b>	<b>4,732</b>

Source: BT, Deloitte analysis

In addition to operating costs we also collected data on BT's total cost of capital. To calculate the cost of capital BT provided us with a breakdown of their assets at NRC. Before adjustment for comparability BT's NRC stood at £17,518 million. This asset base however corresponded to BT Group thus necessitating the following adjustments:

- As BT Group includes BT's retail and international operations, all components/activity groups associated with these areas were excluded;
- In common with the operating costs the standard adjustments for LEC comparability outlined above were made, such as exclusion of costs for interconnection, payphones and so on.

These adjustments led to a reduction in the NRC of 1.8% or £311 million. To calculate the total cost of capital from this, BT assigned the remaining components to either being owned by BT Openreach or by BT Wholesale, since these have different regulated WACCs<sup>32</sup>. Allocating NRC in this way and multiplying by the appropriate WACC yields a total cost of capital of £1,796 million:

$$\begin{aligned}
 CoC_{BT} &= (NRC_{OR} \times WACC_{OR}) + (NRC_{WS} \times WACC_{WS}) \\
 &= (£11,798 \times 10\%) + (£5,409 \times 11.4\%) \\
 &= £1,796
 \end{aligned}
 \tag{Equation 28}$$

Having calculated both the operating and capital costs for BT we converted these into US dollars using a PPP (purchasing power parity) adjusted exchange rates. The use of a PPP-adjusted exchange rate ensures that differences in prices between the UK and the US are eliminated<sup>33</sup>.

<sup>32</sup> Ofcom (2005): 'Ofcom's approach to risk in the assessment of the cost of capital' from [http://www.ofcom.org.uk/consult/condocs/cost\\_capital2/statement/final.pdf](http://www.ofcom.org.uk/consult/condocs/cost_capital2/statement/final.pdf)

<sup>33</sup> We used the PPP rate from the IMF World Economic Outlook Database, October 2007.

### 3.1.2.2 Output and network data

Volume data for each of the products was provided within the financial statements, key performance indicators and supplemented by BT. Table 4 summarises the data source for each of the network and output variables.

**Table 4: Summary of BT output and network data and sources**

Volume	Source
Length of duct (km)	BT
Length of sheath (km)	BT
Local switch minutes	2007 Financial statements
Main switch minutes	2007 Financial statements
Number of switched lines	BT 2006/2007 Q4 KPI
Number of leased lines	BT
Population density	CIA World Factbook, UK government statistics
Temperature spread	UK government statistics

Outputs and network data were broadly deemed to be comparable to the LECs' data with the exception of leased lines which were converted into 64kbps-equivalent:

$$\text{leased lines} = \frac{\text{lines} \times \text{bandwidth}}{64k} \quad (\text{Equation 29})$$

In the LEC data we have weighted special access and private local lines to create a composite leased lines variable. For consistency we have weighted BT's data in the same way by giving a weighting of 1.5 to any line which has one local end (PPCs), and a weighting of 2 to any line with 2 ends.

One final issue is that the measure of leased lines in 64k equivalents is fast becoming outdated, as higher bandwidth lines become common. Ideally, our leased line variable would take into account the number of lines and bandwidth, since cost is not expected to vary linearly with bandwidth. This would also overcome the problem of leased lines appearing to grow by a very large amount in the last few years, as BT has started to provide some very high bandwidth products. However, we are constrained by the data obtained from the US LECs, which is measured in 64k equivalents.

In order to be conservative in our estimates (and therefore, if this were to affect the result it would be to overstate BT's inefficiency), we have taken the bandwidth of BT's products at the lower bound, where bandwidth is not specifically defined.

## 3.2 TFP data

In general, the data collected for our TFP analysis is at a higher level than for the SFA analysis. However, we have also used detailed LEC data for calculating our econometric TFP model.

### 3.2.1 Data for our standard TFP analysis

The following data was used when calculating the indexed TFP:

- 26 US LECs, using regulatory accounting data from 1996 to 2006. This is a lower number of LECs than for the SFA, as data for the purpose of calculating TFP was only available at the parent level (therefore, for example, looking at Verizon South instead of Verizon South Illinois); and
- 10 European telecommunication incumbent operators using financial accounting data from 2002 to 2006 where available to obtain fixed line business data.

We have attempted to run our standard TFP analysis across a panel including European operators, so as to ensure that any results are directly applicable to BT. While we would not intuitively expect there to be a large difference in productivity improvements between operators in Europe and the US, but without testing for this it would not be possible to draw robust conclusions. We are therefore running standard TFP models both including and excluding the European operators, to see what effect, if any, they have on our results.

Since the calculation of TFP growth here looks within a company over time, rather than between companies, it is not necessary to adjust the data for comparability. However, for consistency we use the same set of output and input measures across the companies although the reporting standards may differ between them.

Data was collected on the following types of output:

**Table 5: Input and Output data for indexed TFP analysis**

Output data	Input Data
Leased lines volumes	Materials or operating expenses <sup>34</sup>
PSTN lines volumes, including ISDN lines	Depreciation
Volume of local and internet minutes	Staff costs
Volume of long-distance minutes	Staff numbers
Revenue from Leased lines	Closing NBV
Revenue from PSTN Lines	Closing GBV
Revenue from Local and internet minutes	Asset price trends
Revenue from Long-distance minutes	

For the US LECs, this data was taken directly from our SFA analysis dataset and appended with further data from the ARMIS dataset where necessary. For the European operators data was extracted predominantly from the company annual reports and company fact sheets. Due to this alternate data source, a wider range of output categories were available across the EU operators.

<sup>34</sup> Defined as those operating costs reported above the operating profit or EBITDA line in standard financial accounts.

For the purposes of our analysis these were made consistent with those of the US LECs by aggregating all switched lines, leased lines, and types of call minutes.

The following assumptions were made during the data collection process:

- Among the European operators, where the operator was an integrated fixed and mobile operator, costs and volumes associated with the mobile business were removed using segmental analysis in the accounts. In general, this was calculated using the relative segmental proportions of revenues, EBITDA or staff costs.
- All operating expenses were separated into network and non-network related costs. However, expenses which were viewed as shared between the network and non-network parts of the business, such as land, premises and vehicles, were allocated 40% to network and 60% to non-network expenses based on industry experience.
- Where data was missing for only one particular year among the European operators, the data point was sometimes estimated based either on historic ratios or trends.

### **3.2.2 Data for our econometric TFP analysis**

To calculate TFP using the fixed effects econometric approach we have used the LEC data set used in the index methodology. We have abstracted from inclusion of EU operators due to a lack of data disaggregation of outputs.

#### **3.2.2.1 Output data**

As established in section 2.2.3, in order to estimate the fixed effects model we need to construct a composite output index. To generate this index we have collected data and associated revenues for:

- Local and internet minutes;
- Long distance minutes;
- Leased lines; and
- Switched lines.

#### **3.2.2.2 Factor input data**

The fixed effects model estimates TFP growth without bias from changes in the capital stock, labour and material input. To achieve this we have collected data for the LECs for each of these factors: the capital stock is measured by the closing NBV; labour input by staff costs; and materials by the total opex expenditure.

While we would ideally use the quantity of each input, it is not possible to accurately measure the quantity of materials used; nor is it feasible to count the number of assets of different types. The

costs of these assets, deflated by a price index, therefore give us a good proxy for the relative quantity over time.

While LECs and some European operators do report the number of full-time employees in each year, it is not possible to know how these employees are distributed across the organisational structure. Again, we believe that the total staff cost (deflated by inflation) will act as a good proxy for the relative input of labour across time.

The data in the LEC database are recorded in nominal terms. To prevent price effects affecting our measure of TFP we have deflated all factor inputs to 1996 prices. We deflated both labour and materials inputs by US economy wide inflation, from the IMF, whilst deflating capital stock by the asset price index used to convert the HCA costs to CCA in section 3.1.1.2.

## 4 Results: Comparative efficiency analysis

This section presents the results of our econometric analysis into the current efficiency or inefficiency of BT's network operations. Our conclusions can be summarised as:

- BT lies slightly above the top decile of companies ranked by efficiency, meaning that there is no catch-up component needed.
- Our models are robust and statistically good fits, with the results being relatively insensitive to changes in specification or assumptions.

All models presented in this section have been estimated using Stata, a statistical software package, using time-variant stochastic frontier analysis over a panel dataset as set out in section 2.1.8.

### 4.1 Specification

Our preferred regression has the following specification based on a Cobb Douglas cost function. The exact form of this specification is set out in Equation 5 with a time invariant or time variant specification for the efficiency term.

#### 4.1.1 Output variables

As would be expected, the size of a firm's output appears to be the most significant driver of costs; see Appendix A.1 for graphical representations of the correlations between outputs and costs. Output was measured in terms of the number of switched PSTN lines, leased lines and switch minutes. Of these, it appears that PSTN lines is the most significant driver of costs, and our output variables contribute to a model which has a high degree of explanatory power.

##### 4.1.1.1 Switch minutes and correlation

In many specifications, we find that the number of switch minutes is an insignificant variable, sometimes negatively associated with total cost. This is unintuitive, since we would expect that costs would rise significantly as the number of switch minutes carried across a network increased.

However, as can be seen in Table 6, there is a very high correlation between switched minutes and switched lines (specifically, 0.967 with BT included in the analysis and 0.981 without). There is also a high correlation between leased lines and the other variables, although this is not as pronounced. These high correlations are not surprising, since as the number of switched lines increase we would expect there to be a roughly equal increase in the number of switch minutes (since the number of minutes per line will be approximately constant over time).



**Table 6: Correlation statistics between output variables**

	Log of switched lines	Log of leased lines	Log of switch minutes
Log of switched lines	1		
Log of leased lines	0.9221	1	
Log of switch minutes	0.9672	0.8967	1

Source: Deloitte analysis

Such a high correlation may cause our regressions difficulty in disaggregating the individual impact on costs of the quantity of switched lines from the impact of the quantity of switch minutes. As a result, it is not possible to specify the exact elasticity impact on total costs of changes in one specific output alone, and focus should be placed on the aggregate of the switched minutes and switched lines coefficients.

In order to attempt to correct for this, we have used as a variable the number of switch minutes per line. However, we find that there remains a correlation between this new variable and the number of switched lines. This could be due to network externality effects, where as the number of subscribers changes the number of calls made by existing subscribers will also change since they have more or fewer people to call. Alternatively, given that competition is driving the number of subscribers down for each of the LECs, we could be seeing some sort of selection bias, where those who make the highest number of calls are the first and most likely to move to an alternative network. In either case, we find that we must still consider both the coefficient on switch minutes per line and the coefficient on switch lines when drawing conclusions over cost volume elasticities.

## 4.1.2 Environmental variables

As previously stated, we have attempted to include in our regression a number of variables to allow for differences in the operating environment for each operator. These included:

- Population variables, including the percentage of urban population, population distribution, and cable length per line; and
- Climate variables, including the temperature range.

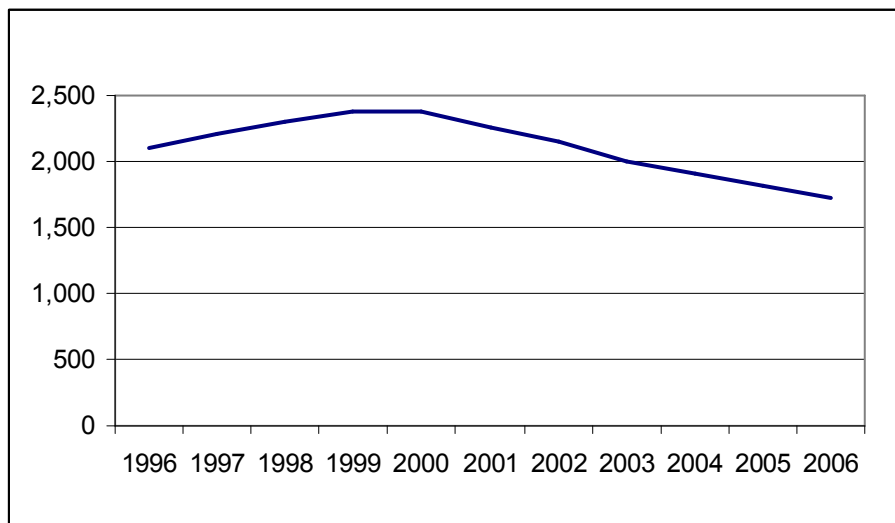
Of these variables only the population variables were found to be a significant driver of costs. We attempted to use three distinct metrics in our regression: the state-wide population density, the average duct per line, and the average total cable (sheath) per line. These three variables were found to be highly correlated, and as such only one could be included into the regression with significance. Of these variables, the model which had the best fit (as measured by log-likelihood tests) used the total sheath per line. Again, this contributes a significant explanatory power to the model.

## 4.1.3 Structural breaks

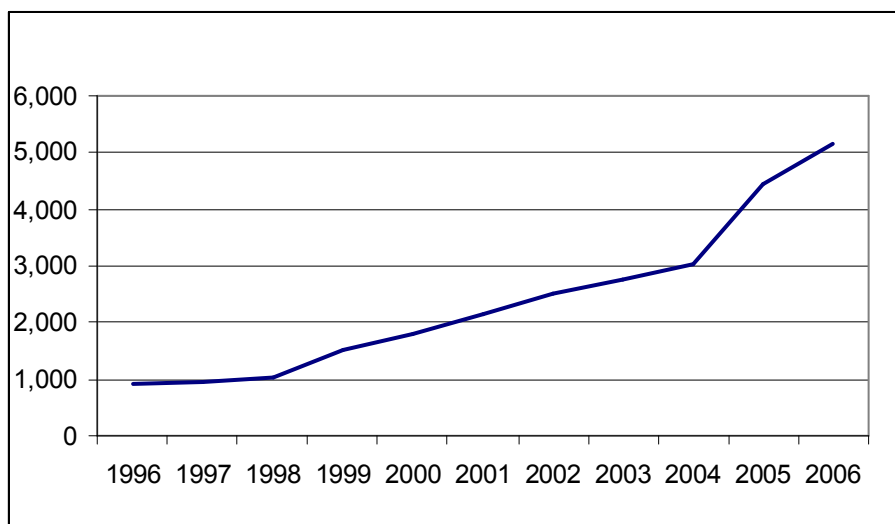
In previous comparative efficiency reports, clear structural breaks have been found in the US LEC data between 1998 and 1999. We have examined this issue further and have found evidence of a

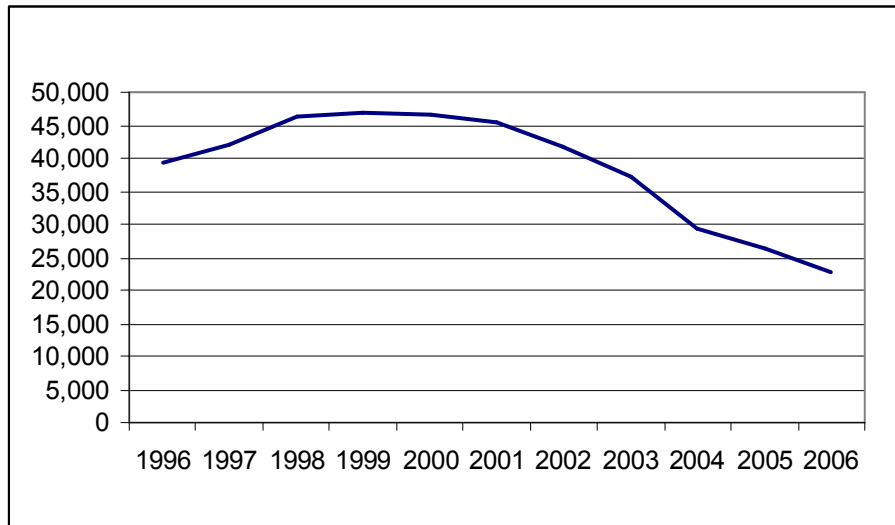
further structural break in the dataset between 2003 and 2004. This would not have been noticeable previously due to a lack of data after 2003. This latter structural break is primarily derived from the large increases in leased lines, clearly illustrated in the figures below which are derived from the US LEC dataset, and recent rises in copper prices.

**Figure 5: Trends in switched lines (averaged across all LECs '000)**



**Figure 6: Trends in leased lines (64k equivalents, averaged across all LECs '000)**



**Figure 7: Trends in switch minutes (averaged across all LECs 000'000)**

The first structural break has been identified in previous studies, including the study used to define the last price control, and is likely caused by a fall in copper prices. Inspection of the graphs above suggests this break could also be caused by changes in output growth, particularly in switched lines and switched minutes.

## 4.2 Overall results

The results of our preferred specification, including two structural breaks along with their interaction terms, is shown in Table 7, whilst the raw output is provided in Appendix A.2.

**Table 7: Regression results of preferred specification**

Explanatory Variables	Coefficient	T-value	Significance <sup>35</sup>
Log of total switched lines	0.848	31.49	0.0000
Log of total switched lines (1996-1998)	-0.006	-0.24	0.8070
Log of total switched lines (2004-2006)	-0.089	-3.51	0.0000
Log of total switch minutes	-0.055	-3.84	0.0000
Log of total switch minutes (1996-1998)	0.049	5.51	0.0000
Log of total switch minutes (2004-2006)	-0.022	-2.34	0.0190
Log of total leased lines	0.174	9.38	0.0000
Log of total leased lines (1996-1998)	-0.063	-4.97	0.0000
Log of total leased lines (2004-2006)	0.106	6.30	0.0000
Log of sheath per line	0.234	6.28	0.0000
Log of sheath per line (1996-1998)	0.041	2.32	0.0210
Log of sheath per line (2004-2006)	0.020	1.11	0.2690
Time	0.01	2.11	0.0350
Time (1996-1998)	-0.01	-1.01	0.3120
Time (2004-2006)	0.04	5.25	0.0000
Constant	0.82	3.42	0.0010
<b>Log likelihood</b>	<b>747.9</b>		

Source: Deloitte analysis

In this table, variable names followed by a time period indicate interaction terms, where the variable is only specified during the period stated. This allows for structural breaks to be analysed.

As stated in section 4.1.1, the correlation between the output variables means that care must be taken in interpreting the coefficient s on any one of these. While it appears that switch lines are the most significant driver of costs, this is likely to include some element of switch minutes effects.

This is a well-fitting model, with a high log-likelihood. Attempting to remove any of the variables included sees a significant fall in the log-likelihood. This model has been run using time-variant stochastic frontier analysis. This allows for each company's efficiency to change over time, rather than remaining constant over the entire eleven-year period. We have tried running the model in the time variant form which leads to similar results, although the model is a less good fit.

As described in section 2.1.10, this regression has been run excluding BT from the analysis.

<sup>35</sup> This is the probability of rejecting the hypothesis that the variable is insignificant when the variable is in fact significant. This known as committing a type one error. In this report we consider a variable significant if the probability of committing a type one error is less than or equal to 0.05.

### 4.2.1 Application of specification to BT

When the specification outlined in Table 7 is applied to the entire dataset, we find estimates of efficiency and rankings as shown in Table 8.

**Table 8: Efficiency estimates and rankings**

Rank	Company Name	Inefficiency against top firm	Inefficiency against top decile	Inefficiency against midpoint
1	Verizon North Illinois	0.00%	-5.76%	-26.91%
2	Verizon South Illinois	1.80%	-4.05%	-25.59%
3	Verizon North Wisconsin	2.81%	-3.11%	-24.86%
<b>4</b>	<b>BT</b>	<b>3.34%</b>	<b>-2.60%</b>	<b>-24.47%</b>
5	Contel Nevada	4.00%	-1.99%	-23.99%
6	Qwest Idaho South	6.10%	0.00%	-22.45%
7	Verizon North Ohio	6.11%	0.00%	-22.45%
8	Verizon North Michigan	8.97%	2.70%	-20.35%
9	Qwest Montana	10.52%	4.16%	-19.22%
10	Verizon North Pennsylvania	13.36%	6.84%	-17.14%
11	Qwest Iowa	14.38%	7.79%	-16.40%
12	Indiana Bell	15.04%	8.42%	-15.91%
13	Wisconsin Bell	15.10%	8.48%	-15.87%
14	Verizon North Indiana	16.48%	9.78%	-14.86%
15	Qwest North Dakota	18.93%	12.08%	-13.08%
16	Verizon Delaware	21.06%	14.09%	-11.52%
17	Qwest South Dakota	23.64%	16.52%	-9.63%
18	Nevada Bell	23.78%	16.65%	-9.53%
19	Verizon Northwest Idaho	26.34%	19.07%	-7.66%
20	Bellsouth Tennessee	28.01%	20.64%	-6.44%
21	Verizon Northwest Oregon	30.43%	22.92%	-4.67%
22	Verizon Northeast Maine	30.50%	22.99%	-4.62%
23	Qwest Minnesota	30.65%	23.13%	-4.51%
24	Contel Arizona	30.89%	23.36%	-4.33%
25	Bellsouth Alabama	32.42%	24.80%	-3.21%
26	Bellsouth Kentucky	32.77%	25.13%	-2.95%
27	Michigan Bell	33.77%	26.07%	-2.23%
28	Ohio Bell	34.21%	26.49%	-1.90%
29	Qwest Wyoming	35.47%	27.67%	-0.98%
30	Qwest New Mexico	35.50%	27.70%	-0.96%
31	Bellsouth South Carolina	35.85%	28.03%	-0.70%
32	Southwestern Oklahoma	35.88%	28.06%	-0.69%

Rank	Company Name	Inefficiency against top firm	Inefficiency against top decile	Inefficiency against midpoint
33	Bellsouth Mississippi	36.44%	28.59%	-0.27%
34	Verizon West Virginia	36.82%	28.94%	0.00%
35	Verizon South Virginia	37.91%	29.97%	0.80%
36	Verizon Northeast Rhode Island	37.96%	30.02%	0.84%
37	Bellsouth North Carolina	38.22%	30.27%	1.03%
38	Southwestern Kansas	39.84%	31.79%	2.21%
39	Verizon Northeast Vermont	39.92%	31.87%	2.27%
40	Verizon Maryland	41.11%	32.99%	3.14%
41	Southwestern Arkansas	41.74%	33.58%	3.59%
42	Verizon Virginia	42.06%	33.89%	3.83%
43	Qwest Utah	42.36%	34.17%	4.05%
44	Verizon Northwest Washington	42.46%	34.26%	4.12%
45	Bellsouth Louisiana	42.73%	34.51%	4.32%
46	Illinois Bell	43.61%	35.34%	4.96%
47	Verizon Northeast New Hampshire	44.16%	35.86%	5.37%
48	Qwest Oregon	45.65%	37.26%	6.45%
49	Verizon New Jersey	47.77%	39.27%	8.01%
50	Verizon Pennsylvania	50.55%	41.88%	10.04%
51	Qwest Washington	50.72%	42.05%	10.16%
52	Southwestern Missouri	51.40%	42.68%	10.66%
53	Bellsouth Florida	51.47%	42.76%	10.71%
54	GTE California	51.80%	43.07%	10.95%
55	Qwest Nebraska	53.49%	44.66%	12.19%
56	Qwest Arizona	53.86%	45.01%	12.46%
57	Verizon South South Carolina	53.92%	45.06%	12.50%
58	Pacific Bell California	54.34%	45.46%	12.81%
59	Verizon Northwest West Coast California	55.27%	46.34%	13.49%
60	Verizon Southwest Texas	56.12%	47.14%	14.11%
61	Bellsouth Georgia	56.36%	47.36%	14.29%
62	Verizon Florida	58.75%	49.61%	16.03%
63	Qwest Colorado	58.94%	49.79%	16.17%
64	Verizon South North Carolina	75.90%	65.77%	28.56%
65	Southwestern Texas	79.41%	69.09%	31.13%
66	Verizon Northeast Massachusetts	83.81%	73.23%	34.35%
67	Verizon New York Telephone	118.14%	105.58%	59.44%

Source: Deloitte analysis

Our results clearly show that BT lies slightly above the top decile of our ranked list of operators. Since Ofcom has previously considered this decile to be the appropriate benchmark for

efficiency<sup>36</sup>, this means that we conclude that BT is 2.6% more efficient than the benchmark, and therefore should not be assumed to be inefficient for the purposes of regulation.

## 4.2.2 Robustness of model

As stated, our model is a good fit to the data, with a high log-likelihood. This, combined with the large number of observations used, plus the general reasonableness of coefficients, shows that our model is robust.

Compared to previous models used by Ofcom, this can again be seen to be a reasonable specification. In particular, the range of efficiency estimates (that is, the difference between the most efficient and least efficient firm) is relatively low compared to some previous studies, indicating that our model is explaining more of the variation in costs. Other than Verizon New York Telephone, there are no significant outliers in terms of efficiency estimates.

Some of the coefficients in our preferred specification appear to be separately insignificant. However taking these variables out was found to lead to a much increased efficiency spread between the top and bottom firms indicative of a poorer specification, and falls in the log-likelihood. This is likely due to correlation issues among the variables.

## 4.3 Sensitivities

In addition to the model specifications presented above, we have performed a number of sensitivity tests, to help us in assessing the robustness of the models above.

### 4.3.1 Fixed effects estimation

While we believe the SFA is the optimum econometric model for our purposes, there is value in testing our final specifications using other methods. One such method is specifying the model as in Equation 5 and estimating via fixed effects<sup>37</sup>.

Estimating in this way, we found the coefficient estimates using this approach to be similar to those reported in Table 7.

### 4.3.2 Inclusion of environmental variables

During our modelling process, we tested the significance of a large number of environmental variables. These included:

- Population density: as discussed in section 4.1.2, we attempted to use a number of different variables to measure population density. We found these variables to be highly correlated, and therefore only one variable should be used; the variable with the best explanatory power was the total sheath per line.

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<sup>36</sup> See NERA (2005)

<sup>37</sup> Estimating by fixed effects additionally allows for a fixed, across time, firm specific idiosyncratic effect.

- Absolute temperature and temperature variance: these were never found to be significant.
- Telecommunications sector wages: these were included to allow for regional differences in pay costs. However, the variable was found to generally to be insignificant across a range of specifications<sup>38</sup>.

Including any of these variables in our models has a negligible effect on the results increasing the fit insufficiently to statistically justify inclusion.

### 4.3.3 Sensitivities on structural breaks

Section 4.1.3 describes how we have settled on the use of two structural breaks for our model; one between 1998 and 1999, and one between 2003 and 2004. We have tested for the effect of this by running the model with one or both of these structural breaks removed.

Altering the number of structural breaks to one has no impact on the size of inefficiency relative to the top decile. Restricting the model to one structural break is, however, rejected on the basis of statistical testing (using a log-likelihood test). The coefficient estimates and summary of efficiency and rankings are presented in Table 9<sup>39</sup>.

**Table 9: Regression results using one structural break**

Explanatory Variables	Coefficient	T-value	Significance <sup>40</sup>
Log of total switched lines	0.860	37.95	0.0000
Log of total switched lines (1996-1998)	0.035	1.52	0.1280
Log of total switch minutes	-0.093	-6.98	0.0000
Log of total switch minutes (1996-1998)	0.042	4.61	0.0000
Log of total leased lines	0.188	11.17	0.0000
Log of total leased lines (1996-1998)	-0.086	-6.52	0.0000
Log of sheath per line	0.150	5.92	0.0000
Log of sheath per line (1996-1998)	0.053	3.10	0.0020
Time	0.011	3.26	0.0010
Time (1996-1998)	-0.011	-1.43	0.1530
Constant	1.123	5.16	0.0000
<b>Log likelihood</b>	<b>702.05</b>		

Source: Deloitte analysis

<sup>38</sup> In the limited specifications wages were found to be borderline significant; in general the overall model fit was not improved substantially and BT's overall inefficiency was unchanged.

<sup>39</sup> Raw output is provided in Appendix A.2.

<sup>40</sup> This is the probability of rejecting the hypothesis that the variable is insignificant when the variable is in fact significant. This known as committing a type one error. In this report we consider a variable significant if the probability of committing a type one error is less than or equal to 0.05.



**Table 10: Efficiency estimates and rankings using one structural break**

Rank	Company Name	Inefficiency against top firm	Inefficiency against top decile	Inefficiency against midpoint
1	Contel Nevada	0.00%	-3.52%	-21.25%
<b>5</b>	<b>BT</b>	<b>2.10%</b>	<b>-1.50%</b>	<b>-19.59%</b>
7	Indiana Bell	3.65%	0.00%	-18.37%
34	Verizon Virginia	26.98%	22.50%	0.00%

Source: Deloitte analysis

Our conclusions, that BT should be considered to be efficient, is unchanged.

When no account for structural breaks is made (that is, we do not include any dummy terms or interaction variables in the model), the model does not converge to a solution. We tested whether this could be a result of combinations of variables in the specification but found a lack of convergence across a variety of combinations. This therefore does not allow us to draw any conclusions over how it would affect the estimated efficiency of BT.

#### 4.3.4 Removal of early years

As shown in our results, we have included variables which allow our explanatory factors to have a different effect in the earlier years of our model. This is because we have found statistical evidence that the general trends in these variables break between the early years and later years. This change in trend could be caused by a number of factors, including changes in reporting definitions, mergers of operators, revised business plans, and so on.

In order to compensate for these structural breaks, we inserted interaction terms which allow the effect of variables to vary between early and later years. We tested the effects of these interaction terms allowing for the breaks in trends to fall in various years; as can be seen in our models presented above, we found the model was more significant when a break was inserted between 1998 and 1999.

An alternative method of dealing with these structural breaks is to remove the earlier years from our model. Removing the years 1996, 1997 and 1998 from our dataset BT was found to be slightly more efficient. However, across a range of specifications a substantial reduction in the log likelihood and a lack of stability was found. Furthermore, this approach is theoretically undesirable as it excludes a substantial quantity of information hence will yield less efficient parameter estimates.

As a result, it is clear that including all years in the model, with interaction terms in the earlier years, should be the preferred methodology.

### 4.3.5 Weighting on switch minutes

The regression set out in Table 7 is run using a total switch minutes variable, comprising an aggregate of local and main switched minutes weighted according to approximate relative costs. If this assumption of differing costs were relaxed, and both local and main switch minutes given a weight of unity, our regression results are as set out below.

**Table 11: Regression results with unweighted switch minutes**

Explanatory Variables	Coefficient	T-value	Significance <sup>41</sup>
Log of total switched lines	0.848	31.43	0.0000
Log of total switched lines (1996-1998)	-0.006	-0.28	0.7790
Log of total switched lines (2004-2006)	-0.089	-3.48	0.0000
Log of total switch minutes	-0.055	-3.82	0.0000
Log of total switch minutes (1996-1998)	0.050	5.53	0.0000
Log of total switch minutes (2004-2006)	-0.022	-2.35	0.0190
Log of total leased lines	0.174	9.37	0.0000
Log of total leased lines (1996-1998)	-0.063	-4.91	0.0000
Log of total leased lines (2004-2006)	0.105	6.28	0.0000
Log of sheath per line	0.234	6.28	0.0000
Log of sheath per line (1996-1998)	0.041	2.33	0.0200
Log of sheath per line (2004-2006)	0.019	1.09	0.2750
Time	0.010	2.12	0.0340
Time (1996-1998)	-0.008	-1.02	0.3080
Time (2004-2006)	0.036	5.27	0.0000
Constant	0.826	3.44	0.0010
<b>Log likelihood</b>	<b>748.1</b>		

Source: Deloitte analysis

Applying this specification to BT's data, we find that BT is 2.1% more efficient than the top decile. Altering the weighting on switch minutes therefore has little effect on our overall conclusions.

### 4.3.6 Non-linear time trends

Within our models we have allowed the cost function to vary linearly with time. This means that we allow costs to gradually fall, in line with our experience in telecommunications markets worldwide.

However, this assumption of linearity may not be true, since the speed of cost changes may vary over time itself. In order to consider the extent to which this is true, we attempted to include

<sup>41</sup> This is the probability of rejecting the hypothesis that the variable is insignificant when the variable is in fact significant. This known as committing a type one error. In this report we consider a variable significant if the probability of committing a type one error is less than or equal to 0.05.

parametric and other functional forms of time (such as including a time-squared variable, and a log-time variable).

When trying these different specifications we found generally that the linear specification was sufficient in capturing the time trend effects. For example, we found including a squared time trend with the linear-trend increased the log likelihood only by 0.6. This increase was insufficient to reject the restriction that the extension to non-linearity is accepted statistically<sup>42</sup>.

Further discussion of the time trend can be found below in section 5.1.

## 4.4 Overall conclusions

We have found that BT's network operations lie on the top decile when compared against a list of US LECs ranked in order of efficiency. Using previous methodologies to calculate the efficiency of BT, this translates to **BT being not inefficient compared to the top decile of US LECs**. Indeed BT appears to be slightly more efficient than the benchmark.

This result is robust, with our core conclusion being unchanged across a number of specifications and modelling methods.

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<sup>42</sup> Replacement of the linear trend for a log-linear trend which was also rejected as the model's fit decreased.

## 5 Results: Productivity analysis

This section presents the results of our analysis into the movement of the efficiency frontier over time. Our conclusions can be summarised as:

- When controlling for exogenous factors such as economies of growth, we can identify a yearly increase in productivity of between 0.5% and 1.1%.
- Due to the rapidly changing market, individual estimates of TFP are very volatile. However, over the entire market and over a number of years, we can identify a clear trend.
- We have tried to identify this trend using a number of different methods; each of these methods gives a consistent result.

### 5.1 Implied time trend from SFA

In previous network charge controls, Ofcom have used the time trend from comparative efficiency analysis to predict the trend in productivity gains. The time trend which we find over the entire period in our SFA analysis in Table 7 is generally consistent with those found in previous studies of BT's efficiency. This shows that costs in nominal terms are only slightly changing over time.

However, this method of estimating the movement in the efficiency frontier does not measure the change in costs directly, and it is quite possible that the time trend could be picking up other external factors which have changed over time. This is beneficial for our SFA regression, as we would not want these external effects being included in a measure of efficiency.

Furthermore, this method looks at costs rather than productivity movements, which may not be directly related. These costs are in nominal terms, so we are introducing inflation fluctuations into the general trend. In addition, estimating movements in productivity in this way does not take into account the capacity utilization effects described in section 2.2.

Finally, the frontier as calculated by SFA is based on the assumption that firms' inefficiency is identically distributed in each time period (therefore, with an identical mean and variance). The frontier itself moves over time only with the inclusion of the time variable in the regression model. However, it is possible that over time firms may become generally more or less efficient, meaning that all firms should move towards or away from the actual efficiency frontier. However, such a movement could lead to SFA incorrectly shifting the estimated frontier, therefore overestimating or underestimating all firms' inefficiency.

We would not expect such a uniform shifting in efficiency, but it is not possible to definitively state that it does not exist. Certainly, the ranges of the LEC inefficiency estimates found in previous studies carried out by NERA fluctuate considerably.

We believe that there exist better methods to estimate the movement in productivity over time. We look to directly compare outputs to inputs through total factor productivity methods. Estimates from such methods can also be compared against the SFA results to consider whether the results from our previous analysis are reasonable.

## 5.2 Standard TFP analysis

Our preferred analysis uses a Törnqvist index for both output and input. When we calculate these indices, it is clear that there is a large amount of volatility in both series, with large positive and negative changes within companies over time. There is, however, no clear trend between companies as to when input or output should be higher or lower.

**Figure 8: Trends in Törnqvist output index across LECs**

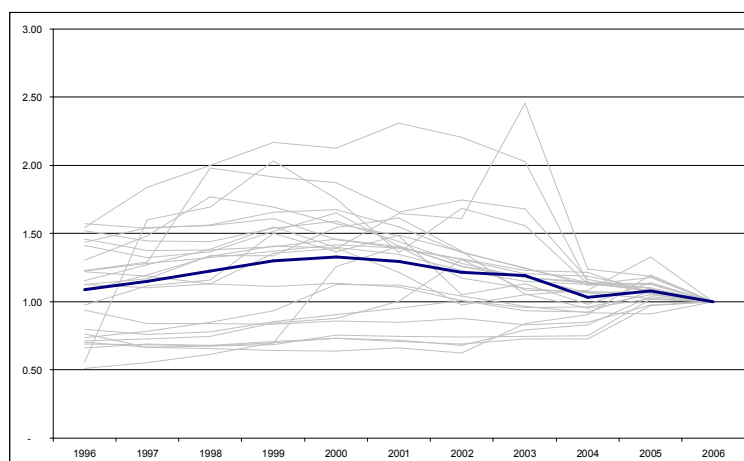


Figure 8 shows how output has varied by operator over the time period, when measured by the Törnqvist index with 2006 set to 1. It can be clearly seen that output is very volatile for a number of firms, although the average trend (represented by the bolder line) is less extreme in its movements, showing instead a slow decline in volumes since 2000.

To calculate the TFP, we firstly average the input index across all operators, and secondly average the output index. This then gives an average TFP across all firms, minimising the volatility.

### 5.2.1 Inclusion of EU companies

As discussed in 3.2.1, for our standard TFP analysis we have attempted to include data from European operators as well as US LECs, to ensure that our analysis is directly applicable to operators outside of the US.

We find that the data for the EU operators is more volatile than that of the US LECs. In particular, since data has been collected from annual reports, which are not always reported on a consistent basis, we experience large swings in output variables in some cases. In order to minimise the effect of these anomalies, we have excluded the largest proportional changes as outliers.

The inclusion of the EU data into our analysis has very little impact on our overall results, but it slightly increases the goodness of fit of our econometric model used to strip out volume effects. While this is partially due to the fact that we have fewer operators in the EU dataset than in the US, it is a clear indication that there are not significant differences in TFP between the US and the EU. Our results below are shown including the EU data.

## 5.2.2 TFP after removing capacity utilisation effects

Once we have removed capacity utilisation effects (as discussed in section 2.2.2.1), we estimate an annual productivity increase of 0.5%. This uses the Törnqvist indexing method for outputs and inputs<sup>43</sup>, and is based on quantity changes, excluding any price effects. If prices were in fact increasing significantly, we would find a positive average change in *nominal* costs.

Generally, individual TFP estimates are relatively volatile both within and between firms. This is caused directly by some rapidly varying volumes. However, the econometric model which we run to take out capacity utilisation effects finds a clear correlation between TFP and volume changes for all outputs. The econometrics is therefore able to strip out these effects to leave the true TFP of 0.5%. Our final specification is shown in Table 12.

**Table 12: Output for standard TFP analysis removing capacity utilisation effects**

Explanatory Variables	Coefficient
Change in leased lines	0.1877
Change in switched lines	-0.2927
Change in local and internet minutes	0.5438
Change in long distance minutes	0.0961
Constant (TFP estimate excluding capacity utilisation effects)	0.0045
<b>Overall R<sup>2</sup></b>	<b>0.54</b>

Source: Deloitte analysis

While this method is designed to separate out any capacity utilisation bias, it may have difficulty fully identifying issues surrounding the economics of scale present. This led us to propose an extension to the specification as established in 2.2.2.1. This extension essentially includes the output variables stated in absolute as well as in changes over time.

However, when including the outputs in absolute terms as the variables in the mode, we found these to be statistically insignificant. This result possibly suggests that economies of scale have been sufficiently abstracted in the original specification.

Finally, our model does not take into account movements of firms towards and away from the frontier. However, since the frontier itself is defined by the average of firms in any year, we would expect these movements relative to the frontier to average out at any time, and so these will not affect our results.

<sup>43</sup> Other indexing methods were tried but produced worse-fitting models, or had difficulty specifying the constant term.

## 5.3 Econometric TFP model

The fixed effects estimation of TFP was based on the specification outlined in Equation 21. We estimated this specification using fixed effects as opposed to random effects given the fixed effect parameter  $\nu_i$  was found to be significant and highly correlated with the explanatory variables.

Our preferred specification results are outlined in Table 13. The coefficient on time shows how TFP productivity changes in annual terms. Our preferred specification therefore suggests compound TFP growth of around 1.1%.

**Table 13: Regression output for econometric TFP**

Explanatory Variables	Coefficient	T-value <sup>44</sup>	Significance
Log of materials	0.0231	0.22	0.828
Log of capital	0.5059	5.01	0.000
Log of staff compensation	0.2007	2.19	0.024
Time	0.0114	2.26	0.024
Constant	-12.6042	-5.15	0.000
<b>Overall R<sup>2</sup></b>	<b>0.8768</b>		

Source: Deloitte analysis

Across several specifications material inputs were found to be statistically insignificant. This is likely a result of high correlation amongst the explanatory variables.

This result is robust to economies of scale bias but is not robust to capacity utilisation bias. However, this is only problematic if inefficient utilisation is consistent across firms and time.

### 5.3.1 Sensitivities

As the data used for this analysis follows closely that used in the SFA analysis, we tried taking account of structural breaks. However, the fit of the model was little changed leading to the joint restriction of no significance on the structural break terms being accepted.

We also further used a range of different output indices including the Paasche, Laspeyres and Fisher. We found that changing the indices made only minor impacts on the TFP growth estimated. However, our preferred specification remains that using the Törnqvist index given the *a priori* theoretical reasons outlined in section 2.2.1.

## 5.4 Overall conclusions

Using the most theoretically appropriate indexation technique, our standard TFP model gives an annual productivity increase of 0.5% once capacity utilisation has been accounted for.

<sup>44</sup> All standard errors were calculated using methods robust to the error term being heteroskedastic or serially correlated.

This contrasts to our 'econometric TFP' model which gives a productivity increase of 1.1%. However, as discussed this may suffer from capacity utilisation bias, so could be less robust.

Therefore, our overall preferred methodology estimates **a movement in the efficiency frontier of between 0.5% and 1.1% per annum.**



## 6 Conclusions

This report has detailed our analysis into BT's current level of efficiency, and the rate at which this can be expected to change over time. We have:

- Estimated the comparative efficiency of BT in 2006 using stochastic frontier analysis, comparing BT's network operations to the US LECs.
- Carried out a standard total factor productivity analysis using data from the US LECs and European fixed-line operators, and ran a basic econometric model on this to remove any productivity changes caused by capacity utilisation.
- Put together a detailed econometric TFP model, which attempts to derive the relationship between our output index and the various inputs, using a fixed effects panel regression.

Through these analyses, we conclude that:

- When compared to the US LECs, BT's network operations lie slightly above the top decile when ranked according to efficiency. Therefore, using the precedence of previous methodology, BT should be considered to be **not comparatively inefficient**.
- Over time, **productivity is increasing by between 0.5% and 1.1% per year**. We believe that the lower end of this range may suffer from capacity utilisation bias, so could be less robust.

## 7 Uses and Applications of Efficiency Estimates

In this chapter we provide an overview on options for including efficiency estimates into network charge control (NCC) models.

### 7.1 Incorporating efficiency adjustments into NCCs

We understand that Ofcom will include efficiency measures within the network charge controls applied on products supplied by BT. As noted in Figure 1, there are a number of ways in which efficiency factors are incorporated into NCCs.

#### 7.1.1 The 'catch-up' effect

Assuming that Ofcom proceeds with the use of this efficiency approach to calculate BT's relative efficiency "the catch-up effect", then there are several ways in which Ofcom could choose to incorporate this factor into NCC.

- As an adjustment to the base year operating costs, including depreciation. For example, if BT is determined to be "5%" inefficient then base year costs could be reduced by a maximum of "5%". This would potentially require BT to immediately change its prices by a similar amount.
- As an additional input into the annual productivity factor. For example, in a 4 year price control then this might result in a 1.25% decrease in allowable costs, assuming compounding is not considered, and hence price in each year. This is independent of any productivity factor to reflect movements in the efficient frontier.
- A partial adjustment to reflect cost controllability: To the extent that some of the inefficient costs cannot be changed immediately then Ofcom could backload the efficiency adjustment or only include a proportion recognising that, particularly when volumes are falling, many costs are sunk and therefore not within BT's control.

Ofcom could choose to use a combination of these methods, for example by requiring a smaller fall in costs in the first year, with an eventual move to efficiency over the period of the price control. This would be a more pragmatic approach, since many of BT's costs (particularly those of cable and installations) are fixed in the short-term. It may also be argued that a number of these costs are uncontrollable over the period of the entire price control, and could be affected only over a much longer period. For example, where capital expenditure has been occurred then depreciation will be incurred over the period and BT can reduce that portion of its cost base.

#### 7.1.2 Annual productivity gains

The productivity factor is typically included in a separate line within the price control and is modelled as an annual change in cost.

It is important that the productivity gain is calculated at constant volumes. Otherwise, when volumes are falling, the NCC model will double count the possible cost reduction by assuming it comes through once in the CVEs but again in the annual productivity change.

It may not be appropriate to apply a productivity gain to investments that are assumed to occur during the price control period if it can be shown that the allowed cost of the investment is efficient at the start of the price control period.

There may be other types of costs, for example legacy assets that will be eventually removed rather than replaced or other sunk costs, to which it would be inappropriate to apply an annual productivity improvement since these costs are effectively outside of BT's control.

## **7.2 Application of our estimates**

Our estimates of BT's comparative efficiency and the movement in the efficient frontier have been calculated over the whole of BT's network operations, not confined to a single product or service. This estimate also applies to the whole of the network business, not simply to BT Wholesale or Openreach alone.

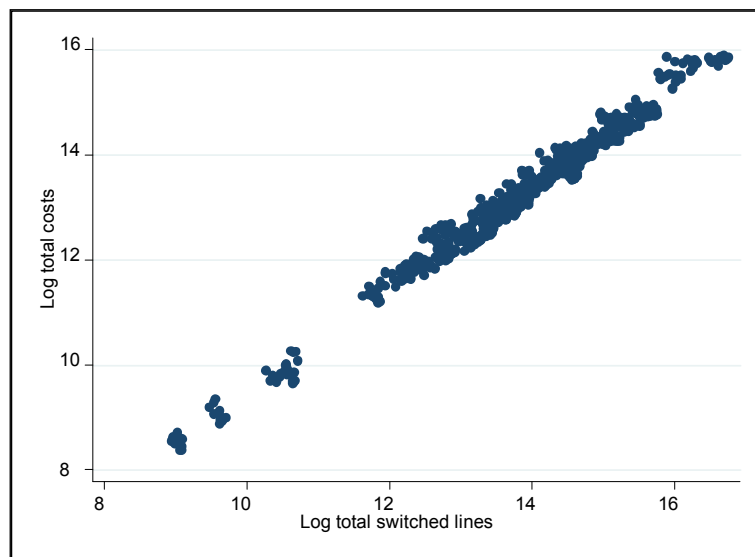
## Appendix A SFA Efficiency output

In this appendix we provide the output from the econometrics package we used (Stata) for each of the SFA econometric models referred to in the previous chapters. We also provide some high-level analysis showing that our measures of outputs are reasonable cost drivers.

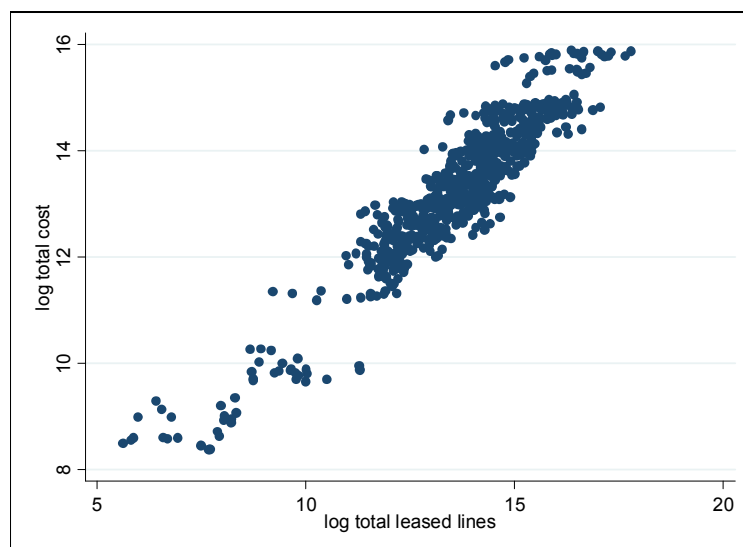
### A.1 Correlations of costs to outputs

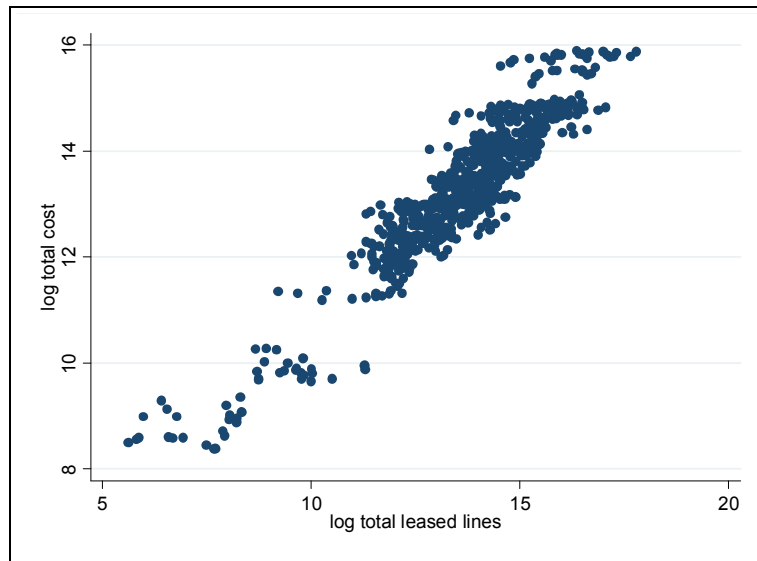
Below we provide scatter plots of the outputs of used in the SFA analysis against total costs, showing data from the US LEC ARMIS database. These clearly show that the outputs chosen are all highly correlated with costs, and as such can be expected to be reasonable cost drivers.

**Figure 9: Log total costs against log total switched lines**



**Figure 10: Log total costs against log total leased lines**



**Figure 11: Log total costs against log total switched minutes**

## A.2 Regression output

This appendix provides the output from the econometrics package we used, Stata, for our preferred SFA regression. We also provide output for our regression looking at the sensitivity of our results to the restriction to one structural break, as described in section 4.3.3.

## A.2.1 Econometric Results: Two structural breaks

```

Iteration 0: log likelihood = -34.548074 (not concave)
Iteration 1: log likelihood = 649.31878 (not concave)
Iteration 2: log likelihood = 711.11611
Iteration 3: log likelihood = 729.41453
Iteration 4: log likelihood = 741.32706
Iteration 5: log likelihood = 747.39467
Iteration 6: log likelihood = 747.88123
Iteration 7: log likelihood = 747.94606
Iteration 8: log likelihood = 747.94647
Iteration 9: log likelihood = 747.94647

```

```

Time-varying decay inefficiency model      Number of obs      =      727
Group variable: company_code               Number of groups   =      67

```

```

Time variable: time                      Obs per group: min =      1
                                           avg =     10.9
                                           max =      11

```

```

Log likelihood = 747.94647                Wald chi2(15)      =    5147.23
                                           Prob > chi2        =      0.0000

```

log_total_~t	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
log_sheath~e	.2339549	.0372759	6.28	0.000	.1608955	.3070144
D_log_shea~e	.0411033	.0177484	2.32	0.021	.006317	.0758895
D2_log_she~e	.019713	.0178291	1.11	0.269	-.0152314	.0546574
log_tota~tes	-.0547509	.0142536	-3.84	0.000	-.0826874	-.0268144
D_log_to~tes	.0494877	.0089836	5.51	0.000	.0318802	.0670953
D2_log_t~tes	-.0221891	.0094908	-2.34	0.019	-.0407907	-.0035875
log_~d_lines	.1741029	.0185611	9.38	0.000	.1377237	.2104821
D_lo~d_lines	-.0632497	.0127264	-4.97	0.000	-.088193	-.0383065
D2_l~d_lines	.1055653	.0167534	6.30	0.000	.0727292	.1384015
l~cess_lines	.8475847	.0269123	31.49	0.000	.7948375	.9003319
D_total_sw~s	-.0055439	.0227472	-0.24	0.807	-.0501276	.0390399
D2_total_s~s	-.0894733	.0255206	-3.51	0.000	-.1394927	-.0394538
time	.009719	.0046007	2.11	0.035	.0007018	.0187361
D_time	-.0078074	.0077167	-1.01	0.312	-.0229318	.007317
D2_time	.0353129	.0067214	5.25	0.000	.0221392	.0484866
_cons	.8190918	.2392691	3.42	0.001	.350133	1.28805
/mu	.3237258	.0608902	5.32	0.000	.2043832	.4430684
/eta	-.0013108	.0078576	-0.17	0.868	-.0167115	.0140899
/lnsigma2	-3.485121	.2470895	-14.10	0.000	-3.969408	-3.000835
/ilgtgamma	1.574709	.3156192	4.99	0.000	.9561064	2.193311
sigma2	.03065	.0075733			.0188846	.0497455
gamma	.8284538	.0448552			.7223416	.8996472
sigma_u2	.0253921	.0076219			.0104535	.0403308
sigma_v2	.0052579	.0003021			.0046658	.00585

## A.2.2 Econometric Results: One structural break

```

Iteration 0: log likelihood = 457.49755 (not concave)
Iteration 1: log likelihood = 524.21999
Iteration 2: log likelihood = 644.14677
Iteration 3: log likelihood = 694.4712
Iteration 4: log likelihood = 700.70596
Iteration 5: log likelihood = 701.99777
Iteration 6: log likelihood = 702.04534
Iteration 7: log likelihood = 702.04565
Iteration 8: log likelihood = 702.04565

```

```

Time-varying decay inefficiency model      Number of obs      =      727
Group variable: company_code              Number of groups   =      67

```

```

Time variable: time                      obs per group: min =      1
                                           avg   =     10.9
                                           max   =     11

```

```

Log likelihood = 702.04565                wald chi2(10)      =    9301.53
                                           Prob > chi2        =     0.0000

```

log_total~t	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
log_sheath~e	.1501057	.0253591	5.92	0.000	.1004028	.1998085
D_log_shea~e	.0531484	.0171302	3.10	0.002	.0195737	.0867231
log_tota~tes	-.0932729	.0133572	-6.98	0.000	-.1194525	-.0670933
D_log_tota~tes	.0415384	.00901	4.61	0.000	.0238791	.0591976
log~d_lines	.1882228	.0168547	11.17	0.000	.1551881	.2212574
D_lo~d_lines	-.0858674	.0131748	-6.52	0.000	-.1116896	-.0600453
l~cess_lines	.8604733	.0226734	37.95	0.000	.8160343	.9049122
D_total_sw~s	.0347046	.0228182	1.52	0.128	-.0100183	.0794274
time	.0111508	.0034234	3.26	0.001	.0044411	.0178605
D_time	-.0110363	.0077268	-1.43	0.153	-.0261806	.0041079
_cons	1.122722	.2174582	5.16	0.000	.6965117	1.548932
-----						
/mu	.2424389	.0474603	5.11	0.000	.1494185	.3354594
/eta	-.0001888	.008953	-0.02	0.983	-.0177364	.0173588
/lnsigma2	-3.556306	.2269175	-15.67	0.000	-4.001056	-3.111555
/ilgtgamma	1.285488	.3000192	4.28	0.000	.6974608	1.873515
-----						
sigma2	.0285441	.0064772			.0182963	.0445316
gamma	.7833825	.0509116			.6676246	.8668644
sigma_u2	.0223609	.0064948			.0096314	.0350905
sigma_v2	.0061831	.0003459			.0055052	.0068611

## Appendix B Rate of change indexation

In this appendix we provide the calculations of our rate of change analysis.

### B.1 Data

Data was collected from US LECs and European operators.

#### B.1.1 US LEC data

As mentioned in section 3.1.1, data for the 26 US LECs used in our dataset were collected from the FCC's Automated Reporting Management Information System (ARMIS) database<sup>45</sup> for the period from 1996 to 2006 inclusive. Table 14 below sets out the inputs and outputs collected for the US LECs.

**Table 14: Input and Output data for US LECs**

Input data	Output Data
Operating expenses	Leased lines volumes
Depreciation	PSTN lines volumes
Staff costs	Volume of local and internet minutes
Staff numbers	Volume of long-distance minutes
Closing Net Book Value	Revenue from Leased lines
Closing Gross Book Value	Revenue from PSTN Lines
	Revenue from Local and internet minutes
	Revenue from Long-distance minutes

For each LEC, a company specific asset price index was constructed using Producer Price Index data from the U.S Bureau of Labour Statistics<sup>46</sup>. The price indices were calculated for each one of the LECs' main asset categories<sup>47</sup> and then multiplied by the relative percentage share that the asset constituted of the total asset portfolio for the years 1996 to 2006. Additionally, US inflation data was collected from the IMF World Economic Outlook Database 2007<sup>48</sup>.

<sup>45</sup> Available at <http://www.fcc.gov/wcb/armis/>

<sup>46</sup> Available at <http://www.bls.gov/data/>

<sup>47</sup> The main asset categories for the US LECs were identified as: Cable and Wire, Poles, Switches, Transmission equipment including payphones, furniture, computing equipment, vehicles, duct, buildings and office equipment.

<sup>48</sup> October 2007 Edition. Available at <http://www.imf.org/external/pubs/ft/weo/2007/02/index.htm>



### **B.1.2 European operators' data**

Data for the European Operators was obtained mainly from annual reports, including company fact sheets<sup>49</sup>, for the years from 2002 until 2006 where available. Furthermore, only data relating to the fixed part of the operator's business have been included in the TFP calculations. Table 15 below shows the input and output data collected for the European operators.

As the amount of available output data varied greatly between the European Operators, mainly due to varying accounting and reporting practices, a larger number of output categories were identified for the European operators than for the US LECs. Although this was the case, only those volume categories with corresponding revenues were included in the TFP analysis.

---

<sup>49</sup> Fact sheets were all publicly available on the operators' websites

**Table 15: Input and Output data for European Telecommunication Operators**

Input data	Output Data
Operating expenses	Leased lines volumes
Depreciation	PSTN lines volumes
Staff costs	Volume of International minutes
Staff numbers	Volume of local and internet minutes
Closing Net Book Value	Volume of long-distance minutes
Closing Gross Book Value	Volume of total domestic minutes
	Volume of local and long-distance
	Volume of total national traffic
	Volume of internet traffic
	Volume of local traffic
	Volume of other minutes
	Volume of total PSTN lines inc ISDN
	Volume of total PSTN lines exc ISDN
	Volume of total voice traffic
	Volume of total broadband
	Volume of access
	Revenue from leased lines
	Revenue from PSTN lines
	Revenue from international minutes
	Revenue from local and internet minutes
	Revenues from long-distance minutes
	Revenues from total domestic traffic
	Revenues from local and long-distance
	Revenues from total national traffic
	Revenues from internet traffic
	Revenues from local traffic
	Revenues from other minutes
	Revenues from total PSTN lines inc ISDN
	Revenues from total PSTN lines exc ISDN
	Revenues from total voice traffic
	Revenues from total broadband
	Revenues from access

Additionally, inflation data for the European economies was collected from the IMF World Economic Outlook Database 2007<sup>50</sup>.

<sup>50</sup> October 2007 Edition. Available at <http://www.imf.org/external/pubs/ft/weo/2007/02/index.htm>

## B.2 Indices

A number of indices were considered for the construction of input and output indices. As discussed in section 2.2.1.2, we believe that the Törnqvist index is most appropriate for our model.

Below we show the various indices considered for output. In all cases,

- $p$  is the price of outputs
- $q$  is the quantity of outputs

These were also used for inputs, substituting payment to inputs for prices.

### B.2.1 Törnqvist index

$$Q_t^T = \prod_{m=1}^M \frac{q_t^m}{q_0^m}^{0.5(w_0^m + w_t^m)} \quad (\text{Equation 30})$$

$$\text{where } w_t^m = \frac{p_t^m q_t^m}{\sum_{m=1}^M p_t^m q_t^m} \text{ is output } m\text{'s nominal output share}$$

The Törnqvist index weights each output by both the current and base year revenue share. This allows the differing importance of outputs over time to be captured.

### B.2.2 Paasche index

$$Q_t^P = \frac{\sum_{m=1}^M p_t^m q_t^m}{\sum_{m=1}^M p_t^m q_0^m} = \left[ \sum_{m=1}^M w_t^m \left( \frac{q_t^m}{q_0^m} \right)^{-1} \right]^{-1} \quad (\text{Equation 31})$$

The Paasche index weights each output by the current year revenue share only. This index thus has the disadvantage of not taking into account the base year mix of outputs. The Paasche index is considered to be a lower bound for the true index value.

### B.2.3 Laspeyres index

$$Q_t^L = \frac{\sum_{m=1}^M p_0^m q_t^m}{\sum_{m=1}^M p_0^m q_0^m} = \sum_{m=1}^M w_0^m \frac{q_t^m}{q_0^m} \quad (\text{Equation 32})$$

The Laspeyres index weights each output by the base year revenue share only. This index thus has the disadvantage of not taking into account the current periods mix of outputs. The Laspeyres index is considered to be an upper bound for the true index value.

## B.2.4 Fisher relative quantity index

$$Q_t^F = (Q_t^L Q_t^P)^{\frac{1}{2}} \quad (\text{Equation 33})$$

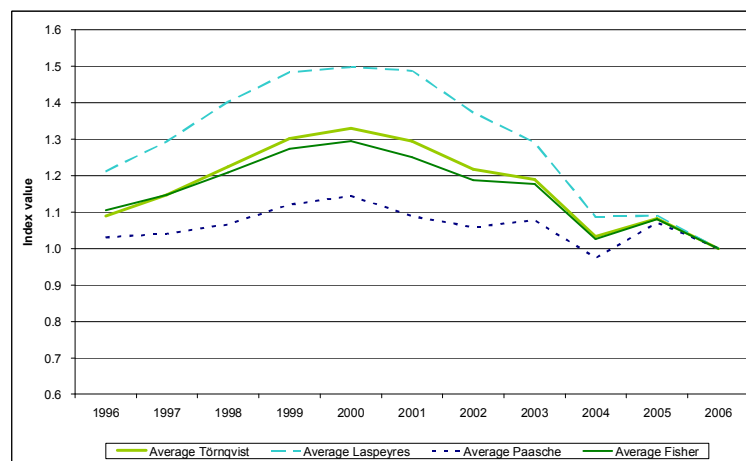
The Fisher index weights each output by both the current and base year revenue share. This allows the differing importance of outputs over time to be captured in a similar way to the Törnqvist.

The choice between the Fisher and Törnqvist index is of secondary significance given they produce very similar results. We have used the Törnqvist index throughout this report largely due to the prevalence of this index in productivity analysis<sup>51</sup>.

## B.2.5 Relationship of indices

Figure 12 below demonstrates how similar the Fisher and Törnqvist output indices are, taking an average across the LECs. Further it demonstrates that Laspeyres provides an upper bound whilst Paasche the lower.

**Figure 12: Varying output indexes**



<sup>51</sup> This point is highlighted by the UN Statistics division at <http://unstats.un.org/unsd/sna1993/tocLev8.asp?L1=16&L2=3>

## Appendix C Fixed effects growth model

In this appendix we provide the output from the econometrics package (Stata) for our preferred fixed effect growth model referred to in section 5.3.

```

Fixed-effects (within) regression              Number of obs   =       286
Group variable: company_code                 Number of groups =        26

R-sq:  within = 0.1115                      Obs per group:  min =        11
        between = 0.9025                      avg   =       11.0
        overall = 0.8768                      max   =        11

corr(u_i, Xb) = 0.5704                      F(4,256)        =        8.03
                                           Prob > F         =       0.0000

```

L_output61	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
L_materials	.0231216	.1064322	0.22	0.828	-.1864726	.2327157
L_staff	.2007008	.0916982	2.19	0.030	.0201219	.3812796
L_cl_NBV	.505918	.1010557	5.01	0.000	.3069117	.7049243
time	.0114339	.0050502	2.26	0.024	.0014887	.0213791
_cons	-12.6042	2.445171	-5.15	0.000	-17.41941	-7.788992
sigma_u	.41950149					
sigma_e	.19601528					
rho	.82079588	(fraction of variance due to u_i)				

```

F test that all u_i=0:      F(25, 256) =      31.04      Prob > F = 0.0000

```